JMP® ACADEMIC CASE STUDY

JMP034: Durability of Mobile Phone Screen - Part 3

Statistical Modeling: Single Variable Logistic Regression

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STATISTICAL DISCOVERY FROM SAS

Durability of Mobile Phone Screen - Part 3

Statistical Modeling: Single Variable Logistic Regression

Key ideas:

This case study requires building logistic regression models to evaluate the durability of different mobile phone screen types in a drop test across various heights.

This case study is an extension of previous case studies (JMP032 - Durability of Mobile Phone Screen - Part 1 and JMP033 - Durability of Mobile Phone Screen - Part 2. While it isn't necessary, it is recommended to complete those case studies first, since together they present a more comprehensive description of each stage of the study and subsequent analyses.

Background

The durability of a product is clearly an important quality characteristic for both the end user and the manufacturer. For end users, durability is especially important for mobile phones. Dropping a phone on a hard surface, for example, can result in the screen cracking or even breaking, rendering the phone unusable. To evaluate the durability of these screens, manufacturers subject a sample of screens to a variety of tests to simulate typical wear and tear by a user, such as dropping the phone onto a concrete surface.

In case JMP032 - Durability of Mobile Phone Screen - Part 1 material scientists for a screen manufacturer experimented with two new formulations of an aluminosilicate glass (A and B). These two formulations were produced by making a change to a final processing step that uses a specific level of potassium nitrate to strengthen the glass.

A sample of 10 screens of each type was developed for testing. Each screen was installed into the same style of phone. The phones were then dropped in a controlled identical manner from a height of 1 meter onto a concrete surface. A binary variable "Success" (no damage) and "Fail" (screen damage) was recorded.

One of company's goals is that 97% of the screens manufactured would be able to experience a drop of 1 meter without becoming damaged (i.e., the Population Success Rate).

The analyses illustrated in case JMP032 - Durability of Mobile Phone Screen - Part 1, which were based on 10 phones tested for each Screen Type, failed to generate the statistical evidence needed to demonstrate the desired Success Rate. The analyses also failed to find any statistically significant difference between the two Screen Types.

The engineers decided to test an additional 40 phones for each of the two Screen Types. Analyzing the results from this expanded test was the objective of the exercises for that case study.

In case JMP033 - Durability of Mobile Phone Screen - Part 2, the material scientists developed a third Screen Type (C), which required a more expensive process. Tests were done for this Screen Type at 1.0m, as had been done for Screen Types A and B. In an effort to gain a comprehensive understanding of the durability of these screens, the engineers conducted the drop test at two additional heights (0.5 and 1.5 meters), which resulted in the following data:

	0.5m	1.0m	1.5m
Type A	n=40 S=36 F=4	n=50 S=41 F=9	n=50 S=28 F=22
Type B	n=40 S=40 F=0	n=50 S=48 F=2	n=50 S=40 F=10
Type C	n=40 S=39 F=1	n=50 S=47 F=3	n=50 S=39 F=11

These data were analyzed in case JMP033 - Durability of Mobile Phone Screen - Part 2.

To continue developing a deeper understanding of the performance of the Screen Types, the engineers performed additional tests at Drop Heights of 0.25m, 1.25m, 2.0m, 2.5m and 3.0m. In addition, a sample of screens from a leading competitor were obtained and included in the testing, resulting in the following data:

	0.25m	0.5m	1.0m	1.25m	1.5m	2.0m	2.5m	3.0m
Type A	n=25 S=25 F=0	n=40 S=36 F=4	n=50 S=41 F=9	n=50 S=35 F=15	n=50 S=28 F=22	n=40 S=4 F=36	n=40 S=0 F=40	n=25 S=0 F=25
Type B	n=25 S=25 F=0	n=40 S=40 F=0	n=50 S=48 F=2	n=50 S=45 F=5	n=50 S=40 F=10	n=40 S=18 F=22	n=40 S=4 F=36	n=25 S=0 F=25
Type C	n=25 S=25 F=0	n=40 S=39 F=1	n=50 S=47 F=3	n=50 S=45 F=5	n=50 S=39 F=11	n=40 S=16 F=14	n=40 S=3 F=38	n=25 S=0 F=25
Competitor	n=10 S=10 F=0	n=20 S=19 F=1	n=20 S=18 F=2	n=20 S=16 F=4	n=20 S=13 F=7	n=20 S=5 F=15	n=20 S=**1** F=19	n=10 S=0 F=10

The Task

The primary objectives of this analysis are to:

1. Describe how the performance of each Screen Type changes as the Drop Height changes.
2. Describe differences in durability between the three Screen Types.
3. Determine how the company's three Screen Types compare to the screens of its competitor.
4. Develop a tool that will estimate the Success Rate at any Drop Height, including those not part of the testing.
5. Determine the Drop Height at which it is estimated that the Success Rate would be at least 97% for each Screen Type.

The Data Drop Test 3.jmp

The data is stored in what's referred to as Outcome/Frequency Table format.

Screen	Four screen types (A, B, C, and Comp)
Height	Height from which screens were dropped (0.25m, 0.5m, 1.0m, 1.25m,1.5m, 2.0m, 2.5m, and 3.0m)
Outcome	Two outcomes (Success, Fail)
Tested	Number of screens tested
Count	Number of specimens that resulted in either Success or Fail
Rate	The percentage of phones that succeeded (didn't experience damage) or failed (did experience damage) in the test

JMP Tips

To perform an analysis on a subset of the data in the table, which will be used throughout these analyses, you'll need to filter the data. It can be done either through the Global Data Filter at the data table level before running an analysis or with the Local Data Filter within the output window from an analysis.

For Global Data Filter, select Rows>Data Filter. To choose a variable to filter, select the variable and click +. Choose Add to filter by additional variables. Select the values of the variables you wish to include in your analysis and then select the Show and Include boxes. Perform the analysis.

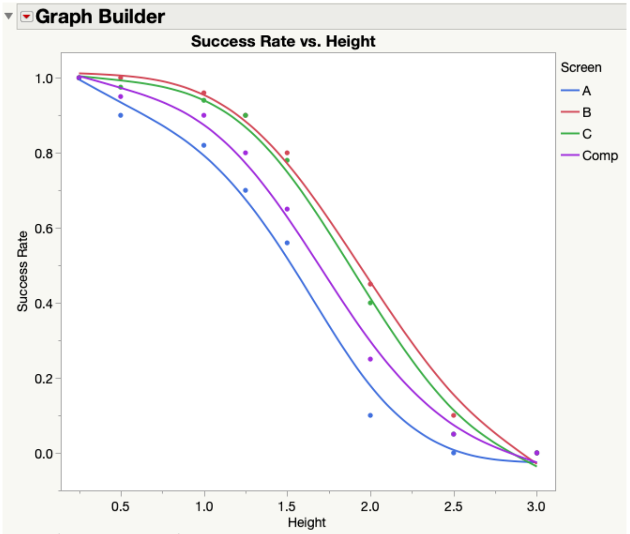
For Local Data Filter, first run the analysis. Then select Local Data Filter under the red triangle at the top of the output. To choose a variable to filter, select the variable and click +. Choose Add to filter by additional variables. Select the values of the variables by which you wish to subset the data. The numerical and graphical results will change to correspond to the data being included in the analysis. The tools and analysis options available may change based on attributes of the data being included.

Analysis

Graphical

We begin by summarizing the data graphically. Exhibit 1 displays a scatter plot showing points for each of the Success Rates for the four Screen Types and eight Drop Heights. In addition, fitted curves are added to visualize how the Success Rate changes across the Drop Heights.

Exhibit 1 Scatter Plot with Smoother



To create this graph from the data table, the data will need to be filtered so that it only uses the rows corresponding to the success outcome. To do so, use the Global Data Filter. Choose Rows>Data Filter. Filter the data by the variable outcome. See JMP Tips on page 4 for instructions.

To create, Graph>Graph Builder. Select Success Rate as the Y Variable, Height as the Freq Variable, and Screen as the Overlay Variable.

This graph provides a nice visualization of the change in Success Rate across the full range of Drop Heights, as well as potential differences between the Screen Types. For example, it appears that the Success Rates for Screen Types B and C are very similar and have the best performance. Screen Type A appears to perform the worst; the Success Rate for the competitor's screen is somewhere in between.

Single variable logistic regression model

Though this graphical display provides a valuable visual description of the data, building statistical models will allow us to describe the data more precisely, providing us with statistical tools to explain, compare, and predict.

Logistic regression is a valuable statistical modeling framework that is well suited for these data.

The structure of the logistic regression is:

$$\log\left(\frac{\mathrm{p}}{1-\mathrm{p}}\right) = \beta_0 + \beta_1 X$$

where p is the probability of the event of interest happening. X is the explanatory variable being studied. β_0, β_1 are the parameters in the model to be estimated from the data, and ε is the random variation in the data that the model does not explain.

For our study, we can define the event of interest as the screen not being damaged in the drop test (Success) and the explanatory variable is the Drop Height.

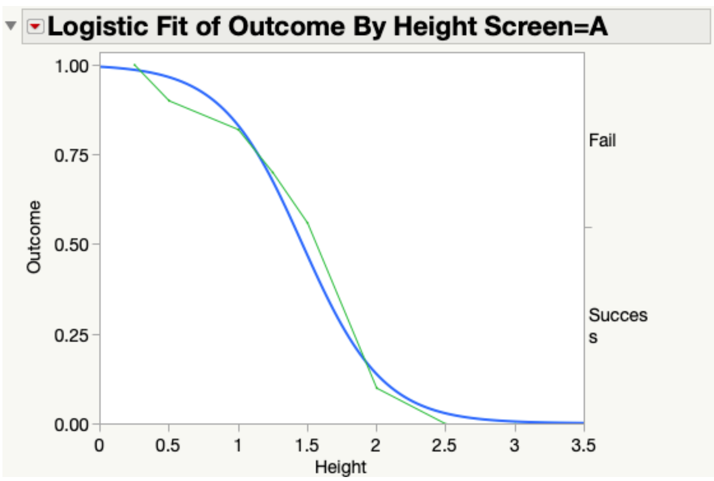
Another, sometimes more convenient, way to express a logistic regression model is in terms of the probability of the event happening.

$$\text{Prob(Event)} = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X)}}$$

We'll begin our model-building process by fitting a logistic regression model to the data from Screen Type A only. You'll be tasked in the exercises to build similar logistic regression models for Screen Types B, C, and the Competitor.

Exhibits 2 through 5 show the output of a building a logistic regression model for Screen Type A.

Exhibit 2 Logistic Regression Plot



For this analysis, the rows corresponding to both Success and Failure outcome need to be used. The data can either be filtered so that it only uses the rows corresponding to Screen A, or the variable Screen Type can be used as a By Variable when running the analysis. Filtering the data can be done via the Global Data Filter prior to running the analysis or afterward with the Local Data Filter. See JMP Tips on page 4 for instructions.

To create, Analyze>Fit Y by X. Select Outcome as the Y Variable, Height as the X Variable, and Count as the Freq Variable. If choosing to use Screen as the By Variable instead of filtering the data, then select Screen as the By Variable. This approach will fit a separate logistic regression model for each Screen.

For these data, it's best to remove the data points from the Logistic Fit Graph and use the Rate Curve instead. Choose Plot Options under the red triangle at the top of the output. Remove the data points by unchecking Show Points. Display the Success Rate values by selecting Show Rate Curve.

This graph is very similar to the one in Exhibit 1. The model fit here is a logistic regression model while the fitted model shown in Exhibit 1 uses a model form known as the cubic spline. They are similar for these data but not identical.

A logistic regression analysis contains other numerical output that can be useful to examine. Exhibit 3 displays some of that output.

Exhibit 3 Logistic Regression

Whole Model Test

Model	-LogLikelihood	DF	ChiSquare	Prob>ChiSq
Difference	102.41021	1	204.8204	<.0001*
Full	118.89037			
Reduced	221.30058			

Parameter Estimates

Term	Estimate	Std Error	ChiSquare	Prob>ChiSq	Unit Odds Ratio	Odds Ratio
Intercept	5.01662299	0.5679002	78.03	<.0001*	.	.
Height	-3.4211238	0.3930324	75.77	<.0001*	0.03267569	8.20575e-5

For log odds of Success/Fail

The Whole Model Test is testing the following hypothesis in the model $\text{Prob(Event)} = \frac{1}{1+e^{-(\beta_0+\beta_1 X)}}$

$$H_0: \beta_1 = 0$$

$$H_A: \beta_1 \neq 0$$

If β_1 is believed to be 0, then there is no significant impact on the probability of success as the Drop Height changes. The p-value for this test is extremely small (< 0.0001), which is very strong statistical evidence that β_1 is not equal to 0. The estimate of β_1 shown in the Parameter Estimate Table is -3.42. A 95% confidence interval for β_1 can be obtained via the Fit Model platform. That interval, not shown in the exhibit, is [-4.19 , -2.65], an interval not containing 0.

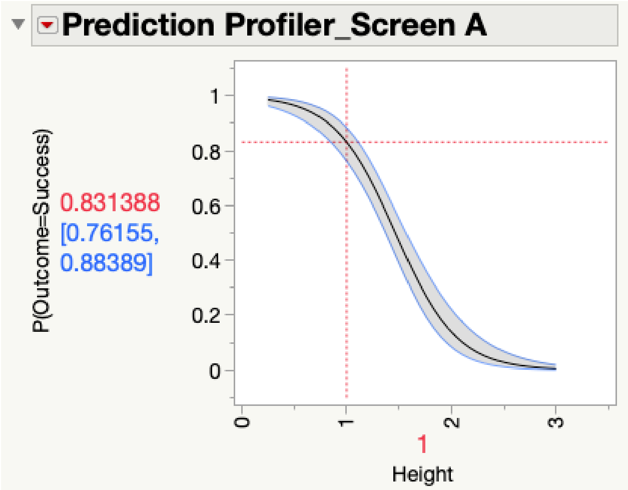
Thus we would conclude that Drop Height has a statistically significant impact on Success Rate, which is quite obviously the result we were expecting as we clearly saw that significant effect in the graphs. Sometimes, however, the impact of a factor on the outcome isn't obvious and a statistical test is needed to determine if that effect is indeed statistically significant.

Using fitted model to make predictions

In addition to describing relationships between variables, statistical models are also useful for making predictions. Using a more general modeling platform in JMP that provides these prediction capabilities, including the addition of confidence intervals, we will illustrate how to make predictions.

Exhibit 4, which uses the Profiler, JMP's interactive tool that displays the fitted model, shows the predicted outcome and 95% confidence intervals for any Drop Height between 0.25 and 3.0 meters.

Exhibit 4 Profiler



For this analysis, the rows corresponding to both Success and Failure outcome needs to be used. The data can either be filtered so that it only uses the rows corresponding to Screen A, or the variable Screen Type can be used as a By Variable when running the analysis. Filtering the data can be done via the Global Data Filter prior to running the analysis or afterward with the Local Data Filter. See JMP Tips on page 4 for instructions.

To create, Analyze>Fit Model. Select Outcome as the Y Variable, Height as the X Variable, Count as the Freq Variable, and Screen as the By Variable. Choose Generalized Linear Model under Personality, Binomial as the Distribution, and Logit as the Link Function. If choosing to use Screen as the By Variable instead of filtering the data, then select Screen as the By Variable. This approach will fit a separate logistic regression model for each Screen.

After running the analysis, choose Profiler under the red triangle at the upper left of the output.

Here we see that the predicted Success Rate at a Height of 1.0m is 83.1%. A 95% confidence interval quantifying the uncertainty in that prediction is [76.2%, 88.4%]. That is, we are 95% confident that the Population Success Rate for Screen Type A at 1.0m ranges from 76.2% to 88.4%.

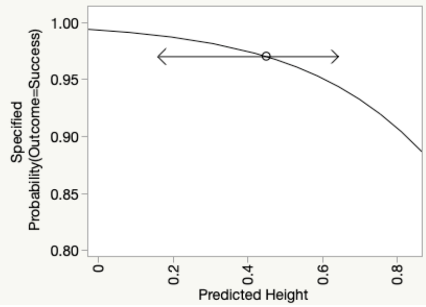
In case JMP032 - Durability of Mobile Phone Screen - Part 1 and case JMP033 - Durability of Mobile Phone Screen - Part 2, we estimated the Success Rate at this Drop Height for the different Screen Types. In the exercises, you'll be asked to compare the precision of these estimates via those analyses compared to the ones produced by the logistic regression model.

Inverse prediction is another useful type of prediction to estimate the Drop Height that would achieve a given Success Rate. This type of prediction is a different way for us to evaluate the durability of the screens. We choose a Success Rate of 0.97, and the model is used to estimate the Drop Height that achieves that Success Rate. Exhibit 5 shows the results of making this inverse prediction.

Exhibit 5 Inverse Prediction

Inverse Prediction

Specified Probability(Outcome=Success)	Predicted Height	Lower 95%	Upper 95%
0.9700000	0.4502977	0.1591076	0.6416494



Choose Inverse Prediction under the red triangle at the upper left of the output. Enter 0.97 for the Probability(Outcome=Success) field.

Note: The above graph was zoomed in to the area of interest.

The predicted Drop Height to achieve 97% Success Rate is 0.45m with a 95% confidence interval of [0.16, 0.64]. Thus, we are 95% confident that a 97% Success Rate occurs between a Drop Height of 0.16m and 0.64m.

The exercises will ask you to build separate logistic regression models for the other three Screen Types, interpret the results and make predictions.

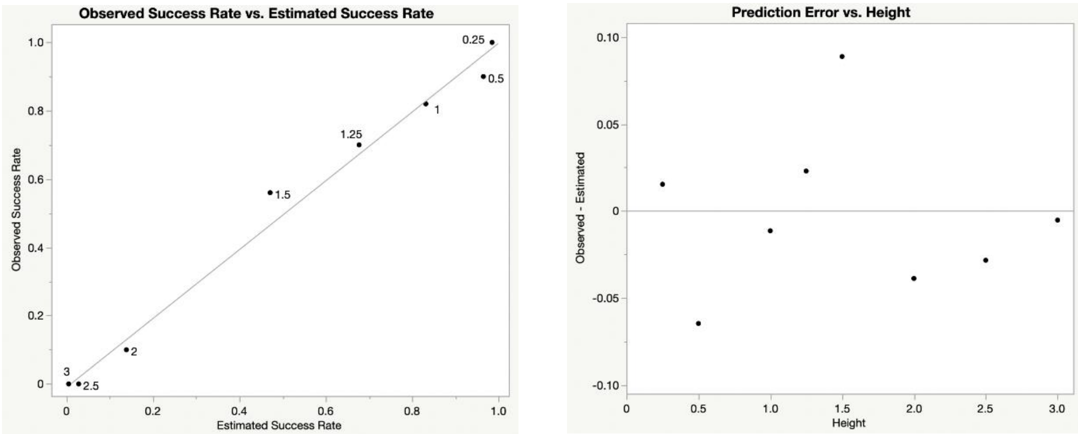
Evaluating model performance

Evaluating model performance can be done by comparing the predicted probability of success from the logistic regression model compared to the actual success rate observed in the data.

Exhibit 6 demonstrates two common ways to visualize prediction error in a logistic regression model. These graphs show how large or small the prediction error is. Points close to the reference lines represent when the Estimated Success Rate from the logistic regression model is close to the Observed Success Rate from the actual data. Points above the line represent when the Observed Success Rate is larger than the Estimated Success Rate; points below the line occur when the estimated is larger than the observed. For example, the prediction error is the greatest for the 1.5m Drop Height. The Observed Success Rate in the data is 100*(28/50) = 56%. The Estimated Success Rate from the model is 47.1%.

Overall the Estimated Success Rates are close to the Observed Success Rates, indicating that the model describes the data quite well.

Exhibit 6 Visualizing Prediction Error



Save the Estimated Success Rates by selecting Save Columns>Predicted Values under the red triangle in the upper left of the output. Create a new column for the difference between the Observed Success Rate and the Estimated Success Rate by selecting Cols>New Column, then selecting Column Properties>Formula. Create the formula Observed – Estimated via the formula: 'Rate' - 'Pred Outcome'.

For the graph on left, the data in the data table will need to be filtered so that it only uses the rows corresponding to the Success Outcome. The best way to do this is via the Global Data Filter. Choose Rows>Data Filter. Filter the data by the variable Outcome. See JMP Tips on page 4 for instructions.

To create the graph on the left, Graph>Graph Builder to plot the Observed Success Rate vs. Estimated Success Rate. A line was added manually via Tools>Line showing when Observed = Predicted. To add the values as data labels, right-click on the Height Column and select Label/Unlabel. Highlight all the data points in the graph. Right-click and choose Rows>Row Label.

To create the graph on the right, Graph>Graph Builder to plot the Observed – Estimated values on the Y axis vs. Drop Height on the X.

Summary

Statistical insights

The study included eight different Drop Heights ranging from 0.25 meters to 3.0 meters. This data allows us to build logistic regression models using an equation to determine that the probability of a screen's success in the drop test is a function of Drop Height.

The logistic regression model fit to the Screen Type A tests provided us with a tool to estimate the probability of a screen being damaged or not across any Drop Height between 0.25 and 3.0 meters, including those not part of the study. We demonstrated that using the logistic regression model, it is estimated (at 95% confidence) that the Population Success Rate for Screen Type A at 1.0m is between 76.2% and 88.4%.

The logistic regression model also provided us with a tool to perform an inverse prediction, specifically, estimating the Drop Height that would result in a given Success Rate. We demonstrated that we estimate (at 95% confidence level) that the Drop Height that would achieve a 97% Success Rate for Screen Type A occurs between 0.16m and 0.64m.

We also demonstrated how the model's Estimated Success Rates can be compared to the Observed Success Rates as a means to evaluate the predictive capabilities of a model.

Implications and next steps

The exercises will ask you to build logistic regression models for the other Screen Types (B, C, and Competitor). You'll use those models to describe the performance of each Screen Type, make predictions, and compare the results.

Exercises

Use the data in file Drop Test 3.jmp data set to answer the following questions:

1. Create separate logistic regressions models for each Screen Type describing the relationship between Success Rate and Drop Height. Show graphs that display the models along with the Observed Success Rates for all four Screen Types.

2. For each model, provide a point estimate and a 95% confidence interval estimate of the Success Rate at a 1.0m Drop Height. Create a table and a visualization that display the point estimate, lower and upper bound of the confidence intervals. Compare the performance of the Screen Types at this Drop Height. How does the performance of the Competitor Screen compare to the three our engineers developed? Compare the precision in the estimates of the Success Rate for each Screen Type. Which ones have the most precision?

3. If case JMP032 - Durability of Mobile Phone Screen - Part 1 or case JMP033 - Durability of Mobile Phone Screen - Part 2 were completed, compare the precision of the estimates of the Success Rates at the 1.0m Drop Height via the confidence intervals from those analyses to the ones done in this case study using the logistic regression model. Which estimates are more precise? What are the possible reasons?

4. For each individual logistic regression model, generate a point estimate and a 95% confidence interval estimate of the Drop Height that would result in a 97% Success Rate. Create a table and a visualization that display the point estimate, confidence intervals, and the width of the CI as a means to present all the results together. Which of the models have the smallest and largest amount of error in these estimates?

5. Create graphs that display the prediction error for each of the individual logistic regression models. Comment on how close the Estimated Success Rates are to the Observed Success Rates.