JMP021: Cost Leadership
Curve Fitting

Produced by:

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Cost Leadership
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Background

Sit Well owns several retail furniture stores in a moderate-sized city. After years of disappointing performance, Sit Well hired a new top management team to oversee the firm's business practices. As its first strategic initiative, the management team decided to engage in cost leadership—a strategy of providing affordable high quality products at a lower cost of operation than competitors. To accomplish this goal, a strong advertising campaign was launched, several smaller furniture stores were gradually acquired to increase capacity, and deeply discounted sale prices were promoted.

Sit Well's accounting department calculated monthly market share, defined as the percentage of revenue captured by Sit Well relative to estimates of all competing furniture stores in the local market.

The Task

Assess the effectiveness of this business strategy.

- Was the strategy successful in increasing market share?
- Is there room for further growth in market share under the current business strategy?
- Can you forecast market shares for the next three months?

The Data   CostLeadership.jmp

The data set contains six years of monthly data, starting with the initiation of the new strategic plan. The variables are:

|	
Month	A monthly counter
Market Share	Sit Well's market share (as defined above) that month

Analysis

The trend analysis shown in Exhibit 1 indicates that market shares have been growing at a rate of 0.49% per month over the last six years, a rate that is statistically significant (p-value < 0.0001). The linear trend analysis accounts for 78% of the observed variability in market shares over this time period (RSquare = 0.78).

By all of the usual statistical measures, the linear trend is a pretty good model of market share gains, until you look at the line relative to the data. Exhibit 1 shows that the linear model:

- *over*predicts market share in the early part of the time-series,
- *under*predicts market share in the middle of the series, and
- *over*predicts market share for the last year or so.

Exhibit 1 Regression with Market Share (Y) and Month (X)
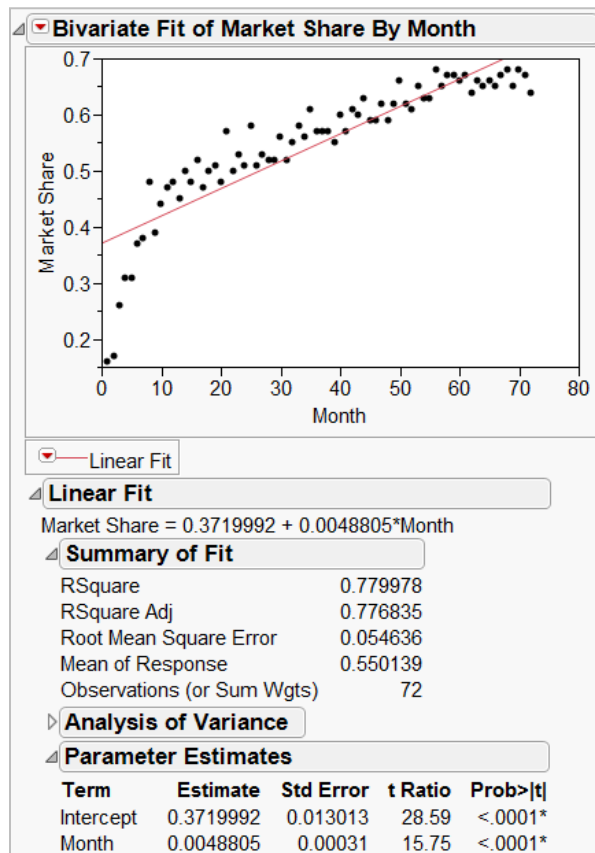
(Analyze > Fit Y by X. Use Market Share as Y, Response and Month as X, Factor. Under the red triangle select Fit Line.)

Bivariate Fit of Market Share By Month

Linear Fit

Linear Fit

Market Share = 0.3719992 + 0.0048805*Month

Summary of Fit

RSquare	0.779978
RSquare Adj	0.776835
Root Mean Square Error	0.054636
Mean of Response	0.550139
Observations (or Sum Wgts)	72

Analysis of Variance

Parameter Estimates

| Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.3719992	0.013013	28.59	<.0001*
Month	0.0048805	0.00031	15.75	<.0001*

The linear model would be expected to generate inaccurate forecasts of future market share, since the *line* will grow at a constant rate of 0.49% (0.0049) every month, whereas the *data* suggest that the growth rate was *faster* than 0.49% initially, then *slower* than 0.49% in the later part of the time-series. Extrapolation along the line will generate forecasts that are higher than we should realistically expect.

Thus, although the linear model has a statistically significant predictor and a reasonably high R-squared, it's really of limited value because it fails to account for the curved pattern shown in the scatterplot.

The flexible spline-fitting in JMP facilitates examination of curved relationships. On the left in Exhibit 2 is a spline with the smallest possible smoothing parameter, lambda = 0.000001. Although this choice of lambda results in an R-squared of 100%, this fit isn't useful for predicting future values since no underlying pattern has been found (we've simply fit every point).

On the right in Exhibit 2 we see a spline with lambda = 100, which produces a smoother fit. Although goodness of fit has been somewhat compromised (the R-squared drops to 0.964), this spline fit captures a curved pattern known as *decreasing returns to scale*; a situation in which market share increases but at a decreasing rate over time).

**Exhibit 2** Exploring the Trend Using Spline-Fitting
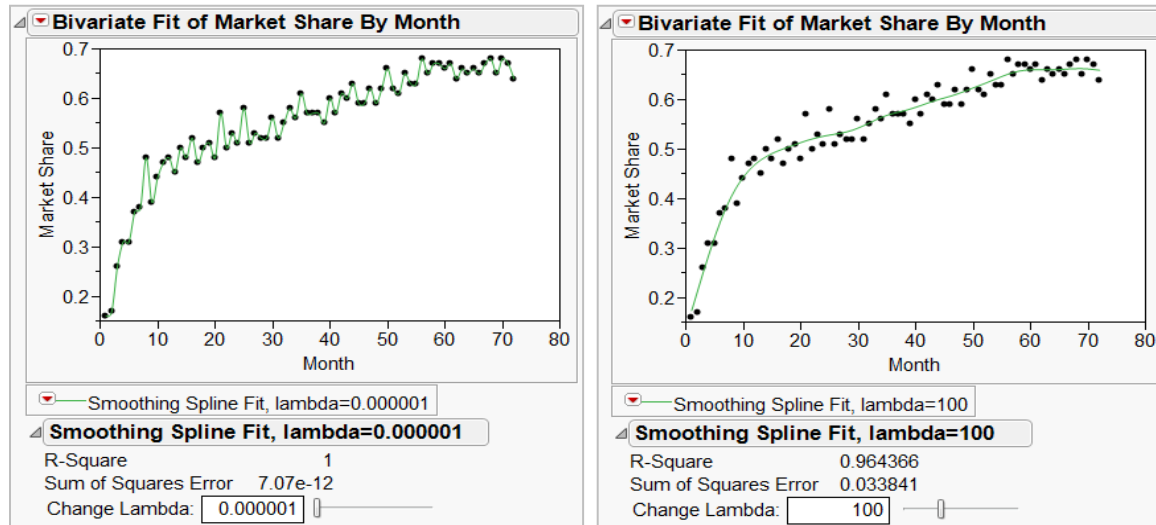
(From the Bivariate Fit output window, click on the red triangle next to Linear Fit and select Remove Fit. Then, select Flexible > Fit Spline under the top red triangle. Enter values in the field, or use the slider to dynamically change lambda. Note that a spline fit is also available from Graph > Graph Builder.)

Decreasing returns to scale can be modeled by using the logarithm of the variable on the horizontal axis. In Exhibit 3, we re-express, the time variable, **Month**, as a logarithm.

With an R-squared of 0.952, more of the variation in market share is captured by the logarithmic curve than the straight line model in Exhibit 1. Further, the RMSE has dropped from 5.46% (0.0546) in the linear model to 2.55% (0.0255) in the logarithmic model. Even a simple visual examination tells us that the curve more closely fits the scatter of points than does the line.

This model, capturing the curved trend over time, will be used for forecasting future market shares.

Exhibit 3 Regression with **Market Share** and **Log(Month)**

◢▼Bivariate Fit of Market Share By Month
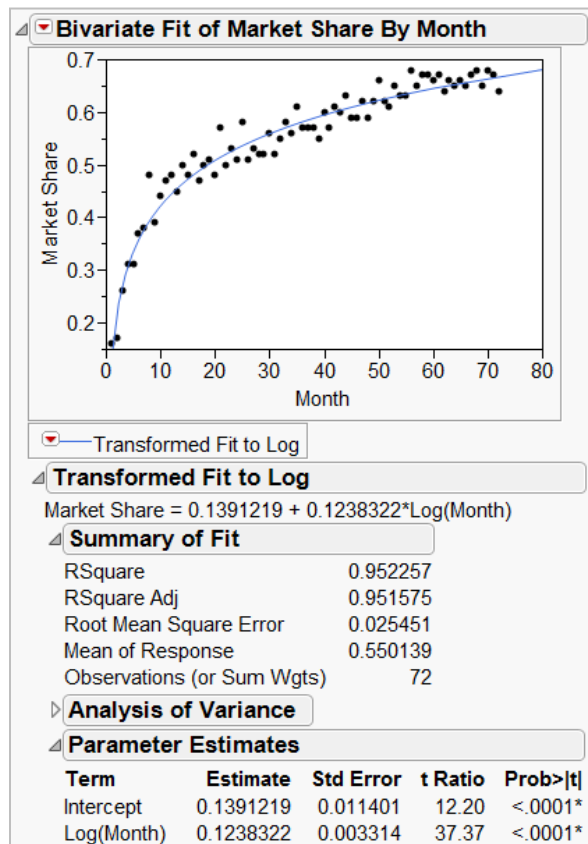
(From the Bivariate Fit output window, click on the red triangle next to Spline and select Remove Fit. Select Fit Special under the top red triangle. Under X Transformation, select Natural Logarithm.)

▼——Transformed Fit to Log

◢ Transformed Fit to Log

Market Share = 0.1391219 + 0.1238322*Log(Month)

◢ Summary of Fit

RSquare	0.952257
RSquare Adj	0.951575
Root Mean Square Error	0.025451
Mean of Response	0.550139
Observations (or Sum Wgts)	72

▷ **Analysis of Variance**

◢ Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.1391219	0.011401	12.20	<.0001*
Log(Month)	0.1238322	0.003314	37.37	<.0001*

The model forecasts that, over the next three months, the market share will be at around 67% (Exhibit 4).

Exhibit 4 Predicted Market Share

Month	Market Share	Predicted Market Share
68	0.68	0.6616328726
69	0.65	0.6634406741
70	0.68	0.6652224631
71	0.67	0.6669789777
72	0.64	0.6687109249
73	•	0.6704189824
74	•	0.6721038002
75	•	0.6737660024

(From the Bivariate Fit output window, click on the red triangle next to Transformed Fit to Log and select Save Predicteds to save the prediction formula. Enter new months in the data table to calculate predicted market shares for those months.)

The prediction intervals (Exhibit 5) provide an estimate of the interval in which the **Market Share** over the next three months may fall.

Exhibit 5 Prediction Intervals for Market Share
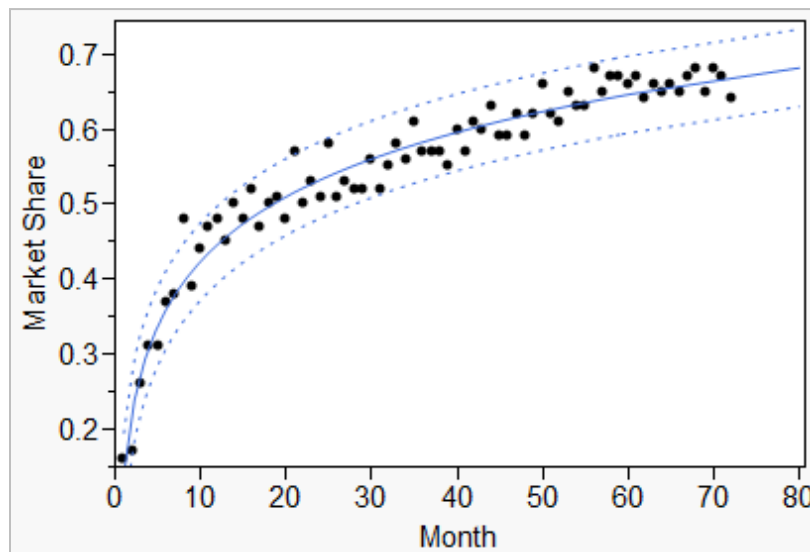
(From the Bivariate Fit output window, click on the red triangle next to Transformed Fit to Log and select Confid Curves Indiv. Use the cross-hair tool to display estimated values for the prediction intervals.)

Summary

Statistical Insights

This case provides an introduction to curve fitting. In this data set, the need for a curve was evident from the scatterplot. In other cases, where the data are more scattered, a spline fit can be used to uncover the need for a model that incorporates curvature.

For the market share data, a logarithmic transformation worked well to capture curvature. Other transformations and models can also be applied to curved relationships. For example, if the pattern exhibits an increase to a certain point, and then a decrease, a quadratic model might have worked better (under Fit Polynomial).

Note that, in assessing the fit of a model, the residuals should be investigated. For any fitted model, select Plot Residuals under the lower red triangle to generate a series of residual plots. Then, check the plots for unusual patterns or observations that might indicate that the fitted model is not appropriate. We'll discuss this formally in a future case.

Managerial Implications

We've been asked three specific questions:

- Was the strategy successful in increasing market share? The data show an increase in market share from 14% at the beginning of the time-series to almost 67% six years later. Without further information, we don't know whether the campaign, defined earlier as "providing quality products at a lower cost of operation than competitors", was responsible for the increased market share. Although market share increased notably early in the campaign, expect *future* increases to be small.
- Is there room for further growth in market shares under the current business strategy? The data suggest that a plateau has been reached. Grabbing a larger percentage of the market isn't likely. Brand loyalty to competitors, for example, may make it impossible to capture 100% of the market. Further gains from here on are expected to be slight.

- The forecasts of market shares for the next three months, shown in Exhibits 4 and 5, are roughly 67%. The prediction intervals are roughly 62% to 72%.

JMP Features and Hints

In this case, we used the flexible smoothing spline in Fit Y by X to explore a curved relationship, and Fit Special to fit a model with a log transformation.

Note that as of JMP 11, variables can be dynamically transformed in any dialog window. To do this, right-click on the variable in the column selection panel, select Transform and then select the transformation of interest. This creates a temporary variable, which will be shown in italics. To save the transformed data to the data table (and create the formula for the transformation), right-click on the transformed variable and select Add to Data Table.

Exercises

Suppose that you've invested $400 in a savings bond that earns interest of 3.5% per year.

1. Calculate the value of the bond at the end of years 1 through 40. Hint: At the end of the first year, the bond is worth $400 x 1.035. At the end of the second year, the bond is worth $400 x 1.035 x 1.03.
2. In JMP, plot the value of the bond over time. Use Fit Line, Fit Polynomial and Fit Special to find the model that best fits the pattern in the data.
3. Use the best model to predict the value of the bond at the end of year 45.

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