

Version 17

Fitting Linear Models

"The real voyage of discovery consists not in seeking new landscapes, but in having new eyes."

Marcel Proust

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17.1

The correct bibliographic citation for this manual is as follows: JMP Statistical Discovery LLC 2022–2023. *JMP® 17 Fitting Linear Models*. Cary, NC: JMP Statistical Discovery LLC

JMP® 17 Fitting Linear Models

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JMP Statistical Discovery LLC, SAS Campus Drive, Cary, North Carolina 27513-2414.

October 2022

March 2023

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Whether you are a first-time or a long-time user, there is always something to learn about JMP.

Visit JMP.com to find the following:

- live and recorded webcasts about how to get started with JMP
- video demos and webcasts of new features and advanced techniques
- details on registering for JMP training
- schedules for seminars being held in your area
- success stories showing how others use JMP
- the JMP user community, resources for users including examples of add-ins and scripts, a forum, blogs, conference information, and so on

jmp.com/getstarted

Contents

Fitting Linear Models

1 Learn about JMP 15

Documentation and Additional Resources

Formatting Conventions in JMP Documentation 17
JMP Help 18
JMP Documentation Library 18
Additional Resources for Learning JMP 25
Search JMP 25
JMP Tutorials 25
Sample Data Tables 26
Learn about Statistical and JSL Terms 26
Learn JMP Tips and Tricks 26
JMP Tooltips 26
JMP User Community 27
Free Online Statistical Thinking Course 27
JMP New User Welcome Kit 27
Statistics Knowledge Portal 27
JMP Training 28
JMP Books by Users 28
The JMP Starter Window 28
JMP Technical Support 28

2 Model Specification 29

Specify Linear Models

Overview of the Fit Model Platform 31
Example of a Regression Analysis Using Fit Model 32
Launch the Fit Model Platform 35
Elements in the Fit Model Launch Window 36
Construct Model Effects 39
Fitting Personalities 46
Model Specification Options 48
Informative Missing 50
Validity Checks 51
Model Specification Templates 52

Simple Linear Regression . . . 52
Polynomial in X to Degree k . . . 53
Polynomial in X and Z to Degree k . . . 53
Multiple Linear Regression . . . 53
One-Way Analysis of Variance . . . 53
Two-Way Analysis of Variance . . . 54
Two-Way Analysis of Variance with Interaction . . . 54
Three-Way Full Factorial . . . 54
Analysis of Covariance, Equal Slopes . . . 54
Analysis of Covariance, Unequal Slopes . . . 55
Two-Factor Nested Random Effects Model . . . 55
Three-Factor Fully Nested Random Effects Model . . . 56
Simple Split Plot or Repeated Measures Model . . . 56
Two-Factor Response Surface Model . . . 57
Knotted Spline Effect . . . 57

3 Standard Least Squares Models . . . 59
Analyze Common Classes of Models

Example Using Standard Least Squares . . . 62
Launch the Standard Least Squares Personality . . . 65
Fit Model Launch Window . . . 66
Standard Least Squares Options in the Fit Model Launch Window . . . 67
Validation in Standard Least Squares . . . 69
Missing Values . . . 69
Fit Least Squares Report . . . 70
Single versus Multiple Responses . . . 70
Report Structure Related to Emphasis . . . 71
Special Reports . . . 71
Least Squares Fit Options . . . 75
Fit Group Options . . . 75
Response Options . . . 76
Regression Reports . . . 77
Summary of Fit . . . 78
Analysis of Variance . . . 79
Parameter Estimates . . . 80
Effect Tests . . . 82
Effect Details . . . 83
Lack of Fit . . . 98
Estimates . . . 100
Show Prediction Expression . . . 102

Sorted Estimates . . . 103
Expanded Estimates . . . 107
Indicator Parameterization Estimates . . . 109
Sequential Tests . . . 110
Custom Test . . . 111
Compare Slopes . . . 114
Joint Factor Tests . . . 114
Inverse Prediction . . . 115
Cox Mixtures . . . 120
Parameter Power . . . 121
Correlation of Estimates . . . 123
Effect Screening . . . 124
Scaled Estimates and the Coding of Continuous Terms . . . 125
Effect Screening Plot Options . . . 126
Normal Plot Report . . . 131
Bayes Plot Report . . . 133
Pareto Plot Report . . . 134
Factor Profiling . . . 135
Profiler . . . 136
Interaction Plots . . . 137
Contour Profiler . . . 138
Mixture Profiler . . . 139
Cube Plots . . . 140
Box Cox Y Transformation . . . 141
Surface Profiler . . . 143
Row Diagnostics . . . 144
Effect Leverage Plots . . . 146
Press . . . 150
Save Columns . . . 151
Prediction Formula . . . 154
Multiple Comparisons . . . 154
Launch the Multiple Comparisons Option . . . 155
Comparisons with Overall Average . . . 159
Comparisons with Control . . . 162
All Pairwise Comparisons . . . 164
Equivalence Tests . . . 168
Effect Summary Report . . . 170
Mixed and Random Effect Model Reports and Options . . . 174
Mixed Models and Random Effect Models . . . 175
Restricted Maximum Likelihood (REML) Method . . . 179

EMS (Traditional) Model Fit Reports ... 184
Models with Linear Dependencies among Model Terms ... 186
Singularity Details ... 186
Parameter Estimates Report ... 187
Effect Tests Report ... 188
Examples of Models with Linear Dependencies ... 188
Statistical Details for the Standard Least Squares Personality ... 191
Statistical Details for Emphasis Rules ... 191
Statistical Details for the Custom Test Example ... 191
Statistical Details for Correlation of Estimates ... 192
Statistical Details for Nominal Effects Coding ... 193
Statistical Details for Leverage Plots ... 194
Statistical Details for the Kackar-Harville Correction ... 197
Statistical Details for Power Analysis ... 198

4 Standard Least Squares Examples ... 211
Analyze Common Classes of Models

Example of Simple Linear Regression ... 213
Example of a Polynomial Effects Model ... 213
Example of One-Way Analysis of Variance ... 213
Example of Two-Way Analysis of Variance ... 216
Example of Two-Way Analysis of Variance with an Interaction ... 217
Example of a Three-Way Full Factorial Model ... 217
Example of Analysis of Covariance with Equal Slopes ... 218
Example of Analysis of Covariance with Unequal Slopes ... 221
Example of a Response Surface Model ... 223
Example of a Two-Factor Nested Random Effects Model ... 231
Example of a Split Plot Design Analysis ... 232
Example of a Simple Repeated Measures Model ... 235
Example of Estimating Random Effect Parameters ... 236
Example of a Knotted Spline Effect ... 238
Example of a Bayes Plot for Active Factors ... 240

5 Stepwise Regression Models ... 243
Find a Model Using Variable Selection

Overview of Stepwise Regression ... 245
Example Using Stepwise Regression ... 245
The Stepwise Report ... 247
Stepwise Platform Options ... 247
Stepwise Regression Control Panel ... 248
Current Estimates Report ... 255

Step History Report ... 256
Models with Crossed, Interaction, or Polynomial Terms ... 257
Models with Nominal and Ordinal Effects ... 257
Construction of Hierarchical Terms ... 258
Perform Binary and Ordinal Logistic Stepwise Regression ... 259
The All Possible Models Option ... 260
The Model Averaging Option ... 260
Validation Options in Stepwise Regression ... 261
Validation Set with Two or Three Values in Stepwise Regression ... 261
K-Fold Cross Validation in Stepwise Regression ... 264
Additional Examples of the Stepwise Personality ... 265
Example of the Combine Rule ... 266
Example of a Model with a Nominal Term ... 267
Example of the Restrict Rule for Hierarchical Terms ... 272
Example of Logistic Stepwise Regression ... 274
Example of the All Possible Models Option ... 275
Example of the Model Averaging Option ... 277

6 Generalized Regression Models ... 279
Build Models Using Variable Selection Techniques

Overview of the Generalized Regression Personality ... 281
Example of Generalized Regression ... 283
Launch the Generalized Regression Personality ... 286
Specify a Distribution ... 288
Generalized Regression Report Window ... 296
Generalized Regression Report Options ... 296
Model Launch Control Panel ... 297
Response Distribution ... 298
Estimation Method Options ... 298
Advanced Controls ... 303
Validation Method Options ... 306
Early Stopping ... 308
Go ... 308
Model Fit Reports ... 309
Regression Plot ... 309
Model Summary ... 310
Estimation Details ... 313
Solution Path ... 313
Parameter Estimates for Centered and Scaled Predictors ... 317
Parameter Estimates for Original Predictors ... 319

Active Parameter Estimates ... 320
Effect Tests ... 320
Model Fit Options ... 321
Self-Validated Ensemble Models ... 332
Overview of Self-Validated Ensemble Models ... 332
Reports for Self-Validated Ensemble Models ... 333
Model Fit Options for Self-Validated Ensemble Models ... 334
Statistical Details for the Generalized Regression Personality ... 336
Statistical Details for Estimation Methods ... 336
Statistical Details for Advanced Controls ... 338
Statistical Details for Distributions ... 339

7 Generalized Regression Examples ... 347
Build Models Using Regularization Techniques

Example of Poisson Generalized Regression ... 349
Example of Binomial Generalized Regression ... 351
Example of Zero-Inflated Poisson Regression ... 353
Example of the Model Comparison Table in Generalized Regression ... 356
Example of Generalized Regression for Wide Data ... 358

8 Mixed Models ... 363
Jointly Model the Mean and Covariance

Overview of the Mixed Model Personality ... 365
Example Using the Mixed Model Personality ... 366
Launch the Mixed Model Personality ... 370
Fit Model Launch Window ... 370
Data Format ... 376
Mixed Model Report and Options ... 376
Random Effects Covariance Parameter Estimates ... 383
Fixed Effects Parameter Estimates ... 385
Repeated Effects Covariance Parameter Estimates ... 386
Random Coefficients ... 386
Random Effects Predictions ... 386
Fixed Effects Tests ... 387
Sequential Tests ... 387
Multiple Comparisons ... 388
Compare Slopes ... 389
Marginal Model Inference ... 389
Actual by Predicted Plot ... 389
Residual Plots ... 389
Marginal Model Profiler ... 390

Variogram ... 391
Conditional Model Inference ... 392
Actual by Conditional Predicted Plot ... 392
Conditional Residual Plots ... 392
Conditional Profilers ... 393
Additional Examples of the Mixed Model Personality ... 394
Example of Repeated Measures ... 394
Example of a Split Plot Experiment ... 409
Example of a Uniformity Trial ... 414
Example of a Correlated Response ... 424
Statistical Details for the Mixed Model Personality ... 431
Statistical Details for the Convergence Score Test ... 431
Statistical Details for the Random Coefficient Model ... 432
Statistical Details for Repeated Measures ... 434
Statistical Details for Repeated Covariance Structures ... 434
Statistical Details for Spatial and Temporal Variability ... 440
Statistical Details for the Kackar-Harville Correction ... 442

9 Generalized Linear Mixed Models ... 445
Fit a Variety of Mixed Models to Non-Normal Response Data

Overview of the Generalized Linear Mixed Models Personality ... 447
Example of a Generalized Linear Mixed Model ... 447
Launch the Generalized Linear Mixed Model Personality ... 450
Fit Model Launch Window ... 450
Data Format ... 454
Generalized Linear Mixed Model Options ... 454
Model Fit Reports ... 455
Fit Statistics and Model Summary ... 455
Random Effects Covariance Parameter Estimates ... 457
Fixed Effects Parameter Estimates ... 457
Random Coefficients ... 458
Fixed Effects Tests ... 458
Model Fit Options ... 459
Additional Example of the Generalized Linear Mixed Model Personality ... 463

10 Multivariate Response Models ... 469
Fit Relationships Using MANOVA

Example of a Multivariate Response Model ... 471
Launch the Manova Personality ... 473
The Manova Fit Report ... 473
The Manova Fit Options ... 474

Response Specification Panel ... 475
Multivariate Response Reports ... 476
Multivariate Tests in Multivariate Response Models ... 478
The Extended Multivariate Report ... 478
Comparison of Multivariate Tests ... 479
Univariate Tests and the Test for Sphericity ... 480
Multivariate Response Models with Repeated Measures ... 481
Discriminant Analysis in Multivariate Response Models ... 482
Additional Examples of the Manova Personality ... 482
Example of a Compound Multivariate Model ... 482
Example of a Repeated Measures Multivariate Model ... 485
Example of the Save Discrim Option ... 486
Example of Univariate and Sphericity Test ... 487
Example of Test Details ... 488
Example of Canonical Correlation Analysis ... 489
Statistical Details for the Manova Personality ... 490
Statistical Details for Multivariate Tests ... 490
Statistical Details for Approximate F-Tests ... 491
Statistical Details for Canonical Calculations ... 492

11 Loglinear Variance Models ... 495
Model the Variance and the Mean of the Response

Overview of the Loglinear Variance Model ... 497
Example Using Loglinear Variance ... 498
Launch the Loglinear Variance Personality ... 500
The Loglinear Variance Fit Report ... 501
Loglinear Variance Fit Report Options ... 502
Additional Examples of Loglinear Variance Models ... 504
Example of Examining the Residuals in a Loglinear Variance Model ... 504
Example of Profiling a Fitted Loglinear Variance Model ... 505

12 Logistic Regression Models ... 507
Fit Regression Models for Nominal or Ordinal Responses

Overview of the Nominal and Ordinal Logistic Personalities ... 509
About Nominal Logistic Regression ... 509
About Ordinal Logistic Regression ... 509
Other JMP Platforms That Fit Logistic Regression Models ... 510
Examples of Logistic Regression ... 510
Example of Nominal Logistic Regression ... 510
Example of Ordinal Logistic Regression ... 512
Launch the Nominal and Ordinal Logistic Personalities ... 515

Validation in Logistic Regression Models ... 516
The Logistic Fit Report ... 516
Whole Model Test ... 518
Fit Details ... 519
Lack of Fit Test ... 520
Logistic Fit Platform Options ... 520
Options for Nominal and Ordinal Fits ... 520
Options for Nominal Fits ... 523
Options for Ordinal Fits ... 524
Additional Examples of Logistic Regression ... 525
Example of Inverse Prediction in Fit Model ... 525
Example of Using Effect Summary for a Nominal Logistic Model ... 527
Example of a Quadratic Ordinal Logistic Model ... 529
Example of Stacking Counts in Multiple Columns ... 532
Statistical Details for the Nominal and Ordinal Logistic Personalities ... 533
Statistical Details for the Logistic Regression Model ... 534
Statistical Details for Odds Ratios ... 534
Statistical Details for Logistic Regression Statistical Tests ... 535

13 Generalized Linear Models ... 537
Fit Models for Nonnormal Response Distributions

Overview of the Generalized Linear Model Personality ... 539
Example of a Generalized Linear Model ... 540
Launch the Generalized Linear Model Personality ... 543
Generalized Linear Model Fit Report ... 545
Whole Model Test ... 546
Generalized Linear Model Fit Report Options ... 547
Additional Examples of the Generalized Linear Models Personality ... 550
Example of Using Contrasts in a Generalized Linear Model ... 550
Example of Poisson Regression with an Offset ... 551
Example of Normal Regression with a Log Link ... 552
Statistical Details for the Generalized Linear Model Personality ... 554
Statistical Details for Generalized Linear Model Construction ... 555
Statistical Details for Model Selection and Deviance ... 557

A Statistical Details ... 561
Fitting Linear Models

The Response Models ... 563
Continuous Responses ... 563
Nominal Responses ... 564
Ordinal Responses ... 565

The Factor Models . . . 567
Continuous Factors . . . 567
Nominal Factors . . . 567
Ordinal Factors . . . 579
Frequencies . . . 585
The Usual Assumptions . . . 585
Assumed Model . . . 585
Relative Significance . . . 585
Multiple Inferences . . . 586
Validity Assessment . . . 586
Alternative Methods . . . 587
Key Statistical Concepts . . . 587
Uncertainty, a Unifying Concept . . . 587
The Two Basic Fitting Machines . . . 588
Likelihood, AICc, and BIC . . . 592
Power Calculations . . . 593
Computations for the LSN . . . 593
Computations for the LSV . . . 594
Computations for the Power . . . 595
Computations for the Adjusted Power . . . 596
Inverse Prediction with Confidence Limits . . . 597

B References . . . 601

C Technology License Notices . . . 607

Chapter 1

Learn about JMP

Documentation and Additional Resources

Learn about JMP documentation, such as book conventions, descriptions of each JMP document, the Help system, and where to find additional support.

Contents

Formatting Conventions in JMP Documentation . . . 17
JMP Help . . . 18
JMP Documentation Library . . . 18
Additional Resources for Learning JMP . . . 25
Search JMP . . . 25
JMP Tutorials . . . 25
Sample Data Tables . . . 26
Learn about Statistical and JSL Terms . . . 26
Learn JMP Tips and Tricks . . . 26
JMP Tooltips . . . 26
JMP User Community . . . 27
Free Online Statistical Thinking Course . . . 27
JMP New User Welcome Kit . . . 27
Statistics Knowledge Portal . . . 27
JMP Training . . . 28
JMP Books by Users . . . 28
The JMP Starter Window . . . 28
JMP Technical Support . . . 28

Formatting Conventions in JMP Documentation

These conventions help you relate written material to information that you see on your screen:

- Sample data table names, column names, pathnames, filenames, file extensions, and folders appear in Helvetica (or sans-serif online) font.
- Code appears in `Lucida Sans Typewriter` (or monospace online) font.
- Code output appears in *`Lucida Sans Typewriter`* italic (or monospace italic online) font and is indented farther than the preceding code.
- **Helvetica bold** formatting (or bold sans-serif online) indicates items that you select to complete a task:
  - buttons
  - check boxes
  - commands
  - list names that are selectable
  - menus
  - options
  - tab names
  - text boxes
- The following items appear in italics:
  - words or phrases that are important or have definitions specific to JMP
  - book titles
  - variables
- Features that are for JMP Pro only are noted with the JMP Pro icon JMP PRO. For an overview of JMP Pro features, visit jmp.com/software/pro.

Note: Special information and limitations appear within a Note.

Tip: Helpful information appears within a Tip.

JMP Help

JMP Help in the Help menu enables you to search for information about JMP features, statistical methods, and the JMP Scripting Language (or *JSL*). You can open JMP Help in several ways:

- Search and view JMP Help on Windows by selecting **Help > JMP Help**.
- On Windows, press the F1 key to open the Help system in the default browser.
- Get help on a specific part of a data table or report window. Select the Help tool from the **Tools** menu and then click anywhere in a data table or report window to see the Help for that area.
- Within a JMP window, click the **Help** button.

Note: The JMP Help is available for users with Internet connections. Users without an Internet connection can search all books in a PDF file by selecting **Help > JMP Documentation Library**. See "JMP Documentation Library" for more information.

JMP Documentation Library

The Help system content is also available in one PDF file called *JMP Documentation Library*. Select **Help > JMP Documentation Library** to open the file. You can also download the Documentation PDF Files add-in if you prefer searching individual PDF files of each document in the JMP library. Download the available add-ins from community.jmp.com.

The following table describes the purpose and content of each document in the JMP library.

Document Title	Document Purpose	Document Content
Discovering JMP	If you are not familiar with JMP, start here.	Introduces you to JMP and gets you started creating and analyzing data. Also learn how to share your results.
Using JMP	Learn about JMP data tables and how to perform basic operations.	Covers general JMP concepts and features that span across all of JMP, including importing data, modifying columns properties, sorting data, and connecting to SAS.

Document Title	Document Purpose	Document Content
Basic Analysis	Perform basic analysis using this document.	Describes these Analyze menu platforms: • Distribution • Fit Y by X • Tabulate • Text Explorer Covers how to perform bivariate, one-way ANOVA, and contingency analyses through Analyze > Fit Y by X. How to approximate sampling distributions using bootstrapping and how to perform parametric resampling with the Simulate platform are also included.
Essential Graphing	Find the ideal graph for your data.	Describes these Graph menu platforms: • Graph Builder • Scatterplot 3D • Contour Plot • Bubble Plot • Parallel Plot • Cell Plot • Scatterplot Matrix • Ternary Plot • Treemap • Chart • Overlay Plot The book also covers how to create background and custom maps.
Profilers	Learn how to use interactive profiling tools, which enable you to view cross-sections of any response surface.	Covers all profilers listed in the Graph menu. Analyzing noise factors is included along with running simulations using random inputs.

Document Title	Document Purpose	Document Content
Design of Experiments Guide	Learn how to design experiments and determine appropriate sample sizes.	Covers all topics in the DOE menu.
Fitting Linear Models	Learn about Fit Model platform and many of its personalities.	Describes these personalities, all available within the Analyze menu Fit Model platform: • Standard Least Squares • Stepwise • Generalized Regression • Mixed Model • Generalized Linear Mixed Model • MANOVA • Loglinear Variance • Nominal Logistic • Ordinal Logistic • Generalized Linear Model

Document Title	Document Purpose	Document Content
Predictive and Specialized Modeling	Learn about additional modeling techniques.	Describes these Analyze > Predictive Modeling menu platforms: • Neural • Partition • Bootstrap Forest • Boosted Tree • K Nearest Neighbors • Naive Bayes • Support Vector Machines • Model Comparison • Model Screening • Make Validation Column • Formula Depot Describes these Analyze > Specialized Modeling menu platforms: • Fit Curve • Nonlinear • Functional Data Explorer • Gaussian Process • Time Series • Matched Pairs Describes these Analyze > Screening menu platforms: • Explore Outliers • Explore Missing Values • Explore Patterns • Response Screening • Predictor Screening • Association Analysis • Process History Explorer

Document Title	Document Purpose	Document Content
Multivariate Methods	Read about techniques for analyzing several variables simultaneously.	Describes these Analyze > Multivariate Methods menu platforms: • Multivariate • Principal Components • Discriminant • Partial Least Squares • Multiple Correspondence Analysis • Structural Equation Models • Factor Analysis • Multidimensional Scaling • Multivariate Embedding • Item Analysis Describes these Analyze > Clustering menu platforms: • Hierarchical Cluster • K Means Cluster • Normal Mixtures • Latent Class Analysis • Cluster Variables

Document Title	Document Purpose	Document Content
Quality and Process Methods	Read about tools for evaluating and improving processes.	Describes these Analyze > Quality and Process menu platforms: • Control Chart Builder and individual control charts • Measurement Systems Analysis (EMP and Type 1 Gauge) • Variability / Attribute Gauge Charts • Process Screening • Process Capability • Model Driven Multivariate Control Chart • Legacy Control Charts • Pareto Plot • Diagram • Manage Limits • OC Curves
Reliability and Survival Methods	Learn to evaluate and improve reliability in a product or system and analyze survival data for people and products.	Describes these Analyze > Reliability and Survival menu platforms: • Life Distribution • Fit Life by X • Cumulative Damage • Recurrence Analysis • Degradation • Repeated Measures Degradation • Destructive Degradation • Reliability Forecast • Reliability Growth • Reliability Block Diagram • Repairable Systems Simulation • Survival • Fit Parametric Survival • Fit Proportional Hazards

Document Title	Document Purpose	Document Content
Consumer Research	Learn about methods for studying consumer preferences and using that insight to create better products and services.	Describes these Analyze > Consumer Research menu platforms: • Categorical • Choice • MaxDiff • Uplift • Multiple Factor Analysis
Genetics	Learn about methods that are available in JMP to help you analyze your genetic data and use that data to simulate a breeding program to predict the optimum genetic crosses to make.	Describes these Analyze > Genetics menu platforms: • Marker Statistics • Marker Simulation
Scripting Guide	Learn about taking advantage of the powerful JMP Scripting Language (JSL).	Covers a variety of topics, such as writing and debugging scripts, manipulating data tables, constructing display boxes, and creating JMP applications.
JSL Syntax Reference	Read about many JSL functions on functions and their arguments, and messages that you send to objects and display boxes.	Includes syntax, examples, and notes for JSL commands.

Additional Resources for Learning JMP

In addition to reading JMP help, you can also learn about JMP using the following resources:

- "Search JMP"
- "JMP Tutorials"
- "Sample Data Tables"
- "Learn about Statistical and JSL Terms"
- "Learn JMP Tips and Tricks"
- "JMP Tooltips"
- "JMP User Community"
- "Free Online Statistical Thinking Course"
- "JMP New User Welcome Kit"
- "Statistics Knowledge Portal"
- "JMP Training"
- "JMP Books by Users"
- "The JMP Starter Window"

Search JMP

If you are not sure where to find a statistical procedure, do a search across JMP. Results are tailored to the window that you launch the search from, such as a data table or report.

1. Click **Help** > **Search JMP**. Or, press Ctrl+comma.
2. Enter your search text.
3. Click the result that contains the procedure that you want.

   On the right, you can see a description and the location of the procedure.
4. Click the corresponding button to open or go to a result.

JMP Tutorials

You can access JMP tutorials by selecting **Help > Tutorials**.

If you are not familiar with JMP, start with the **Beginners Tutorial**. It steps you through the JMP interface and explains the basics of using JMP.

The rest of the tutorials help you with specific aspects of JMP, such as designing an experiment and comparing a sample mean to a constant.

Sample Data Tables

All of the examples in the JMP documentation suite use sample data. Select **Help > Sample Data Folder** to open the sample data directory.

To view an alphabetized list of sample data tables or view sample data within categories, select **Help > Sample Index**.

Sample data tables are installed in the following directory:

On Windows: C:\Program Files\SAS\JMP\17\Samples\Data

On macOS: \Library\Application Support\JMP\17\Samples\Data

In JMP Pro, sample data is installed in the JMPPRO (rather than JMP) directory.

To view examples using sample data, select **Help > Sample Index** and navigate to the Teaching Resources section.

Learn about Statistical and JSL Terms

For help with statistical terms, select Help > Statistics Index. For help with JSL scripting and examples, select **Help > Scripting Index**.

Statistics Index Provides definitions of statistical terms.

Scripting Index Lets you search for information about JSL functions, objects, and display boxes. You can also edit and run sample scripts from the Scripting Index and get help on the commands.

Learn JMP Tips and Tricks

When you first start JMP, you see the Tip of the Day window. This window provides tips for using JMP.

To turn off the Tip of the Day, clear the **Show tips at startup** check box. To view it again, select **Help > Tip of the Day**. Or, you can turn it off using the Preferences window.

JMP Tooltips

JMP provides descriptive tooltips (or *hover labels*) when you hover over items, such as the following:

- Menu or toolbar options
- Labels in graphs
- Text results in the report window (move your cursor in a circle to reveal)

- Files or windows in the Home Window
- Code in the Script Editor

Tip: On Windows, you can hide tooltips in the JMP Preferences. Select **File > Preferences > General** and then deselect **Show menu tips**. This option is not available on macOS.

JMP User Community

The JMP User Community provides a range of options to help you learn more about JMP and connect with other JMP users. The learning library of one-page guides, tutorials, and demos is a good place to start. And you can continue your education by registering for a variety of JMP training courses.

Other resources include a discussion forum, sample data and script file exchange, webcasts, and social networking groups.

To access JMP resources on the website, select **Help > JMP on the Web > JMP User Community** or visit https://community.jmp.com.

Free Online Statistical Thinking Course

Learn practical statistical skills in this free online course on topics such as exploratory data analysis, quality methods, and correlation and regression. The course consists of short videos, demonstrations, exercises, and more. Visit jmp.com/statisticalthinking.

JMP New User Welcome Kit

The JMP New User Welcome Kit is designed to help you quickly get comfortable with the basics of JMP. You'll complete its thirty short demo videos and activities, build your confidence in using the software, and connect with the largest online community of JMP users in the world. Visit jmp.com/welcome.

Statistics Knowledge Portal

The Statistics Knowledge Portal combines concise statistical explanations with illuminating examples and graphics to help visitors establish a firm foundation upon which to build statistical skills. Visit jmp.com/skp.

JMP Training

SAS offers training on a variety of topics led by a seasoned team of JMP experts. Public courses, live web courses, and on-site courses are available. You might also choose the online e-learning subscription to learn at your convenience. Visit jmp.com/training.

JMP Books by Users

Additional books about using JMP that are written by JMP users are available on the JMP website. Visit jmp.com/books.

The JMP Starter Window

The JMP Starter window is a good place to begin if you are not familiar with JMP or data analysis. Options are categorized and described, and you launch them by clicking a button. The JMP Starter window covers many of the options found in the Analyze, Graph, Tables, and File menus. The window also lists JMP Pro features and platforms.

- To open the JMP Starter window, select **View** (**Window** on macOS) **> JMP Starter**.
- To display the JMP Starter automatically when you open JMP on Windows, select **File > Preferences > General**, and then select **JMP Starter** from the Initial JMP Window list. On macOS, select **JMP > Preferences > General > Initial JMP Starter Window**.

JMP Technical Support

JMP technical support is provided by statisticians and engineers educated in SAS and JMP, many of whom have graduate degrees in statistics or other technical disciplines.

Many technical support options are provided at jmp.com/support, including the technical support phone number.

Chapter 2

Model Specification

Specify Linear Models

The Fit Model platform enables you to specify a variety of complex models using different fitting techniques or *personalities*. This chapter focuses on elements that are common to most personalities.

Fit Model personalities enable you to fit the following types of models:

- simple and multiple linear regression
- analysis of variance and covariance
- random effect, nested effect, mixed effect, repeated measures, and split plot models
- nominal and ordinal logistic regression
- multivariate analysis of variance (MANOVA)
- canonical correlation and discriminant analysis
- loglinear variance (to model the mean and the variance)
- generalized linear models (GLM)
- parametric survival and proportional hazards
- response screening, for studying a large number of responses

JMP PRO In JMP Pro, you can also fit the following models:

- generalized regression models including the elastic net, lasso, and ridge regression
- mixed models with a range of covariance structures
- generalized linear mixed models (GLMM)
- partial least squares

Contents

Overview of the Fit Model Platform . . . 31
Example of a Regression Analysis Using Fit Model . . . 32
Launch the Fit Model Platform . . . 35
Elements in the Fit Model Launch Window . . . 36
Construct Model Effects . . . 39
Fitting Personalities . . . 46
Model Specification Options . . . 48
Informative Missing . . . 50
Validity Checks . . . 51
Model Specification Templates . . . 52
Simple Linear Regression . . . 52
Polynomial in X to Degree k . . . 53
Polynomial in X and Z to Degree k . . . 53
Multiple Linear Regression . . . 53
One-Way Analysis of Variance . . . 53
Two-Way Analysis of Variance . . . 54
Two-Way Analysis of Variance with Interaction . . . 54
Three-Way Full Factorial . . . 54
Analysis of Covariance, Equal Slopes . . . 54
Analysis of Covariance, Unequal Slopes . . . 55
Two-Factor Nested Random Effects Model . . . 55
Three-Factor Fully Nested Random Effects Model . . . 56
Simple Split Plot or Repeated Measures Model . . . 56
Two-Factor Response Surface Model . . . 57
Knotted Spline Effect . . . 57

Overview of the Fit Model Platform

The Fit Model platform provides an efficient way to specify models that have complex effect structures. These effect structures are linear in the model parameters. Once you have specified your model, you can select the appropriate fitting technique from a number of fitting personalities. Once you choose a personality, the Fit Model window provides choices that are relevant for the chosen personality. This chapter focuses on the elements of the Model Specification window that are common to most personalities. For a description of all personalities, see "Elements in the Fit Model Launch Window".

Fit Model can be used to specify a wide variety of models that can be fit using various methods. Table 2.1 lists some typical models that can be defined using Fit Model. In the table, the effects X and Z represent columns with a continuous modeling type, while A, B, and C represent columns with a nominal or ordinal modeling type.

See "Model Specification Templates" for the clicking sequences that produce these model effects, plots of the model fits, and some examples.

Table 2.1 Standard Model Types

Type of Model	Model Effects
Simple Linear Regression	X
Polynomial in X to Degree k	X, X*X,..., X^k
Polynomial in X and Z to Degree k	X, X*X,..., X^k, Z, Z*Z,..., Z^k
Multiple Linear Regression	X, Z, and other continuous columns
One-Way Analysis of Variance	A
Two-Way Analysis of Variance	A, B
Two-Way Analysis of Variance with Interaction	A, B, A*B
Three-Way Full Factorial	A, B, C, A*B, A*C, B*C, A*B*C
Analysis of Covariance, Equal Slopes	A, X
Analysis of Covariance, Unequal Slopes	A, X, A*X
Two-Factor Nested Random Effects Model	A, B[A]&Random
Three-Factor Fully Nested Random Effects Model	A, B[A]&Random, C[A,B]&Random

Table 2.1 Standard Model Types *(Continued)*

Type of Model	Model Effects
Simple Split Plot or Repeated Measures Model	A, B[A]&Random, C, C*A
Two-Factor Response Surface Model	X&RS, Z&RS, X*X, X*Z, Z*Z

Example of a Regression Analysis Using Fit Model

You have data resulting from an aerobic fitness study, and you want to predict the oxygen uptake from several continuous variables.

1. Select **Help > Sample Data Folder** and open Fitness.jmp.
2. Select **Analyze > Fit Model**. Note that the Personality box is empty.
3. Select Oxy and click **Y**.

   When you specify a continuous response, the Personality defaults to Standard Least Squares, but you are free to choose another personality. Also, the Emphasis defaults to Effect Leverage.
4. Press Ctrl and select Sex, Age, Weight, Runtime, RunPulse, RstPulse, and MaxPulse. Click **Add** to add these to the Construct Model Effects list. Note that you can select **Keep dialog open** if you want to have this window available later on. Your Model Specification window should appear as shown in Figure 2.1.
5. Click **Run**. Figure 2.2 gives a partial view of the report.

Figure 2.1 Model Specification Window for Fitness Regression Model

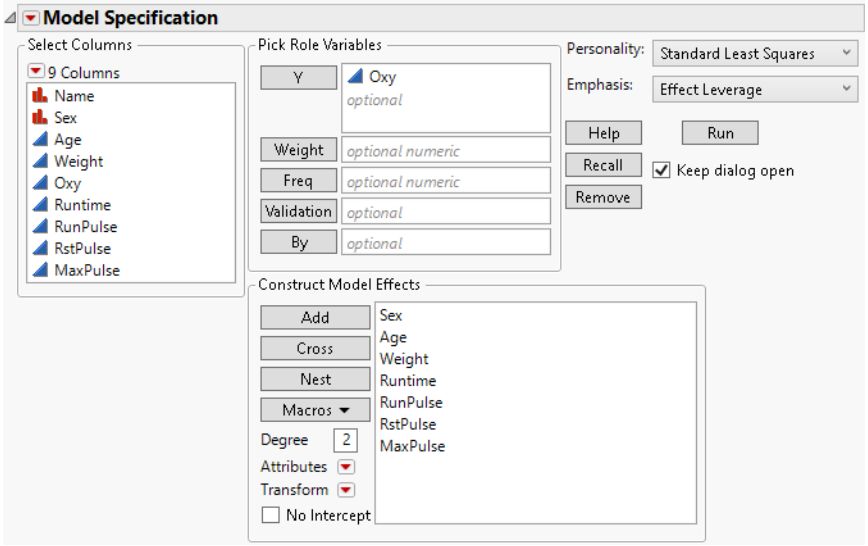
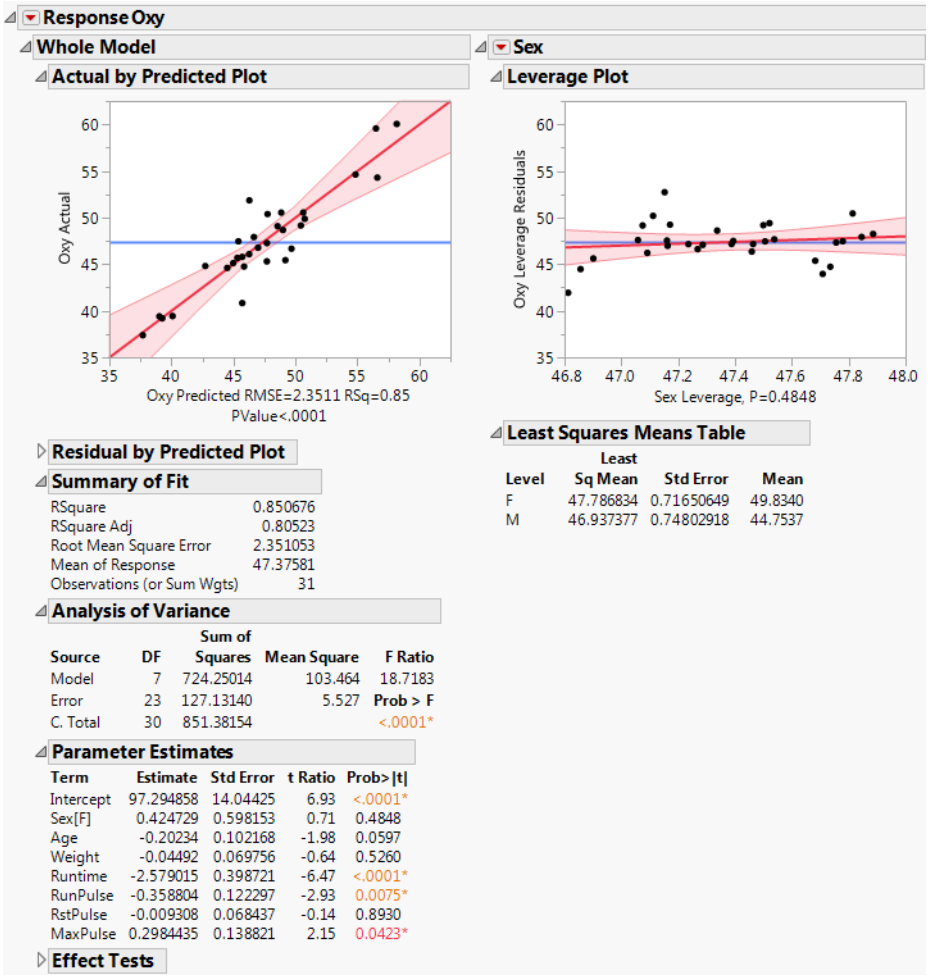


Figure 2.2 Partial View of Standard Least Squares Report for Fitness Data



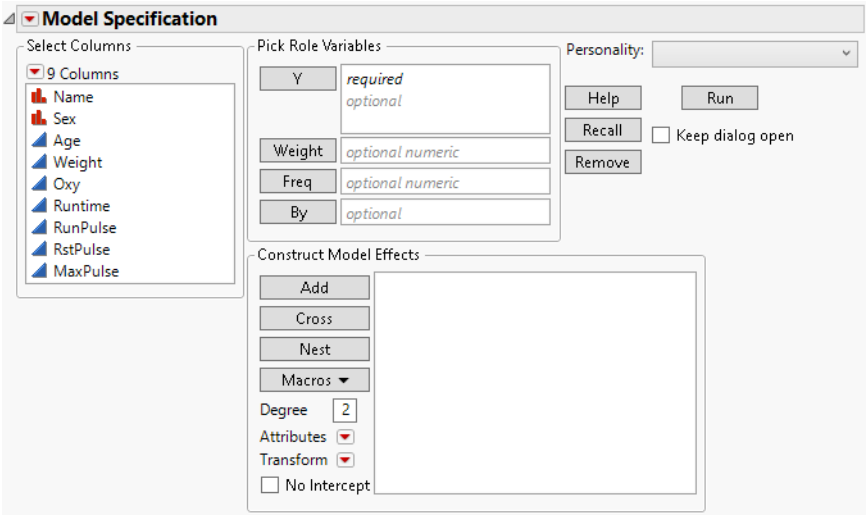
The plot and reports for the whole model appear in the left-most report column. The columns to the right show leverage plots for each of the effects that you specified in the model. Due to space limitations, Figure 2.2 shows only the column for Sex, but the report shows columns for the other six effects as well. The red triangle menus contain additional options that add reports and plots to the report window. For more information about the Standard Least Squares report window, see "Fit Least Squares Report".

Looking at the *p*-values in the Parameter Estimates report, you can see that Runtime, RunPulse, and MaxPulse appear to be significant predictors of oxygen uptake. The next step might be to reduce the model by removing insignificant predictors. See "Stepwise Regression Models".

Launch the Fit Model Platform

Launch the Fit Model platform by selecting **Analyze > Fit Model**.

Figure 2.3 Fit Model Launch Window



Note: When you select **Analyze > Fit Model** in a data table that has a script named *Model* (or *model*), the launch window is automatically filled in based on the script.

The Fit Model launch window contains the following major areas:

- Select Columns is a list of the columns in the current data table. If the current data table contains excluded columns, they do not appear in the list. For more information about the options in the Select Columns red triangle menu, see *Using JMP*.
- Pick Role Variables contains standard JMP launch window buttons. For a description of these buttons, see "Elements in the Fit Model Launch Window".
- Construct Model Effects contains options that you use to enter effects into your model. See "Construct Model Effects".
- Personality is a list of the model types that you can choose from. Once you have selected a personality, different options appear, depending on the personality that you have selected. See "Fitting Personalities".

Elements in the Fit Model Launch Window

The following elements in the Fit Model launch window are common to most personalities:

Model Specification The Model Specification menu contains the following options:

Center Polynomials Centers the effects when polynomials are included in the model.

Informative Missing Creates a coding system that accommodates missing values in effects.

Suppress Coding Ignores any Coding column properties for the effects columns and fits a model using the uncoded effects.

Set Alpha Level Sets the alpha level for confidence intervals in the model reports.

Save to Data Table Saves the model specifications to a script in the current data table.

Save to Script Window Saves the model specifications to a script window.

Create SAS Job Saves SAS code for the current model specification to a SAS Program window.

Submit to SAS Submits SAS code for the current model specification to a SAS session.

Convergence Settings Contains options for setting the maximum iterations and convergence limit used in the convergence of models in some personalities.

For more information about any of these options, see "Model Specification Options".

Select Columns Lists the unexcluded columns in the current data table.

Y Identifies one or more response variables (the dependent variables) for the model.

Note: Response columns with vector values are not supported in Fit Model.

Weight Identifies a column whose values assign a weight to each row for the analysis. See "Weight".

Freq Identifies a column whose values assign a frequency to each row for the analysis. In general terms, the effect of a frequency column is to expand the data table, so that any row with integer frequency *k* is expanded to *k* rows. You are allowed to specify fractional frequencies. See "Frequency".

Switch Specifies columns that can be switched, one at a time, into the model.

JMP PRO **Validation** In JMP Pro, for some personalities, you can enter a Validation column. See the appropriate Personality chapter for more information about how each personality handles a Validation column. If you click the Validation button with no columns selected

in the Select Columns list, you can add a validation column to your data table. For more information about the Make Validation Column utility, see *Predictive and Specialized Modeling*. For more information about how a Validation column is used in JMP modeling platforms, see *Predictive and Specialized Modeling*.

By Identifies a column that creates a report consisting of separate analyses for each level of the variable. If more than one By variable is assigned, a separate analysis is produced for each possible combination of the levels of the By variables.

Add Adds effects to the model. See "Add".

Cross Creates interaction and polynomial effects by crossing two or more variables. See "Cross".

Nest Creates nested effects. See "Nest".

Macros Generates effects for commonly used models. See "Macros".

Degree Applies the specified degree to models with factorial or polynomial effects generated using Macros. See **Factorial to Degree** and **Polynomial to Degree** in "Macros".

Attributes Applies attributes to model effects. These attributes determine how the effects are treated. See "Attributes".

Transform Transforms selected continuous effects or *Y* columns. See "Transform".

No Intercept Excludes the intercept term from the model.

Personality Specifies the fitting methodology. See "Elements in the Fit Model Launch Window". Different options appear depending on the personality that you select.

Target Level (Available only in certain personalities and when *Y* is binary and has a nominal modeling type.) Specifies the level whose probability you want to model. The default value is the higher of the two levels based on the order of the levels.

Help Takes you to Help topics for the Fit Model launch window.

Recall Populates the launch window with the last model specification that you ran.

Remove Removes the selected variable from the assigned role. Alternatively, double-click the effect or select the effect and press Backspace.

Run Generates the report window for the specified model and personality.

Keep dialog open Keeps the launch window open after you run the analysis, enabling you to alter and re-run the analysis at any time.

Frequency

Frequency variables, entered in the **Freq** text box, are supported in most Fit Model personalities. In general, a frequency is interpreted in the following manner. Suppose that a row has a frequency *f*. Then the computed results are identical to those for a data table containing *f* copies of that row, each having a frequency of one.

Rows with zero or missing frequency values are excluded from analyses. Rows with negative frequency values are permitted only for censored observations, otherwise they are excluded from analyses. When used with censored observations, negative frequency values can be used to fit truncated distributions.

Frequency values do not need to be integers. The technical details describing how frequency columns, including those with non-integer values, are handled are given in "Frequencies".

Weight

Weight variables can be useful in situations where there are observations with different variances. For example, this can happen when one performs regression modeling on data where each row consists of pre-summarized means. Here, rows involving a larger number of observations (smaller variance) should contribute more heavily to the loss function than rows involving a smaller number of observations (larger variance). You can ensure that this occurs by using appropriately defined weights.

Weight variables are supported in many Fit Model personalities. Each personality that supports weight variables uses one of the following methods:

- Variance Scaling
- Frequency Symmetry

Variance Scaling

When estimation is performed using least squares or normal theory maximum likelihood, the weight w for a given row scales that row's contribution to the loss function by $w^{-1/2}$.

Weight variables have an impact on estimates and standard errors. However, unlike frequency variables, they do not affect the degrees of freedom used in hypothesis tests.

Rows with negative or zero values for Weight are excluded from analyses.

Frequency Symmetry

In the Nominal Logistic and Ordinal Logistic personalities, weight variables are handled as if they were frequency variables. If weight and frequency variables are both specified, these personalities handle each observation as if it has a frequency value equal to the product of the weight and frequency values.

Construct Model Effects

In the Fit Model launch window, enter your model effects under Construct Model Effects. For examples of how to obtain specific types of models, see "Model Specification Templates".

Add

Adds effects to the model. These effects can either be added directly from the Select Columns list or they can be selected in that list and modified using Macros or Attributes. Effects can also be created and added, or modified, using Cross and Nest. The modeling types of the variables involved in the effect, as well as any Attribute assigned to the effect, determine how that effect is treated in the model.

Note: To remove an effect from the Construct Model Effects list, double-click the effect, or select it and click **Remove** or press Backspace or Delete.

Cross

Creates interaction or polynomial effects. Select two or more variables in the Select Columns list and click **Cross**. Or, select one or more variables in the Select Columns list and one or more effects in the Construct Model Effects list and click **Cross**.

See "Statistical Details", for a discussion of how crossed effects are parameterized and coded.

Note: You can construct effects that combine up to ten columns as crossed and nested.

Example of Crossed Effects

Suppose that a product coating requires a dye to be applied. Both Dye pH and Dye Concentration are suspected to have an effect on the coating color. To understand their effects, you design an experiment where Dye pH and Dye Concentration are each set at a high and low level. It is possible that the effect of Dye pH on the color is more pronounced at the high level of Dye Concentration than at its low level. This is known as an *interaction*. To model this possible interaction, you include the crossed term, Dye pH * Dye Concentration, in the Construct Model Effects list. This enables JMP to test for an interaction.

Nest

Creates nested effects. If the levels of one effect (B) occur only within a single level of another effect (A), then B is said to be *nested* within A. The notation B[A], which is read as "B nested within A," is typically used. Note that nesting defines a hierarchical relationship. A is called the *outside* effect and B is called the *inside* effect. Nested terms must be categorical.

Note: The nesting terms must be specified in order from outer to inner. For example, if B is nested within A, and C is nested within B, then the model is specified as A, B[A], C[B,A] (or, equivalently, A, B[A], C[A,B]). You can construct effects that combine up to ten columns as crossed and nested.

Example of Nested Effects

As an illustration of nesting, consider the math teachers in each of two schools. One school has three math teachers; the other school has two math teachers. Each teacher in each school teaches two or three classes consisting of non-overlapping groups of students. In this example, classes (C) are nested within teachers (B), and teachers (B) are nested within schools (A). Enter these effects in the Fit Model launch window:

1. Add both A and B to the Construct Model Effects panel.
2. In the Construct Model Effects panel, select B.
3. In the Select Columns list, select A.
4. Click **Nest**. This converts B to the effect B[A].
5. Add C to the Construct Model Effects panel.
6. In the Construct Model Effects panel, select C.
7. In the Select Columns list, select A and B.
8. Click **Nest**. The converts C to the effect C[A, B].

Macros

In the Macros list, select options to automatically generate the effects for commonly used models and enter them into the Construct Model Effects list:

Full Factorial Creates all main effects and interactions for the columns selected in the Select Columns list. These are entered in an order that is based on the order in which the main effects are listed in the Select Columns list. For an alternate ordering, see **Factorial Sorted**, in this table.

Factorial to Degree Creates all main effects, but only interactions up to a specified degree (order). Specify the degree in the **Degree** box beneath the **Macros** button.

Factorial Sorted Creates the same set of effects as the **Full Factorial** option but lists them in order of degree. All main effects are listed first, followed by all two-way interactions, then all three-way interactions, and so on.

Response Surface Creates main effects, two-way interactions, and quadratic terms. The selected main effects are given the response surface attribute, denoted **RS**. When the RS attribute is applied to main effects and the Standard Least Squares personality is selected,

a Response Surface report is provided. This report gives information about critical values and the shape of the response surface.

Note: This option does not create quadratic terms for categorical effects.

See also Response Surface Effect in "Attributes" and the *Design of Experiments Guide*.

Mixture Response Surface Creates main effects and two-way interactions. Main effects have the response surface (**RS**) and mixture (**Mixture**) attributes. In the Standard Least Squares personality, the **Mixture** attribute causes a mixture model to be fit. The **RS** attribute creates a Response Surface report that is specific to mixture models.

See also Mixture Effect in "Attributes" and the *Design of Experiments Guide*.

Polynomial to Degree Creates main effects and polynomial terms up to a specified degree. Specify the degree in the **Degree** box beneath the **Macros** button.

Note: This option does not create higher degree terms for categorical effects.

Scheffé Cubic Creates main effects, interactions, and Scheffé cubic terms, which are useful in specifying response surfaces for mixture experiments. This macro creates a complete cubic model.

When you fit a third-degree polynomial model to a mixture, you must not introduce even-powered terms, for example, X1*X1*X2, because they are not estimable. However, it turns out that a complete polynomial specification of the surface can include terms of the form X1*X2*(X1 – X2), which are called *Scheffé cubic* terms.

Scheffé cubic terms are also included if you enter a 3 in the **Degree** box and then select the **Mixture Response Surface** macro command.

Partial Cubic Creates main effects, two-way interactions, quadratic terms, and interactions between main effects and quadratic terms. This macro is equivalent to adding interactions between main effects and quadratic terms to the terms defined by the Response Surface macro.

Note: This option does not create quadratic terms for categorical effects.

JMP PRO **Grouped Regressors** Creates a single effect for a set of continuous factors that are treated as one effect in various parts of the report. You can add or remove the grouped effect in the Effect Summary. There is one leverage plot and one effect test for each group effect, rather than individual plots and tests for each factor in the group. This can be useful if your data table contains indicator columns that represent levels of a categorical effect.

Note: Non-continuous factors included in a group of regressors are not included in the analysis.

Attributes

In the Attributes list, select attributes that you can assign to an effect selected in the Construct Model Effects list.

Random Effect Assigns the Random attribute to an effect. For more information about random effects, see "Specifying Random Effects and Fitting Method".

Response Surface Effect Assigns the **RS** attribute to an effect. Note that the relevant model terms must be included in the Construct Model Effects list. The Response Surface option in the Macros list automatically generates these terms and assigns the RS attribute to the main effects. To obtain the Response Surface report, interaction and polynomial terms do not need to have the RS attribute assigned to them. You need only assign this attribute to main effects.

LogVariance Effect Assigns the LogVariance attribute to an effect. This attribute indicates that the effect is to be included in a model of the variance of the response.

To include an effect in models for *both* the mean and variance of the response, you must specify the effect twice. In the tabbed interface, it must appear on both the Mean Effects and Variance Effects tabs. Otherwise, you can enter it twice on the Mean Effects tab, once without the LogVariance Effect attribute and once with the LogVariance Effect attribute.

Mixture Effect Assigns the Mixture attribute to main effects. This is used to specify the main effects involved in the mixture. Note that the Mixture Response Surface option in the Macros list automatically assigns the mixture attribute to selected effects, and provides a Response Surface report when possible.

Excluded Effect Assigns the Excluded attribute to an effect. This excludes the effect from the model fit. However, the effect is used to group observations for lack-of-fit tests. In the Standard Least Squares personality, a table of least square means is provided for this effect.

Knotted Spline Effect Assigns the Knotted attribute to a continuous main effect. This implicitly adds cubic splines for the effect to the model specification. See "Knotted Spline Effect".

Knotted Spline Effect

Knotted splines are used to fit a response Y using a flexible function of a predictor. Consider the single predictor X. When the Knotted Spline Effect is assigned to X, and k knots are specified, then k - 2 additional effects are implicitly added to the set of predictors. Each of these effects is a piecewise cubic polynomial spline whose segments are defined by the knots. See Stone and Koo (1985).

The number of splines is determined by the number of knots, which you are asked to specify. The coefficients associated with the splines are estimated based on the method used by the personality.

The placement of knots follows guidance given in the literature. In particular, if there are 100 or fewer points, the first and last knots are the fifth smallest and largest points, respectively. Otherwise, the first and last knots are placed at the 0.05 and 0.95 quantiles for 5 or fewer knots, or the 0.025 and 0.975 quantiles for more than 5 knots. The default number of knots is 5 for more than 30 observations, and 3 for fewer than 30 observations.

Note: Knotted splines are implemented only for main-effect continuous terms.

Transform

The Transform options transform selected Y columns or main effects that are selected in the Construct Model Effects text box.

Note: You can also transform a column by right-clicking it in the Select Columns list and selecting Transform. A reference to the transformed column appears in the Select Columns list. You can then use the column in the Fit Model window as you would any data table column. See *Using JMP*.

None Removes any Transform options that have been applied.

Log Applies the natural logarithm transformation to the selected variable.

Sqrt Applies the square root of the values of the selected variable.

Square Applies the square of the values of the selected variable.

Reciprocal Applies the transformation 1/X to the variable X.

Exp Applies the exponential transformation to the selected variable.

Arrhenius Applies the Arrhenius transformation to the variable T (temperature in degrees Centigrade):

$$X = \frac{11604.5181215503}{T + 273.15}$$

This is the component of the Arrhenius relationship that is multiplied by the activation energy.

ArrheniusInv Applies the inverse of the Arrhenius transformation to the variable X:

$$T = \frac{11604.5181215503}{X} - 273.15$$

Logit Calculates the inverse of the logistic function for the selected column (where p is in the range of 0 to 1):

$$\text{Logit}(p) = \log\left(\frac{p}{1-p}\right)$$

Logistic Calculates the logistic (also known as Squish and Logist) function for the selected column (where the result is in the range of 0 to 1):

$$\text{Logistic}(x) = \frac{1}{(1+e^{-x})}$$

LogitPct Calculates the logit as a percent for the selected column (where pct is a percent in the range of 0 to 100):

$$\text{LogitPct}(pct) = \log\left(\frac{\left(\frac{pct}{100}\right)}{1-\left(\frac{pct}{100}\right)}\right)$$

LogisticPct Calculates the logistic (or logist) as a percent for the selected column (where the result is in the range of 0 to 100):

$$\text{LogisticPct}(x) = \frac{100}{(1+e^{-x})}$$

No Intercept

Select No Intercept if you want to fit a model with no intercept term. Certain modeling structures require no intercept models. For these, the No Intercept box is checked by default.

Construct Model Effects Tabs

For the following personalities, you can enter model effects using a tabbed interface:

Note: If you apply Attributes to effects on the first (main) tab, the attributes determine how the effects are treated in the model. If you run the model and then request Model Dialog from the report's red triangle menu, you find that those effects appear on the appropriate tabs.

Standard Least Squares Enter model effects:

Fixed Effects tab Enter effects to be modeled as fixed effects. A fixed effect is one whose specific treatment levels are of interest. You want to compare the mean response across its treatment levels.

Random Effects tab Enter effects to be modeled as random effects. A random effect is one whose levels are considered a random sample from a larger population. You want to estimate the variation in the response that is attributable to this effect.

Mixed Model Enter model effects:

Fixed Effects tab Enter effects to be modeled as fixed effects. See Standard Least Squares in this table.

Random Effects tab Enter effects to be modeled as random effects. Use for variance component models and random coefficients models.

Repeated Structure tab Use to select a covariance structure for repeated effects.

Loglinear Variance Enter model effects:

Mean Effects tab Enter effects for which you want to model expected values.

Variance Effects tab Enter effects for which you want to model variance.

If you want to model both the expected value and variance of an effect, you must enter it on both tabs.

Parametric Survival Enter model effects:

Location Effects tab Enter effects that you want to use in modeling the location parameter, mu, or in the case of the Weibull distribution, the shape parameter.

Scale Effects tab Enter effects that you want to use in modeling the scale parameter.

Fitting Personalities

In the Fit Model launch window, select the fitting and analysis method by specifying a personality. Based on the response (or responses) and the factors that you enter, JMP makes an initial guess at the desired personality, but you can alter this selection in the Personality menu.

The following fitting personalities are available:

Standard Least Squares Fits models where the response is continuous. Techniques include regression, analysis of variance, analysis of covariance, mixed models, and analysis of designed experiments. See "Standard Least Squares Models" and "Emphasis Options for Standard Least Squares".

Stepwise Facilitates variable selection for standard least squares and ordinal logistic analyses (or nominal with a binary response). For continuous responses, cross validation, *p*-value, BIC, and AICc criteria are provided. Also provided are options for fitting all possible models and for model averaging. For logistic fits, *p*-value, BIC, and AICc criteria are provided. See "Stepwise Regression Models".

JMP PRO **Generalized Regression** Fits generalized linear models using regularized, also known as penalized, regression techniques. The regularization techniques include ridge regression, the lasso, the adaptive lasso, the elastic net, and the adaptive elastic net. The response distributions include the normal, binomial, Poisson, zero-inflated Poisson, negative binomial, zero-inflated negative binomial, and gamma. See "Generalized Regression Models" and "Specify a Distribution".

JMP PRO **Mixed Model** Fits a wide variety of linear models for continuous-responses with complex covariance structures. The situations addressed include:

- Split plot experiments
- Random coefficients models
- Repeated measures designs
- Spatial data
- Correlated response data

See "Mixed Models".

JMP PRO **Generalized Linear Mixed Model** Fits generalized linear mixed models for non-Gaussian response variables with random effects, such as blocking. The response distributions include the binomial and Poisson. See "Generalized Linear Mixed Models".

Manova Fits models that involve multiple continuous Y variables. Techniques include multivariate analysis of variance, repeated measures, discriminant analysis, and canonical correlations. See "Multivariate Response Models".

Loglinear Variance For a continuous Y variable, constructs models for both the mean and the variance. You can specify different sets of effects for the two models. See "Loglinear Variance Models".

Nominal Logistic Fits a logistic regression model to a nominal response. See "Logistic Regression Models".

Ordinal Logistic Fits a logistic regression model to an ordinal response. See "Logistic Regression Models".

Proportional Hazard Fits a semiparametric regression model (the Cox proportional hazards model) to assess the effect of explanatory variables on survival times, taking censoring into account.

You can also launch this personality by selecting **Analyze > Reliability and Survival > Fit Proportional Hazards**. See *Reliability and Survival Methods*.

Parametric Survival Fits a general linear regression model to survival times. Use this option if you have survival times that can be expressed as a function of one or more explanatory variables. Takes into account various survival distributions and censoring.

You can also launch this personality by selecting **Analyze > Reliability and Survival > Fit Parametric Survival**. See *Reliability and Survival Methods*.

Generalized Linear Model Fits generalized linear models using various distribution and link functions. Techniques include logistic, Poisson, and exponential regression. See "Generalized Linear Models".

JMP PRO **Partial Least Squares** Fits models to one or more *Y* variables using latent factors. This permits models to be fit when explanatory variables (*X* variables) are highly correlated, or when there are more *X* variables than observations.

You can also launch a partial least squares analysis by selecting **Analyze > Multivariate Methods > Partial Least Squares**. See *Multivariate Methods*.

Response Screening Automates the process of conducting tests for linear model effects across a large number of responses. Test results and summary statistics are presented in data tables and plots. A False-Discovery Rate (FDR) approach guards against incorrect declarations of significance. A robust estimation method reduces the sensitivity of tests to outliers.

Note: This personality allows only continuous responses. Response Screening for individual factors is also available by selecting **Analyze > Screening > Response Screening**. This platform supports categorical responses, and also provides equivalence tests and tests of practical significance. See *Predictive and Specialized Modeling*.

Model Specification Options

In the Fit Model launch window, the Model Specification red triangle menu contains the following options:

Center Polynomials Centers by its mean any continuous term that is involved in an effect with a degree greater than one. This option is checked by default, except when a term involved in the effect is assigned the Mixture Effect attribute or has the Mixture column property. Terms with the Coding column property are centered midway between their specified High and Low values.

Centering is useful in making regression coefficients more interpretable and in reducing collinearity between low-order and high-order effects.

Informative Missing Provides a coding system for missing values. This system allows estimation of a predictive model despite the presence of missing values. It is useful in situations where missing data are informative. See "Informative Missing".

This option is available for the following personalities: Standard Least Squares, Stepwise, Generalized Regression, MANOVA, Loglinear Variance, Nominal Logistic, Ordinal Logistic, Proportional Hazard, Parametric Survival, Generalized Linear Model, and Response Screening.

Suppress Coding (Available only if one or more of the effects columns contains a Coding column property.) Ignores any Coding column properties for the effects columns and fits a model using the uncoded effects. Estimates in the report are therefore shown on the original scale. This option is not recommended unless you need it.

Set Alpha Level Sets the alpha level for confidence intervals in the Fit Model analysis. The default alpha level is 0.05.

Error Specification (Available only for the Standard Least Squares personality when there are no random effects.) Specifies the error variance and the error degrees of freedom that are used for standard errors and tests in the Fit Least Squares report. Note that the Studentized Residuals plot and the Box Cox Transformations report are not affected by changing the Error Specification. When the Error Specification is Pure Error or Specified, an additional column appears in the Analysis of Variance report. See "Analysis of Variance".

Default Estimate Uses the standard root mean square error and error degrees of freedom from the model to calculate all tests and standard errors.

Pure Error Uses the Pure Error mean square and associated degrees of freedom from the Lack of Fit report to calculate all tests and standard errors. See "Lack of Fit".

Caution: If the pure error degrees of freedom is 1, a warning message is displayed indicating that tests are weak and confidence limits are large.

Specified Uses user-specified values for the error variance and error degrees of freedom to calculate all tests and standard errors.

Save to Data Table Saves your Fit Model launch window specifications as a script that is attached to the data table. The script is named Model. When a table contains a script called Model, this script automatically populates the launch window when you select **Analyze > Fit Model**. (Simply rename the script if this is not desirable.)

For more information about JSL scripting, see the *Scripting Guide*.

Save to Script Window Copies your Fit Model launch window specifications to a script window. You can save the script window and re-create the model at any time by running the script.

Create SAS job Creates a SAS program that can re-create the current analysis and data table in SAS in a script window. Once created, you have several options for submitting the code to SAS.

1. Copy and paste the code into the SAS Program Editor. This method is useful if you are running an older version of SAS (pre-version 8.2).
2. Select **Edit > Submit to SAS**.
3. Save the file and double-click it to open it in a local copy of SAS. This method is useful if you would like to take advantage of SAS ODS options, such as generating HTML or PDF output from the SAS code.

See *Using JMP*.

Submit to SAS Submits code to SAS and displays the results in JMP. If you are not connected to a SAS server, prompts guide you through the connection process.

See *Using JMP*.

Convergence Settings The Convergence Settings menu contains the following options:

Maximum Iterations Specifies the maximum number of iterations that are used in the model fitting. By default, the maximum number of iterations is 100. If your model does not readily converge, you might want to increase the number of iterations. If you have a very large data set or a complicated model, you might want to limit the number of iterations.

Convergence Limit Specifies the convergence limit for the model fitting. If you model does not readily converge, you might want to increase the convergence limit. By default, the convergence limit is 0.00000001.

Note: The Convergence Settings menu appears only for certain personalities. In the Standard Least Squares personality, the Convergence Settings menu appears only when there is a random effect and REML is selected as the Method in the launch window.

Options for Many Responses The Options for Many Responses menu contains options that enable you to suppress and consolidate results when a model has many responses:

Suppress Reports Specifies that the individual model reports are hidden. When there are thousands of responses, this option reduces computation time. The fitting objects and some menu items are still available.

Tip: Use the Results in Data Table option to collect the results from the model reports.

Results in Data Table Saves the individual model results across many responses into data tables. The contents and number of output data tables depends on the model being fit.

Dispose Reports Specifies that no individual model reports are shown and that they are removed from memory after fitting. When there are many thousands of responses, this option reduces computation time and saves memory.

Tip: Use this option with the Results in Data Table option to collect the results from the fitted models.

Note: The Options for Many Responses menu appears only when there are multiple responses in certain personalities.

Informative Missing

In the Fit Model launch window, the Informative Missing option constructs a coding system that allows estimation of a predictive model despite the presence of missing values. It codes both continuous and categorical model effects.

Continuous Effects

When a continuous main effect has missing values, a new design matrix column is created. This column is an indicator variable, with values of one if the main effect column is missing and zero if it is not missing. In addition, missing values for the continuous main effect are replaced with the mean of the nonmissing values for rows included in the analysis. The mean is a neutral value that maintains the interpretability of parameter estimates.

The parameter associated with the indicator variable estimates the difference between the response predicted by the missing value grouping and the predicted response if the covariate is set at its mean.

For a higher-order effect, missing values in the covariates are replaced by the covariate means. This makes the higher-order effect zero for rows with missing values, assuming that Center Polynomials is checked (the default setting). This is because Center Polynomials centers the individual terms involved in a polynomial by their means.

In the Effect Tests report, each continuous main effect with missing values has Nparm = 2. In the Parameter Estimates report, the parameter for a continuous main effect with missing values is labeled <colname> Or Mean if Missing and the indicator parameter is labeled <colname> Is Missing. Prediction formulas that you save to the data table are given in terms of expressions corresponding to these model parameters.

Categorical Effects

When a nominal or ordinal main effect has missing values, the missing values are coded as a separate level of that effect. As such, in the Effect Tests report, each categorical main effect with missing values has one additional parameter.

In the Parameter Estimates report, the parameter for a nominal effect is labeled <colname>[ ]. For an ordinal effect, the parameter is labeled <colname>[-x], where x denotes the level with highest value ordering.

As with continuous effects, prediction formulas that you save to the data table are given in terms of expressions corresponding to the model parameters.

Coding Table

When you are using the Standard Least Squares personality, you can view the design matrix columns used in the Informative Missing model by selecting **Save Columns > Save Coding Table.**

Validity Checks

The Fit Model platform checks your model for errors, such as duplicate effects or missing effects in a hierarchy. If you receive an alert message, you can either click **Continue** to proceed with fitting, or click **Cancel** to stop the fitting process.

Model Specification Templates

To obtain specific types of models, enter the correct effects in the Construct Model Effects panel of the Fit Model launch window.

In this section, the model effects X and Z represent continuous columns and the model effects A, B, and C represent nominal or ordinal columns.

Basic steps for constructing model effects are available for the following models:

- "Simple Linear Regression"
- "Polynomial in X to Degree k"
- "Polynomial in X and Z to Degree k"
- "Multiple Linear Regression"
- "One-Way Analysis of Variance"
- "Two-Way Analysis of Variance"
- "Two-Way Analysis of Variance with Interaction"
- "Three-Way Full Factorial"
- "Analysis of Covariance, Equal Slopes"
- "Analysis of Covariance, Unequal Slopes"
- "Two-Factor Nested Random Effects Model"
- "Three-Factor Fully Nested Random Effects Model"
- "Simple Split Plot or Repeated Measures Model"
- "Two-Factor Response Surface Model"
- "Knotted Spline Effect"

Simple Linear Regression

Effects to be entered: X

1. In the Select Columns list, select X.
2. Click **Add**.

Note: For an example of a simple linear regression model, see "Example of Simple Linear Regression".

Polynomial in X to Degree k

Effects to be entered: X, X*X,..., X^k

1. Type *k* into the text box for **Degree**.
2. In the Select Columns list, select X.
3. Select **Macros > Polynomial to Degree**.

Note: For an example of a polynomial model in X to degree k, see "Example of a Polynomial Effects Model".

Polynomial in X and Z to Degree k

Effects to be entered: X, X*X,..., X^k, Z, Z*Z,..., Z^k

1. Type *k* into the text box for **Degree**.
2. In the Select Columns list, select X and Z.
3. Select **Macros > Polynomial to Degree**.

Multiple Linear Regression

Effects to be entered: Selected columns

1. In the Select Columns list, select the continuous effects of interest.
2. Click **Add**.

Note: For an example of multiple linear regression with several predictors, see "Example of a Regression Analysis Using Fit Model".

One-Way Analysis of Variance

Effects to be entered: A

1. In the Select Columns list, select one nominal or ordinal effect, A.
2. Click **Add**.

Note: For an example, of one-way analysis of variance (ANOVA), see "Example of One-Way Analysis of Variance".

Two-Way Analysis of Variance

Effects to be entered: A, B

1. In the Select Columns list, select two nominal or ordinal effects, A and B.
2. Click **Add**.

Note: For an example of a two-way analysis of variance model, see "Example of Two-Way Analysis of Variance".

Two-Way Analysis of Variance with Interaction

Effects to be entered: A, B, A*B

1. In the Select Columns list, select two nominal or ordinal effects, A and B.
2. Select **Macros > Full Factorial**.

Or:

1. In the Select Columns list, select two nominal or ordinal effects, A and B.
2. Click **Add**.
3. In the Select Columns list, select A and B again and click **Cross**.

Note: For an example of a two-way analysis of variance model with an interaction effect, see "Example of Two-Way Analysis of Variance with an Interaction".

Three-Way Full Factorial

Effects to be entered: A, B, C, A*B, A*C, B*C, A*B*C

1. In the Select Columns list, select three nominal or ordinal effects, A, B, and C.
2. Select **Macros > Full Factorial**.

Note: For an example of a three-way full factorial model, see "Example of a Three-Way Full Factorial Model".

Analysis of Covariance, Equal Slopes

Test for the effect of A with X as a covariate. Suppose that you have reason to believe that the effect of X on the response does not depend on the level of A.

Effects to be entered: A, X

1. In the Select Columns list, select one nominal or ordinal effect, A, and one continuous effect, X.
2. Click **Add**.

Note: For an example of analysis of covariance with equal slopes, see "Example of Analysis of Covariance with Equal Slopes".

Analysis of Covariance, Unequal Slopes

Test for the effect of A with X as a covariate. Suppose that you construct your model to allow the possibility that the effect of X on the response depends on the level of A.

Effects to be entered: A, X, A*X

1. In the Select Columns list, select one nominal or ordinal effect, A, and one continuous effect, X.
2. Select **Macros > Full Factorial**.

Or:

1. In the Select Columns list, select one nominal or ordinal effect, A, and one continuous effect, X.
2. Click **Add**.
3. In the Select Columns list, select A and X again and click **Cross**.

Note: For an example of analysis of covariance with unequal slopes, see "Example of Analysis of Covariance with Unequal Slopes".

Two-Factor Nested Random Effects Model

Consider a model with two factors, A and B, but where B is nested within A. Although there are situations where a nested effect is treated as a fixed effect, in most situations a nested effect is treated as a random effect. For this reason, in the model described below, the nested effect is entered as a random effect.

Effects to be entered: A, B[A]&Random

1. In the Select Columns list, select two nominal or ordinal effects, A and B.
2. Click **Add**.
3. To nest B within A: In the Construct Model Effects list, select B. In the Select Columns list, select A. The two effects should be highlighted.

4. Click **Nest**.
5. With B[A] highlighted in the Construct Model Effects list, select **Attributes > Random Effect**.

Note: For an example of a two-factor nested random effects model, see "Example of a Two-Factor Nested Random Effects Model".

Three-Factor Fully Nested Random Effects Model

Consider a model with three factors, A, B, and C, but where B is nested within A and C is nested within both A and B. Also consider B and C to be random effects.

Effects to be entered: A, B[A]&Random, C[A,B]&Random

1. In the Select Columns list, select three nominal or ordinal effects, A, B, and C.
2. Click **Add**.
3. To nest B within A: In the Construct Model Effects list, select B. In the Select Columns list, select A. The two effects should be highlighted.
4. Click **Nest**.
5. To nest C within A and B: In the Construct Model Effects list, select C. In the Select Columns list, select A and B. The three effects should be highlighted.
6. Click **Nest**.
7. With both B[A] and C[A,B] highlighted in the Construct Model Effects list, select **Attributes > Random Effect**.

Simple Split Plot or Repeated Measures Model

Effects to be entered: A, B[A]&Random, C, C*A where A is the whole plot variable, B[A] is the whole plot ID, and C is the split plot, or repeated measures, variable.

1. In the Select Columns list, select two nominal or ordinal effects, A and B.
2. Click **Add**.
3. To nest B within A: In the Construct Model Effects list, select B. In the Select Columns list, select A. The two effects should be highlighted.
4. Click **Nest**.
5. In the Construct Model Effects list, select B[A].
6. Select **Attributes > Random Effect**.
7. In the Select Columns list, select a third nominal or ordinal effect, C.
8. Click **Add**.

9. In the Construct Model Effects list, select C. In the Select Columns list, click A. Both effects should be highlighted.
10. Click **Cross**.

Note: For an example of a simple repeated measures model, see "Example of a Simple Repeated Measures Model".

Two-Factor Response Surface Model

Effects to be entered: X&RS, Z&RS, X*X, X*Z, Z*Z

1. In the Select Columns list, select two continuous effects, X and Z.
2. Select **Macros > Response Surface**.

Knotted Spline Effect

Effects to be entered: X&Knotted

1. In the Select Columns list, select a continuous effect, X.
2. Click **Add**.
3. Select X in the Construct Model Effects list.
4. Select **Attributes > Knotted Spline Effect**.
5. In the menu that appears, specify the number of knots or accept the default number.
6. Click **OK**.

Note: For an example of a model with a knotted spline effect, see "Example of a Knotted Spline Effect".

Chapter 3

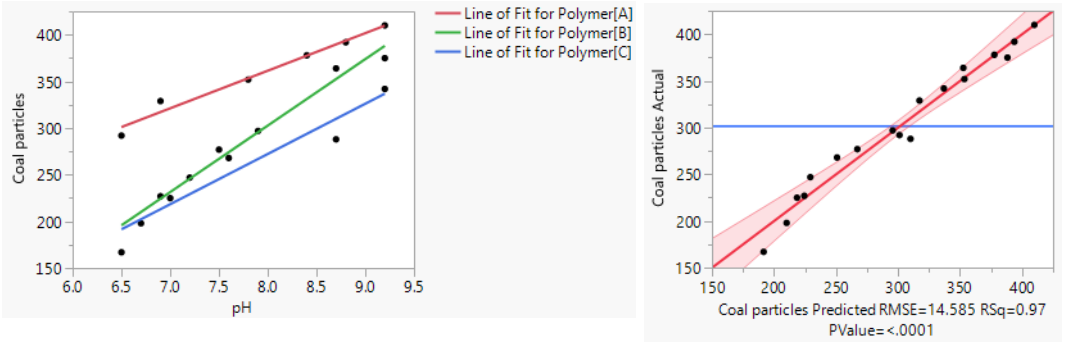
Standard Least Squares Models

Analyze Common Classes of Models

The Standard Least Squares personality of the Fit Model platform fits a wide spectrum of standard models. These models include regression, analysis of variance, analysis of covariance, and mixed models, as well as the models typically used to analyze designed experiments. Use the Standard Least Squares personality to construct linear models for continuous-response data using least squares or, in the case of random effects, restricted maximum likelihood (REML).

Analytic results are supported by compelling dynamic visualization tools such as profilers, contour plots, and surface plots (see *Profilers*). These visual displays stimulate, complement, and support your understanding of the model. They enable you to optimize several responses simultaneously and to explore the effect of noise.

Figure 3.1 Examples of Standard Least Squares Plots



Contents

Example Using Standard Least Squares . . . 62
Launch the Standard Least Squares Personality . . . 65
Fit Model Launch Window . . . 66
Standard Least Squares Options in the Fit Model Launch Window . . . 67
Validation in Standard Least Squares . . . 69
Missing Values . . . 69
Fit Least Squares Report . . . 70
Single versus Multiple Responses . . . 70
Report Structure Related to Emphasis . . . 71
Special Reports . . . 71
Least Squares Fit Options . . . 75
Fit Group Options . . . 75
Response Options . . . 76
Regression Reports . . . 77
Summary of Fit . . . 78
Analysis of Variance . . . 79
Parameter Estimates . . . 80
Effect Tests . . . 82
Effect Details . . . 83
Lack of Fit . . . 98
Estimates . . . 100
Show Prediction Expression . . . 102
Sorted Estimates . . . 103
Expanded Estimates . . . 107
Indicator Parameterization Estimates . . . 109
Sequential Tests . . . 110
Custom Test . . . 111
Compare Slopes . . . 114
Joint Factor Tests . . . 114
Inverse Prediction . . . 115
Cox Mixtures . . . 120
Parameter Power . . . 121
Correlation of Estimates . . . 123
Effect Screening . . . 124
Scaled Estimates and the Coding of Continuous Terms . . . 125
Effect Screening Plot Options . . . 126
Normal Plot Report . . . 131
Bayes Plot Report . . . 133
Pareto Plot Report . . . 134

Factor Profiling . . . 135
Profiler . . . 136
Interaction Plots . . . 137
Contour Profiler . . . 138
Mixture Profiler . . . 139
Cube Plots . . . 140
Box Cox Y Transformation . . . 141
Surface Profiler . . . 143
Row Diagnostics . . . 144
Effect Leverage Plots . . . 146
Press . . . 150
Save Columns . . . 151
Prediction Formula . . . 154
Multiple Comparisons . . . 155
Launch the Multiple Comparisons Option . . . 155
Comparisons with Overall Average . . . 159
Comparisons with Control . . . 162
All Pairwise Comparisons . . . 164
Equivalence Tests . . . 168
Effect Summary Report . . . 170
Mixed and Random Effect Model Reports and Options . . . 174
Mixed Models and Random Effect Models . . . 175
Restricted Maximum Likelihood (REML) Method . . . 179
EMS (Traditional) Model Fit Reports . . . 184
Models with Linear Dependencies among Model Terms . . . 186
Singularity Details . . . 186
Parameter Estimates Report . . . 187
Effect Tests Report . . . 188
Examples of Models with Linear Dependencies . . . 188
Statistical Details for the Standard Least Squares Personality . . . 191
Statistical Details for Emphasis Rules . . . 191
Statistical Details for the Custom Test Example . . . 191
Statistical Details for Correlation of Estimates . . . 192
Statistical Details for Nominal Effects Coding . . . 193
Statistical Details for Leverage Plots . . . 194
Statistical Details for the Kackar-Harville Correction . . . 197
Statistical Details for Power Analysis . . . 198

Example Using Standard Least Squares

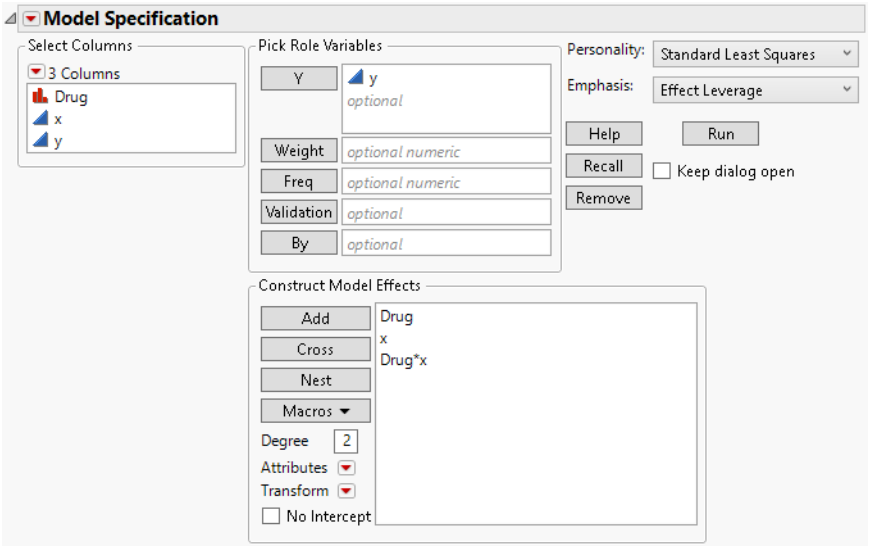
This example shows how to use the Standard Least Squares personality of the Fit Model platform to fit an analysis of covariance model. In a study of the effect of drugs in treating a disease, thirty patients are randomly divided into three groups of ten. Two of these groups are administered drugs (Drug a and Drug d), whereas the third group is administered a placebo (Drug f). A pretreatment measure, x, is taken on each patient, as well as a posttreatment measure, y. The pretreatment score, x, is included as a covariate, to account for differences in the stage of the disease among patients.

You are interested in determining if there is a difference in the three Drug groups. You construct a model with response y and model effects Drug, x, and the interaction of Drug and x. The interaction might account for a situation where drugs have differential effects, based on the stage of the disease. For background on the Fit Model window and the various personalities, see "Model Specification".

1. Select **Help > Sample Data Folder** and open Drug.jmp.
2. Select **Analyze > Fit Model**.
3. Select y and click **Y**.

   When you add this column as **Y**, the fitting **Personality** becomes Standard Least Squares. An **Emphasis** option is added with a selection of Effect Leverage, which you can change if desired.
4. Select Drug and x. With these two effects highlighted in the Select Columns list, click **Macros** and select **Full Factorial**. The macro adds the two effects and their two-way interaction to the Construct Model Effects list.

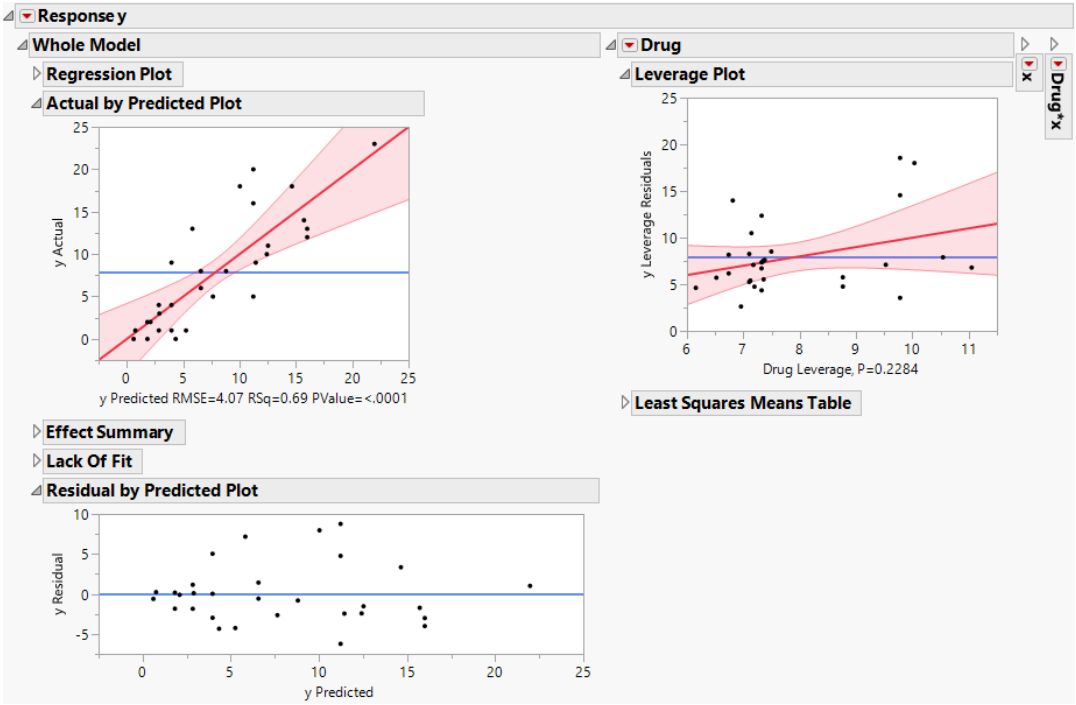
Figure 3.2 Completed Fit Model Launch Window



5. Click **Run**.

The Fit Least Squares report is shown in Figure 3.3. Note that some of the constituent reports are closed because of space considerations. The Actual by Predicted, Residual by Predicted, and Leverage plots show no discrepancies in terms of model fit and underlying assumptions.

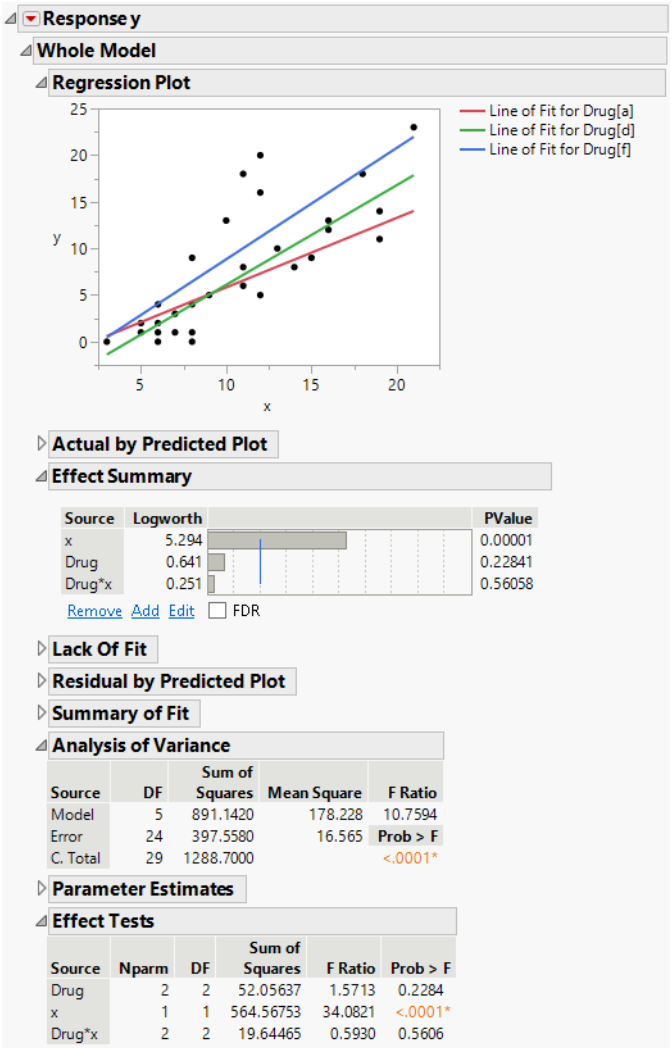
Figure 3.3 Fit Least Squares Report Showing Plots to Assess Model Fit



Since there are no apparent problems with the model fit, you can now interpret the statistical tests. Figure 3.4 shows the relevant reports. The overall model is significant, as shown in the Analysis of Variance report.

Although the Regression Plot suggests that Drug and the pretreatment measure, x, interact, the Prob > F value in the Effect Tests report does not support that conclusion. The Effect Tests report also shows that x is significant in explaining y, but Drug is not significant. The study does not detect a difference among the three groups. However, you cannot conclude that Drug has no effect. The drugs might have different effects, but the study size was not large enough to detect that difference.

Figure 3.4 Fit Least Squares Report Showing Reports to Assess Significance



Launch the Standard Least Squares Personality

Standard least squares is one of several analytic techniques that you can select in the Fit Model launch window. This section describes how you select standard least squares as your fitting methodology in the Fit Model launch window. Options that are specific to this selection are also covered. For more information about the options in the Select Columns red triangle menu, see *Using JMP*.

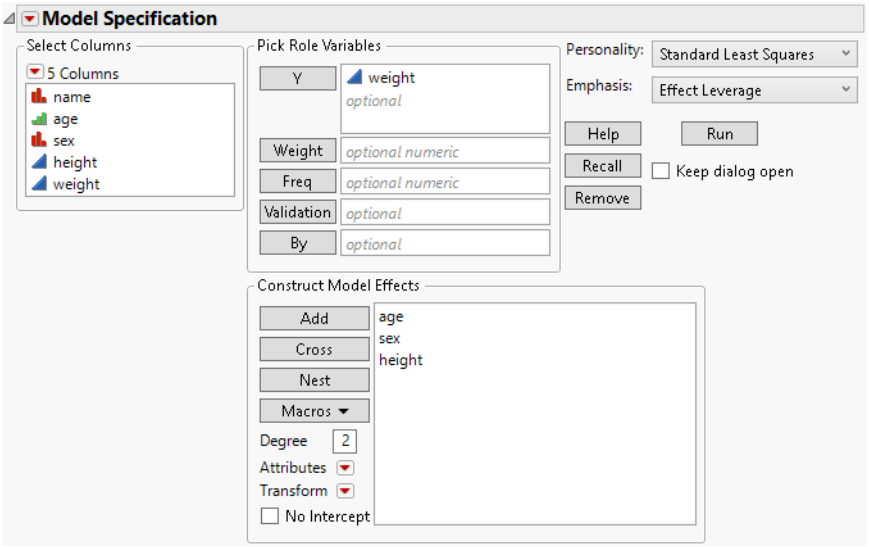
Fit Model Launch Window

You can specify models with both fixed and random effects in the Fit Model launch window. The options differ based on the nature of the model that you specify.

Fixed Effects Only

To fit models using the standard least squares personality, select **Analyze > Fit Model** and then select **Standard Least Squares** from the Personality list. When you enter one or more continuous variables in the Y list, the Personality defaults to Standard Least Squares. Note, however, that other selections are available for continuous Y variables. When you specify only fixed effects for a Standard Least Squares fit, the Fit Model launch window appears as shown in Figure 3.5. This example illustrates the launch window using the Big Class.jmp sample data table.

Figure 3.5 Fit Model Launch Window for a Fixed Effects Model



When the Standard Least Squares personality is selected in the Personality list, an Emphasis option also appears. Emphasis options control the reports that are provided in the initial report window. Based on the model effects that are included, JMP infers which reports you are likely to want. However, any report not shown as part of the initial report can be shown by selecting the appropriate option from the default report's red triangle menu.

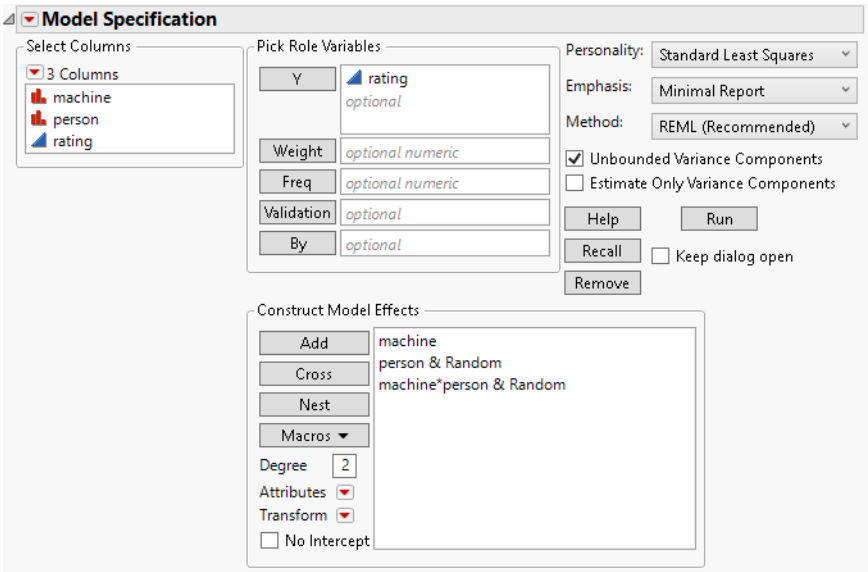
For more information about reports that are available for each Emphasis option, see "Emphasis Options for Standard Least Squares".

Random Effects

If the specified model contains one or more random effects, then additional options become available in the Fit Model launch window. Consider the Machine.jmp sample data table. Each of six randomly chosen workers performs work at each of three machines and their output is rated. You are interested in estimating the variation in ratings across the workforce, rather than in determining whether these six specific workers' ratings differ. You need to treat person and machine*person as random effects when you specify the model.

The Fit Model launch window for this model is shown in Figure 3.6. When the Random Effect attribute is applied to person, a Method option and two options relating to variance components appear in the Fit Model Launch window.

Figure 3.6 Fit Model Launch Window for a Model Containing a Random Effect



Standard Least Squares Options in the Fit Model Launch Window

In the Fit Model launch window, the following options are specific to the Standard Least Squares personality.

Emphasis Controls the types of reports and plots that appear in the initial report window. See "Emphasis Options for Standard Least Squares".

Method (Appears only when random effects are specified.) Estimates the model using one of these methods:

REML See "REML Variance Component Estimates".

EMS Expected Mean Squares, also called the Method of Moments. See "EMS (Traditional) Model Fit Reports".

Unbounded Variance Components (Appears only when REML is selected as the Method.) Allows variance component estimates to be negative. This option is selected by default. This option should remain selected if you are interested in fixed effects, since bounding the variance estimates at zero leads to bias in the tests for fixed effects. See "Negative Variances".

Estimate Only Variance Components (Appears only when REML is selected as the Method.) Provides a report that shows only variance component estimates. See "Estimate Only Variance Components".

Fit Separately (Appears only for models with multiple Y variables and no random effects.) Fits a separate model for each Y variable using all rows that are nonmissing. See "Missing Values".

Emphasis Options for Standard Least Squares

The three options in the Emphasis list control the types of plots and reports that you see as part of the initial report for the Standard Least Squares personality. See the descriptions below. JMP chooses a default emphasis based on the number of rows in the data table, the number of effects entered in the Construct Model Effects list, and the attributes applied to effects. You can change this choice of emphasis based on your needs. For more information about how JMP chooses the emphasis, see "Statistical Details for Emphasis Rules".

After the initial report opens, you can add other reports and plots from the red triangle menu in the platform report window.

The following emphasis options are available:

Effect Leverage Shows leverage and residual plots, as well as reports with details about the model fit. This option is useful when your main focus is model fitting.

Effect Leverage is the most comprehensive option. This emphasis divides reports into those that relate to the Whole Model and those that relate to individual model effects. The Whole Model reports are in the left corner of the report window under the Whole Model title, with effect reports to the right.

Effect Screening Shows a sorted or scaled parameter estimates report along with a graph (when appropriate), the Prediction Profiler, and reports with details about the model fit. This option is useful when you have many effects and your initial focus is to discover which effects are active, as in screening designs.

When Effect Screening is selected, a Box-Cox transformation is calculated. If the confidence interval for the estimated λ does not contain 1, the Box-Cox Transformations report appears. See "Box Cox Y Transformation".

Minimal Report Shows only the regression plot and reports with details about the model fit. This Emphasis is the default when the Random Effect attribute is applied to any model effect.

This option is the least detailed and most concise. You can request reports of specific interest to you from the red triangle menus.

To change which reports or plots appear for all of the Emphasis options, use platform preferences. Go to **File > Preferences > Platforms > Fit Least Squares**, and use the **Set** check boxes:

- To prevent an option from appearing in the report, next to an option, select **Set** but do not select the option.
- To ensure that an option appears in the report, select **Set** and select the option.

JMP PRO Validation in Standard Least Squares

In JMP Pro, you can specify a Validation column in the Fit Model window. A validation column must have a numeric data type and should contain at least two distinct values.

- If the column contains two values, the smaller value defines the training set and the larger value defines the validation set.
- If the column contains three values, the values define the training, validation, and test sets in order of increasing size.
- If the column contains four or more distinct values, the Validation column is ignored.

The Standard Least Squares personality of the Fit Model platform in JMP Pro supports the use of a Validation column. If you enter a Validation column, a Crossvalidation report is provided. See "Crossvalidation Report".

If you enter a Validation column, observations from the Validation and Test sets are marked as 'v' and 't', respectively, in plots in the report.

For more information about how a Validation column is used in JMP modeling platforms, see *Predictive and Specialized Modeling*.

Missing Values

By default, rows that have missing values for Y or any model effects are excluded from the Standard Least Squares analysis.

Note: JMP provides an Informative Missing option in the Fit Model window in the Model Specification red triangle menu. Informative Missing enables you to fit models using rows where model effects are missing. See "Informative Missing".

When your model contains a random effect, Y values are fit separately by default. The individual reports appear in the Fit Group report.

Suppose that your model contains only fixed effects, and the following statements are true:

- You specified more than one Y response.
- Some of these Y responses have missing values.
- You did not select the Fit Separately option in the Fit Model launch window.

Then, JMP prompts you to select one of the following options:

- Fit Separately fits each Y using all rows that are nonmissing for that particular Y.
- Fit Together fits each Y uses only those rows that are nonmissing for all of the Y variables.

When you select Fit Separately, a Fit Group report contains the individual reports for the Y variables. You can select profilers from the Fit Group red triangle menu to view all the Y variables in the same profiler. Alternatively, you can select a profiler from an individual Y variable report to view only that variable in the profiler.

When you select Fit Together, a Fit Least Squares report contains individual reports for each of the Y variables. However, some parts of the report are combined for all Y variables: the Effect Summary and the Profilers.

Fit Least Squares Report

When you fit a model using the Standard Least Squares personality, you obtain a Fit Least Squares report. The content of the report is driven by the nature of the data and your selections in the Fit Model launch window.

Tip: To always see reports that do not appear by default, select them using **File > Preferences > Platforms > Fit Least Squares**.

Single versus Multiple Responses

When you fit a single response variable Y, the Fit Least Squares window organizes detailed reports in a report entitled “Response Y”. When you fit several responses, reports for individual responses are usually organized in a report entitled “Least Squares Fit”. However, if there is missing response data, and you select the option to Fit Separately, reports for individual responses are organized in a report titled “Fit Group”.

Report Structure Related to Emphasis

When you select the Effect Leverage Emphasis option for Standard Least Squares in the Fit Model launch window, the report for a given response is arranged in columns. The left column consists of the Whole Model report, which contains additional reports that pertain to the model. Reports for each effect in the model are shown in the columns to the right of the Whole Model report.

When you select either the Effect Screening or Minimal Report Emphasis in the Fit Model launch window, all reports for each response are arranged in the left column.

Special Reports

In the Fit Least Squares report, the Special Reports section contains reports that are available based on the data structure or choices that you made regarding effect attributes.

Singularity Details

When there are linear dependencies among model effects, the Singularity Details report appears as the first report under the Response report title. It contains a table of the linear functions that the model terms satisfy. These functions define the aliasing relationships among model terms. Figure 3.7 shows an example for the Singularity.jmp sample data table.

Figure 3.7 Singularity Details Report

Singularity Details

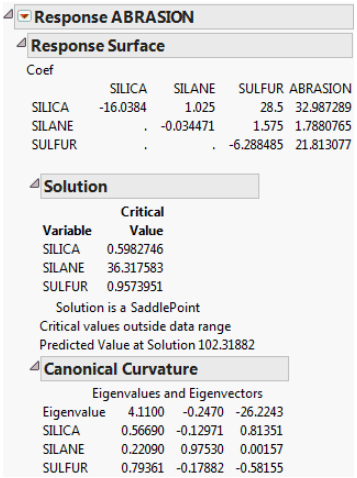
Term	Details
X1	= - X2 + X3=2*Intercept + 3*X2 - 4*A[a] + 2*A[b]= - X2 + 2*A[a] - 2*A[c]

When there are linear dependencies among effects, estimates of some model terms are not unique. See “Models with Linear Dependencies among Model Terms”.

Response Surface Report

When an effect in a model has the response surface (&RS) or mixture response surface (&RS&Mixture) attribute, a Response Surface report is provided. See Figure 3.8 for an example of a Response Surface report for the Tiretread.jmp sample data table.

Figure 3.8 Response Surface Report



Coef Table

The Coef table shown as the first part of the Response Surface report gives a concise summary of the estimated model parameters. The first columns give the coefficients of the second-order terms. The last column gives the coefficients of the linear terms. To see the prediction expression in its entirety, select **Estimates** > **Show Prediction Expression** from the report's red triangle menu.

Solution Report

The Solution report gives a critical value (maximum, minimum, or saddle point), if one exists, along with the predicted value at that point. It also alerts you if the solution falls outside the range of the data.

Canonical Curvature Report

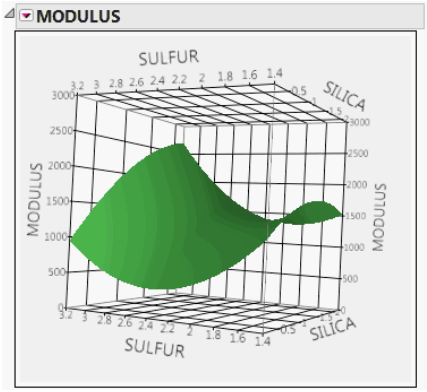
The eigenvalues and eigenvectors of the matrix of second-order parameter estimates determine the type of curvature. The eigenvectors show the principal directions of the surface, including the directions of greatest and smallest curvature.

The eigenvalues are provided in the first row of the Canonical Curvature table.

- If the eigenvalues are negative, the response surface curves downward from a maximum.

- If the eigenvalues are positive, the surface shape curves upward from a minimum.
- If there are both positive and negative eigenvalues, the surface is saddle shaped, curving up in one direction and down in another direction. See Figure 3.9 for an example using the Tiretread.jmp sample data table.

Figure 3.9 Surface Profiler Plot with Saddle-Shaped Surface



The eigenvectors listed below the eigenvalues show the orientation of the principal axes. The larger the absolute value of an eigenvalue, the greater the curvature of the response surface in its associated direction. Sometimes a zero eigenvalue occurs. This eigenvalue means that, along the direction described by the corresponding eigenvector, the fitted surface is flat.

Note: The response surface report is not shown for response surface models consisting of more than 20 factors. No error message or alert is given. For more information about response surface designs, see the *Design of Experiments Guide*.

Mixed and Random Effect Model Reports

When you specify a random effect in the Fit Model launch window, the Method list appears. This list provides two fitting methods: REML (Recommended) and EMS (Traditional). Additional reports as well as Save Columns and Profiler options are shown, based on the model and the method that you select.

For more information about the REML method reports, see "Restricted Maximum Likelihood (REML) Method". For more information about the EMS method reports, see "EMS (Traditional) Model Fit Reports".

Crossvalidation Report

JMP PRO When you enter a Validation column in the Fit Model launch window, a Crossvalidation report is provided. The report gives the following for each of the sets used in validation:

Source Identifies the set as the Training, Validation, or Test set.

RSquare The RSquare value calculated for observations in the given set relative to the model derived using the Training Set. For the Training Set, this is the usual RSquare value.

For each of the Training, Validation, and Test sets, the RSquare value is computed as follows:

- For each observation in the given set, compute the prediction error. This is the difference between the actual response and the response predicted by the Training set model.
- Square and sum the prediction errors to obtain SSE_{Source}, where the subscript *Source* denotes any of the Training, Validation, or Test sets.
- Square and sum the differences between the actual responses for observations in the *Source* set and their mean. Denote this value by SST_{Source}.
- RSquare for the *Source* set is:

$$\text{RSquare}_{Source} = 1 - \frac{SSE_{Source}}{SST_{Source}}$$

Note: It is possible for RSquare values for the Validation and Test sets to be negative.

RASE The square root of the mean squared prediction error. For each of the Training, Validation, and Test sets, RASE is computed as follows:

- For each observation in the given set, calculate the prediction error. This is the difference between the actual response and the response predicted by the Training set model.
- Square and sum the prediction errors to obtain SSE_{Source}, where the subscript *Source* denotes any of the Training, Validation, or Test sets.
- Denote the number of observations by n.
- RASE is:

$$\text{RASE}_{Source} = \sqrt{\frac{SSE_{Source}}{n}}$$

Freq The number of observations in the Source set.

Least Squares Fit Options

In the Fit Model launch window, when you select the Fit Together option for Standard Least Squares, the responses are grouped in a report called Least Squares Fit. This is useful if you have more than one Y and no missing response values, or more than one Y with missing values. The Least Squares Fit red triangle menu includes the following options:

Profilers Shows all responses in a single profiler. You can view the effects of model terms on all responses simultaneously and perform multiple optimization. See "Factor Profiling".

Model Dialog Shows the completed launch window for the current analysis.

Effect Summary Shows the interactive Effect Summary report that enables you to add or remove effects from the model. See "Effect Summary Report".

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Save By-Group Script Contains options that enable you to save a script that reproduces the platform report for all levels of a By variable to several destinations. Available only when a By variable is specified in the launch window.

Fit Group Options

In the Fit Model launch window, when you have more than one Y and you select the Fit Separately option for Standard Least Squares, the responses are grouped in a report called Fit Group. The red triangle menu includes the following options:

Profiler Shows all responses in a single profiler. You can view the effects of model terms on all responses simultaneously and perform multiple optimization. See "Profiler".

Contour Profiler Shows all responses in a single contour profiler. You can explore the effects of model terms on all responses simultaneously.

Surface Profiler Shows separate surface profiler reports for each response.

Arrange in Rows Rearranges the reports for the platform analyses in a specified number of rows. This would be used mostly to arrange reports so that more reports fit in a window or on the page of output.

Order by Goodness of Fit Sorts the reports by significance of fit (RSquare). This option is available only for platforms that surface the RSquare to the platform level. For example, if you generate hundreds of Oneway analyses from one launch window, they appear in a `FitGroup` and you can sort them so that the strongest relationships appear first.

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Response Options

In the Fit Least Squares report, the red triangle menu options for the response give you the ability to customize reports according to your needs.

Regression Reports Provides basic reports and report options. See "Regression Reports".

Estimates Provides options for further analyses relating to parameter estimates. See "Estimates".

Effect Screening Provides reports and plots for identifying significant effects. See "Effect Screening".

Factor Profiling Provides profilers, interaction, and cube plots to examine how the response is related to the model terms. Also provides a plot and report for fitting a Box-Cox transformation. See "Factor Profiling".

Row Diagnostics Provides plots and reports for examining residuals. Also reports the PRESS statistic and provides a Durbin-Watson test. See "Row Diagnostics".

Save Columns Saves model results as columns in the data table, except for Save Coding Table, which saves results in a separate data table. See "Save Columns".

Model Dialog Shows the completed Fit Model launch window for the current analysis.

Effect Summary Shows the interactive Effect Summary report that enables you to add or remove effects from the model. See "Effect Summary Report".

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Save By-Group Script Contains options that enable you to save a script that reproduces the platform report for all levels of a By variable to several destinations. Available only when a By variable is specified in the launch window.

Regression Reports

In the Fit Least Squares report, the Regression Reports options (accessed from the Response red triangle menu) provide summary information about model fit, effect significance, and model parameters.

Summary of Fit Shows or hides a summary of model fit. See "Summary of Fit".

Analysis of Variance Shows or hides calculations for comparing the fitted model to a simple mean model. See "Analysis of Variance".

Parameter Estimates Shows or hides a report containing the parameter estimates and *t* tests for the hypothesis that each parameter is zero. See "Parameter Estimates".

Effect Tests Shows or hides tests for the fixed effects in the model. See "Effect Tests".

Effect Details Shows or hides a report containing details, plots, and tests for individual effects. See "Effect Details".

When the Effect Leverage Emphasis option is selected, each effect has its own report at the top of the Fit Least Squares report window. This report includes effect details options as well as a leverage plot. See "Effect Leverage Plots".

Lack of Fit Shows or hides a test assessing if the model has the appropriate effects, when that test can be conducted. See "Lack of Fit".

Show All Confidence Intervals Shows or hides confidence intervals for the following statistics:

- Parameter estimates in the Parameter Estimates report
- Least squares means in the Least Squares Means Table

AICc Shows or hides AICc and BIC values in the Summary of Fit report. See "Likelihood, AICc, and BIC".

Summary of Fit

In the Fit Least Squares report, the Summary of Fit option provides details such as RSquare calculations and the AICc and BIC values.

RSquare Estimates the proportion of variation in the response that can be attributed to the model rather than to random error. Using quantities from the corresponding Analysis of Variance table, RSquare (also called the *coefficient of multiple determination)* is calculated as follows:

$$\frac{\text{Sum of Squares(Model)}}{\text{Sum of Squares(C. Total)}}$$

An RSquare closer to 1 indicates a better fit to the data than does an RSquare closer to 0. An RSquare near 0 indicates that the model is not a much better predictor of the response than is the response mean.

Note: A low RSquare value suggests that there might be variables not in the model that account for the unexplained variation. However, if your data are subject to a large amount of inherent variation, even a useful regression model can have a low RSquare value. Read the literature in your research area to learn about typical RSquare values.

Rsquare Adj Adjusts the RSquare statistic for the number of parameters in the model. Rsquare Adj facilitates comparisons among models with different numbers of parameters. The computation uses the degrees of freedom. Using quantities from the corresponding Analysis of Variance table, RSquare Adj is calculated as follows:

$$1 - \frac{\text{Mean Square(Error)}}{\text{Sum of Squares (C. Total)}/\text{DF(C. Total)}}$$

Root Mean Square Error Estimates the standard deviation of the random error. This quantity is the square root of the Mean Square for Error in the Analysis of Variance report.

Note: Root Mean Square Error is commonly known as *RMSE*.

Mean of Response Shows the overall mean of the response values.

Observations (or Sum Wgts) Shows the number of observations used in the model.

- This value is the same as the number of rows in the data table under the following conditions: there are no missing values, no excluded rows, and no column assigned to the role of Weight or Freq.
- This value is the sum of the positive values in the Weight column if there is a column assigned to the role of Weight.
- This value is the sum of the positive values in the Freq column if there is a column assigned to the role of Freq.

AICc (Appears only if you have selected the AICc option from the Regression Reports menu or if you have set AICc as a Fit Least Squares preference.) Shows the corrected Akaike Information Criterion value (AICc). See "Likelihood, AICc, and BIC".

BIC (Appears only if you have selected the AICc option from the Regression Reports menu or if you have set AICc as a Fit Least Squares preference.) Shows the Bayesian Information Criterion value (BIC). See "Likelihood, AICc, and BIC".

Analysis of Variance

In the Fit Least Squares report, the Analysis of Variance option provides the calculations for comparing the fitted model to a model where all predicted values equal the response mean.

Note: If either a Frequency or a Weight variable is entered in the Fit Model launch window, the entries in the Analysis of Variance report are adjusted in keeping with the descriptions in "Frequency" and "Weight".

The Analysis of Variance report contains the following columns:

Source The three sources of variation: Model, Error, and C. Total (Corrected Total).

DF The associated *degrees of freedom* (DF) for each source of variation. The C. Total DF is always one less than the number of observations, and it is partitioned into degrees of freedom for the Model and Error as follows:

- The Model DF is the number of parameters (other than the intercept) used to fit the model.
- The Error DF is the difference between the C. Total DF and the Model DF.

Sum of Squares The associated Sum of Squares (SS) for each source of variation.

- The total (C. Total) SS is the sum of the squared differences between the response values and the sample mean. It represents the total variation in the response values.
- The Error SS is the sum of the squared differences between the fitted values and the actual values. It represents the variability that remains unexplained by the fitted model.
- The Model SS is the difference between C. Total SS and Error SS. It represents the variability explained by the model.

Mean Square The mean square statistics for the Model and Error sources of variation. Each Mean Square is the sum of squares divided by its corresponding DF.

Note: The square root of the Mean Square for Error is the same as RMSE in the Summary of Fit report.

MSE Used (Appears only when the Error Specification is Pure Error or Specified.) The mean square error used when the default error specification is not selected. This value is used to calculate the F Ratio instead of the mean square error in the Mean Square column.

DFE Used (Appears only when the Error Specification is Pure Error or Specified.) The error degrees of freedom used when the default error specification is not selected. This value is used to calculate the Prob > F value instead of the Error DF in the DF column.

F Ratio The model mean square divided by the error mean square. The F Ratio is the test statistic for a test of whether the model differs significantly from a model where all predicted values are the response mean.

Prob > F The *p*-value for the test. The Prob > F value measures the probability of obtaining an F Ratio as large as what is observed, given that all parameters except the intercept are zero. Small values of Prob > F indicate that the observed *F* Ratio is unlikely. Such values are considered evidence that there is at least one significant effect in the model.

Parameter Estimates

In the Fit Least Squares report, the Parameter Estimates option shows the estimates of the model parameters. For each parameter, a *t* test is given for the hypothesis that it equals zero.

Note: Estimates are obtained and tested, if possible, even when there are linear dependencies among the model terms. Such estimates are labeled Biased or Zeroed. See "Models with Linear Dependencies among Model Terms".

Term The model term corresponding to the estimated parameter. The first term is always the intercept, unless the No Intercept option was checked in the Fit Model launch window. Continuous effects appear with the name of the data table column. Note that continuous columns that are part of higher order terms might be centered. Nominal or ordinal effects appear with values of levels in brackets. See "Statistical Details for Nominal Effects Coding" and "The Factor Models" for information about the coding of nominal and ordinal terms.

Estimate The parameter estimates for each term. These are the estimates of the model coefficients. When there are linear dependencies among model terms, these might be labeled as Biased or Zeroed. See "Models with Linear Dependencies among Model Terms".

Std Error The estimates of the standard errors for each of the estimated parameters.

t **Ratio** The tests of whether the true value of each parameter is zero. The *t* Ratio is the ratio of the estimate to its standard error. Given the usual assumptions about the model, the *t* Ratio has a Student's *t* distribution under the null hypothesis.

Prob>|t| The *p*-value for the test that the true parameter value is zero, against the two-sided alternative that it is not.

Lower 95% The lower 95% confidence limit for the parameter estimate. This column appears only if you have the Regression Reports > Show All Confidence Intervals option selected or if you right-click in the report and select Columns > Lower 95%.

Upper 95% The upper 95% confidence limit for the parameter estimate. This column appears only if you have the Regression Reports > Show All Confidence Intervals option selected or if you right-click in the report and select Columns > Upper 95%.

Std Beta The parameter estimates for a regression model where all of the terms have been standardized to a mean of 0 and a variance of 1. This column appears only if you right-click in the report and select Columns > Std Beta.

VIF The variance inflation factor for each term in the model. High VIFs indicate a collinearity issue among the terms in the model.

The VIF for the i^{th} term, x_i, is defined as follows:

$$VIF_i = \frac{1}{1 - R_i^2}$$

where R_i^2 is the RSquare, or *coefficient of multiple determination*, for the regression of x_i as a function of the other explanatory variables. This column appears only if you right-click in the report and select Columns > VIF.

Design Std Error The square roots of the *relative variances* of the parameter estimates (Goos and Jones 2011, p. 25):

$$\sqrt{diag(X'X)^{-1}}$$

These are the standard errors divided by RMSE. This column appears only if you right-click in the report and select Columns > Design Std Error.

Effect Tests

In the Fit Least Squares report, the Effect Tests option appears only when there are fixed effects in the model. The effect test for a given effect tests the null hypothesis that all parameters associated with that effect are zero. An effect might have only one parameter as for a single continuous explanatory variable. In this case, the test is equivalent to the *t* test for that term in the Parameter Estimates report. A nominal or ordinal effect can have several associated parameters, based on its number of levels. The effect test for such an effect tests whether all of the associated parameters are zero.

Note the following:

- Effect tests are conducted, when possible, for effects whose terms are involved in linear dependencies. See "Models with Linear Dependencies among Model Terms".
- Parameterization and handling of singularities differ from the SAS GLM procedure. For more information about parameterization and handling of singularities, see "The Factor Models".

The Effects Test report contains the following columns:

Source The effects in the model.

Nparm The number of parameters associated with the effect. A continuous effect has one parameter. The number of parameters for a nominal or ordinal effect is one less than its number of levels. The number of parameters for a crossed effect is the product of the number of parameters for each individual effect.

DF The degrees of freedom for the effect test. Ordinarily, Nparm and DF are the same. They can differ if there are linear dependencies among the predictors. In such cases, DF might be less than Nparm, indicating that at least one parameter associated with the effect is not testable. Whenever DF is less than Nparm, the note LostDFs appears to the right of the line in the report. If there are degrees of freedom for error, the test is conducted. See "Effect Tests Report".

Sum of Squares The sum of squares for the hypothesis that the effect is zero.

Mean Square (Hidden column.) The mean square for the effect, which is the sum of squares for the effect divided by its DF.

F Ratio The F statistic for testing that the effect is zero. The F Ratio is the ratio of the mean square for the effect divided by the mean square for error. The mean square for the effect is the sum of squares for the effect divided by its degrees of freedom.

Prob > F The *p*-value for the effect test.

η^2 **Effect Size** (Hidden column.) The η^2 (eta squared) effect size index statistic for the effect. This value is calculated as the effect sum of squares divided by the total sum of squares. See Albers and Lakens (2018).

ω^2 **Effect Size** (Hidden column.) The ω^2 (omega squared) effect size index statistic for the effect. This statistic is a less biased alternative to η^2. See Albers and Lakens (2018).

Note: To make the hidden columns visible in the table, right-click in the table and select the column name from the Columns submenu.

Effect Details

In the Fit Least Squares report, the Effect Details option provides details, plots, and tests for individual effects. It consists of separate reports based on the emphasis that you select in the Fit Model launch window.

- Effect Leverage emphasis: Each effect has its own report at the top of the Fit Least Squares report window to the right of the Whole Model report. In this case, the report includes a Leverage Plot for the effect.
- Effect Screening or Minimal Report emphases: The Effect Details report is provided but is initially closed. Click the disclosure icon to show the report.

The initial content of the report is the Table of Least Squares Means. Depending on the nature of the effect, this table might not be appropriate, and the default report might initially show no content. However, certain red triangle options are available.

Table of Effect Options

The red triangle menu next to an effect name provides the following options. For certain modeling types, some of these options might not be appropriate and are therefore not available.

LSMeans Table Shows the statistics that are compared when effects are tested. See "LSMeans Table".

This option is not enabled for continuous effects.

LSMeans Plot Shows plots of least squares means for nominal and ordinal effects. If the effect is an interaction, this option displays the Least Squares Means Plot Options window. See "LSMeans Plot".

LSMeans Contrast Shows the Contrast Specification window, which enables you to specify and test contrasts to compare levels for nominal and ordinal effects and their interactions. See "LSMeans Contrast".

LSMeans Student's t Shows tests and confidence intervals for pairwise comparisons of least squares means using Student's *t* tests. See "LSMeans Student's t and LSMeans Tukey HSD".

Note: The significance level applies to individual comparisons and *not* to all comparisons collectively. The error rate for the collection of comparisons is greater than the error rate for individual tests.

LSMeans Tukey HSD Shows tests and confidence intervals for pairwise comparisons of least squares means using the *Tukey-Kramer HSD* (Honestly Significant Difference) test (Tukey 1953; Kramer 1956). See "LSMeans Student's t and LSMeans Tukey HSD".

Note: The significance level applies to the collection of pairwise comparisons. The significance level is exact if the sample sizes are equal and conservative if the sample sizes differ (Hayter 1984).

LSMeans Dunnett Shows tests and confidence intervals for pairwise comparisons against a control level that you specify. Also provides a plot of test results. See "LSMeans Dunnett".

Test Slices For each level of each column in the interaction, jointly tests pairwise comparisons among all the levels of the other classification columns in the interaction. See "Test Slices".

Note: Available only for interactions involving nominal and ordinal effects.

Power Analysis Shows the Power Details report, which enables you to analyze the power for the effect test. See "Power Analysis".

LSMeans Table

Least squares means are values predicted by the model for the levels of a categorical effect where the other model factors are set to *neutral* values. The neutral value for a continuous effect is defined to be its sample mean. The neutral value for a nominal effect that is not involved in the effect of interest is the average of the coefficients for that effect. The neutral value for an uninvolved ordinal effect is defined to be the first level of the effect in the value ordering.

Least squares means are also called *adjusted means* or *population marginal means*. Least squares means can differ from simple means when there are other effects in the model. In fact, it is common for the least squares means to be closer together than the sample means. This situation occurs because of the nature of the neutral values where these predictions are made.

Because least squares means are predictions at specific values of the other model factors, you can compare them. When effects are tested, comparisons are made using the least squares means. For more information about least squares means, see "Least Squares Means across Nominal Factors" and "Ordinal Least Squares Means".

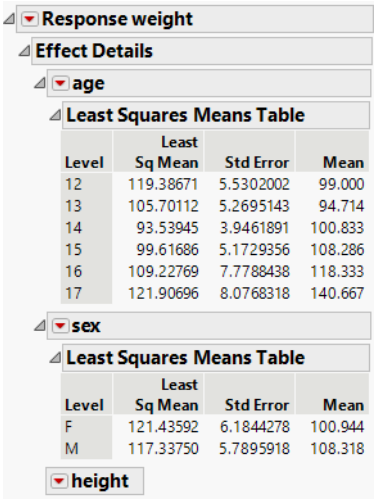
For main effects, the Least Squares Means Table also includes the sample mean (Figure 3.10).

Example of a Least Squares Means Table

1. Select **Help > Sample Data Folder** and open Big Class.jmp.
2. Select **Analyze > Fit Model**.
3. Select weight and click **Y**.
4. Select age, sex, and height and click **Add**.
5. From the **Emphasis** list, select **Effect Screening**.
6. Click **Run**.
7. The Effect Details report appears near the bottom of the Fit Least Squares report and is initially closed. Click the disclosure icon next to the Effect Details report title to show the report.

The Effect Details report, shown in Figure 3.10, shows reports for each of the three effects. Least Squares Means tables are given for age and sex, but not for the continuous effect height. Notice how the least squares means differ from the sample means.

Figure 3.10 Least Squares Mean Table



Response weight

Effect Details

age

Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
12	119.38671	5.5302002	99.000
13	105.70112	5.2695143	94.714
14	93.53945	3.9461891	100.833
15	99.61686	5.1729356	108.286
16	109.22769	7.7788438	118.333
17	121.90696	8.0768318	140.667

sex

Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
F	121.43592	6.1844278	100.944
M	117.33750	5.7895918	108.318

height

The Least Squares Means report contains the following columns:

Level The categorical levels or combination of levels.

Least Sq Mean An estimate of the least squares mean for each level.

Estimability (Appears only when a least squares mean is not estimable.) A warning if a least squares mean is not estimable.

Std Error Shows the standard error of the least squares mean for each level.

Lower 95% Shows the lower 95% confidence limit for the least squares mean. This column appears only if you have the Regression Reports > Show All Confidence Intervals option selected or if you right-click in the report and select Columns > Lower 95%.

Upper 95% Shows the upper 95% confidence limit for the least squares mean. This column appears only if you have the Regression Reports > Show All Confidence Intervals option selected or if you right-click in the report and select Columns > Upper 95%.

Mean Shows the response sample mean for the given level. This mean differs from the least squares mean if the values for other effects in the model do not balance out across this effect.

LSMeans Plot

The LSMeans Plot option produces a Least Squares Means Plot for nominal and ordinal main effects and their interactions. If the effect is an interaction, this option displays the Least Squares Means Plot Options window. See "Least Squares Means Plot Options".

The Least Squares Means Plot red triangle menu contains the following options:

Show Confidence Limits Shows or hides confidence limits for each estimate in the plot.

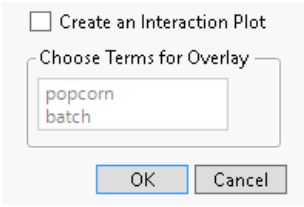
Show Connected Points Shows or hides one or more lines that connect the least squares means for each level in the plot.

Remove Removes the Least Squares Means Plot report for the specified effect.

Least Squares Means Plot Options

When you select the LSMeans Plot option from the red triangle menu of an interaction effect, the Least Squares Means Plot Options window appears.

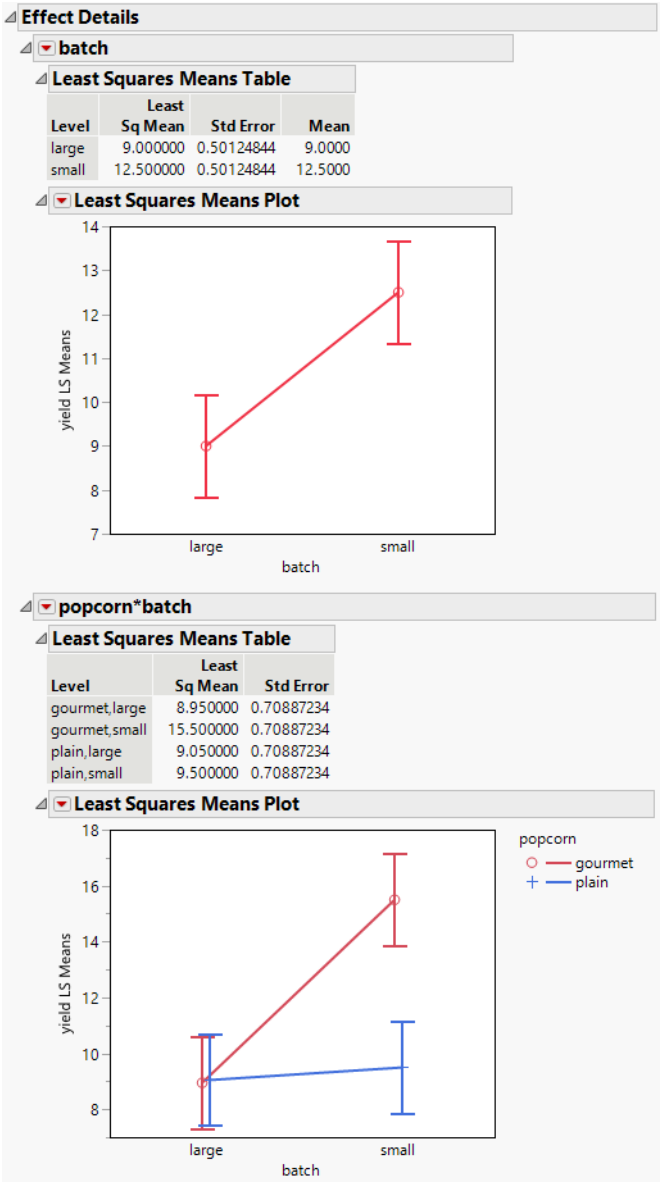
Figure 3.11 Least Squares Means Plot Options Window



If you click OK without selecting anything in the window, one LSMeans Plot appears. The horizontal axis of the plot consists of the levels of the factors nested to obtain a separate effect for each combination. To create an interaction plot, select the box next to Create an Interaction Plot. The Choose Terms for Overlay option enables you to select which effect is displayed as the overlay variable in the interaction plot.

For a three-way interaction term, a second panel of options appears after you choose an overlay variable for the interaction plot. If you click OK without selecting anything in the second panel, one interaction plot appears. Alternatively, use the second panel of options to create separate interaction plots for each level of the effect that you select under Choose Terms for Separate Plots.

Figure 3.12 Least Squares Means Tables and Plots for Two Effects



Example of an LS Means Plot

To create the report in Figure 3.12, follow these steps:

1. Select **Help > Sample Data Folder** and open Popcorn.jmp.
2. Select **Analyze > Fit Model**.
3. Select yield and click **Y**.

4. Select popcorn, oil amt, and batch and click **Macros > Full Factorial**. Note that the Emphasis changes to Effect Screening.
5. Click **Run**.
6. Click the Effect Details disclosure icon to show the details for the seven model effects.
7. Click the batch red triangle and select **LSMeans Plot**.
8. Click the popcorn*batch red triangle and select **LSMeans Plot**. The Least Squares Means Plot Options window appears.
9. In the Least Squares Means Plot Options window, click the box next to Create an Interaction Plot.
10. Under Choose Terms for Overlay, select popcorn.
11. Click **OK**.
12. To transpose the factors in the plot for popcorn*batch, repeat step 8 and step 9.
13. Under the Choose Terms for Overlay, select batch and click **OK**.

Figure 3.13 LSMeans Plot for Interaction with Factors Transposed

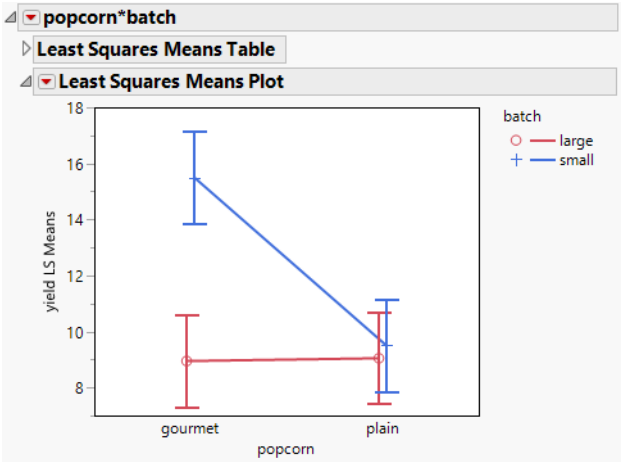


Figure 3.13 shows the popcorn*batch interaction plot with the factors transposed. Compare it with the plot in Figure 3.12. These plots depict the same information but, depending on your interest, one might be more intuitive than the other.

LSMeans Contrast

A *contrast* is a linear combination of parameter values. In the Contrast Specification window, you can specify multiple contrasts and jointly test whether they are zero (Figure 3.14).

JMP builds contrasts in terms of the least squares means of the effect. Each column of the contrast is normalized to have sum zero and so that the sum of the absolute values equals two. If a contrast involves a covariate, you can specify the value of the covariate at which to test the contrast.

The Contrast Specification box shows the name of the effect and the names of the levels in the effect. The contrast values are initially set to zero and appear next to cells containing + and - signs. Click these buttons to compare levels.

Each time you click the + or - button, the contrast coefficients are normalized to make their sum zero and their absolute sum equal to two, if possible. To compare additional levels, click the **New Column** button. A new column appears in which you define a new contrast. After you are finished, click **Done**. The Contrast report appears (Figure 3.15). The overall test is a joint *F* test for all contrasts.

Note: If you attempt to specify more than the maximum number of contrasts possible, the test automatically evaluates.

The Contrast report provides the following details about the joint *F* test:

SS The sum of squares for the joint test.

NumDF The numerator degrees of freedom.

DenDF The denominator degrees of freedom.

F Ratio The ratio of SS divided by NumDF divided by the mean square error.

Prob > F The *p*-value for the significance test.

Test Detail Report

The Test Detail report (Figure 3.15) shows a column for each contrast that you tested. For each contrast, the report gives its estimated value, its standard error, a *t* ratio for a test of that single contrast, the corresponding *p*-value, its sum of squares, and a confidence interval for the contrast estimate.

Parameter Function Report

The Parameter Function report (Figure 3.15) shows the contrasts that you specified expressed as linear combinations of the terms of the model.

Example of LSMeans Contrast

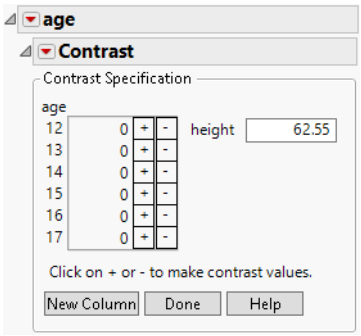
To illustrate the LSMeans Contrast option, form a contrast that compares the first two age levels with the next two levels.

Follow these steps to create the report shown in Figure 3.14.

1. Select **Help > Sample Data Folder** and open Big Class.jmp.
2. Select **Analyze > Fit Model**.
3. Select weight and click **Y**.
4. Select age, sex, and height, and click **Add**.
5. Select age in the Select Columns list, select height in the Construct Model Effects list, and click **Cross**.
6. Click **Run**.

   The Fit Least Squares report appears.
7. Click the age red triangle and select **LSMeans Contrast**.

Figure 3.14 LSMeans Contrast Specification for age

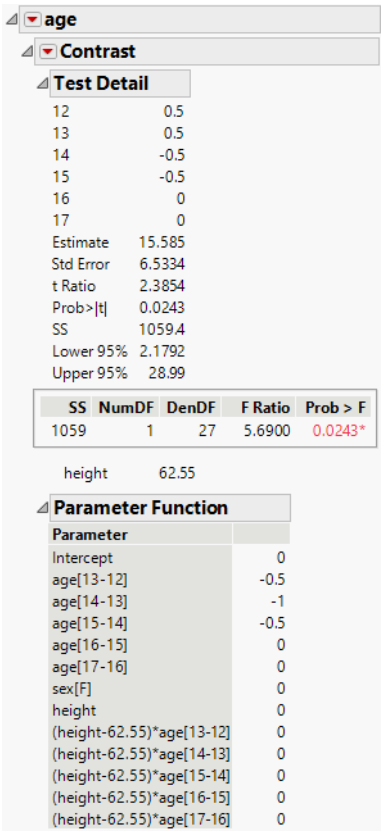


8. Click "+" for the ages 12 and 13.
9. Click "-" for ages 14 and 15.

   This contrast tests whether the mean weights differ for the two age groups, based on predicted values at a height of 62.55.
10. Note that there is a text box next to the continuous effect height. The default value is the mean of the continuous effect.
11. Click **Done**.
12. Open the Test Detail and Parameter Function reports.

The Contrast report is shown in Figure 3.15. The test for the contrast is significant at the 0.05 level. You conclude that the predicted weight for age 12 and 13 children differs statistically from the predicted weight for age 14 and 15 children at the mean height of 62.55.

Figure 3.15 LSMeans Contrast Report



age

Contrast

Test Detail

12	0.5
13	0.5
14	-0.5
15	-0.5
16	0
17	0
Estimate	15.585
Std Error	6.5334
t Ratio	2.3854
Prob>|t|	0.0243
SS	1059.4
Lower 95%	2.1792
Upper 95%	28.99

SS	NumDF	DenDF	F Ratio	Prob > F
1059	1	27	5.6900	0.0243*

height 62.55

Parameter Function

Parameter	
Intercept	0
age[13-12]	-0.5
age[14-13]	-1
age[15-14]	-0.5
age[16-15]	0
age[17-16]	0
sex[F]	0
height	0
(height-62.55)*age[13-12]	0
(height-62.55)*age[14-13]	0
(height-62.55)*age[15-14]	0
(height-62.55)*age[16-15]	0
(height-62.55)*age[17-16]	0

LSMeans Student's t and LSMeans Tukey HSD

The LSMeans Student's t and LSMeans Tukey HSD (*honestly significant difference*) options test pairwise comparisons of model effects.

- The LSMeans Student's t option is based on the usual independent samples, equal variance *t* test. Each comparison is based on the specified significance level. The overall error rate resulting from conducting multiple comparisons exceeds that specified significance level.
- The LSMeans Tukey HSD option conducts Tukey HSD tests. For these comparisons, the significance level applies to the entire collection of pairwise comparisons. For this reason, confidence intervals for LS Means Tukey HSD are wider than those for LSMeans Student's t. The significance level is exact if the sample sizes are equal and conservative if the sample sizes differ (Hayter 1984).

Figure 3.16 shows the LSMeans Tukey report for the effect age in the Big Class.jmp sample data table. (To obtain this report, run the **Fit Model** data table script, click the age red triangle, and select **LS Means Tukey HSD**.) By default, the report shows the Crosstab Report and the Connecting Letters Report.

Figure 3.16 LSMeans Tukey HSD Report

LSMeans Differences Tukey HSD

α= 0.050 Q= 3.02917

LSMean[j]

LSMean[i]	Mean[i]-Mean[j] Std Err Dif Lower CL Dif Upper CL Dif	12	13	14	15	16	17
	12	0 0 0 0	13.6856 6.96659 -7.4174 34.7886	25.8473 7.24776 3.89261 47.8019	19.7698 8.04937 -4.613 44.1527	10.159 9.96733 -20.034 40.3517	-2.5203 10.5955 -34.616 29.5754
	13	-13.686 6.96659 -34.789 7.41737	0 0 0 0	12.1617 6.82786 -8.521 32.8444	6.08426 7.59798 -16.931 29.0998	-3.5266 9.69566 -32.896 25.8432	-16.206 10.0859 -46.758 14.346
	14	-25.847 7.24776 -47.802 -3.8926	-12.162 6.82786 -32.844 8.52105	0 0 0 0	-6.0774 6.28014 -25.101 12.9462	-15.688 8.57237 -41.655 10.2789	-28.368 8.65355 -54.581 -2.1545
	15	-19.77 8.04937 -44.153 4.61302	-6.0843 7.59798 -29.1 16.9313	6.07741 6.28014 -12.946 25.101	0 0 0 0	-9.6108 9.23282 -37.579 18.3569	-22.29 9.21068 -50.191 5.61058
	16	-10.159 9.96733 -40.352 20.0337	3.52657 9.69566 -25.843 32.8963	15.6882 8.57237 -10.279 41.6554	9.61083 9.23282 -18.357 37.5786	0 0 0 0	-12.679 10.8916 -45.672 20.3131
	17	2.52026 10.5955 -29.575 34.6159	16.2058 10.0859 -14.346 46.7577	28.3675 8.65355 2.15448 54.5806	22.2901 9.21068 -5.6106 50.1908	12.6793 10.8916 -20.313 45.6716	0 0 0 0

Level		Least Sq Mean
17	A	121.90696
12	A	119.38671
16	A B	109.22769
13	A B	105.70112
15	A B	99.61686
14	B	93.53945

The Crosstab Report

Both options display a matrix, called the Crosstab Report, where each cell contains the difference in means, the standard error of the difference, and lower and upper confidence limits. The significance level and corresponding critical value are given above the matrix. The default significance level is 0.05, but you can specify a different significance level in the Fit Model launch window. Cells that correspond to pairs of means that differ statistically are shown in red.

The Connecting Letters Report

A Connecting Letters Report appears by default beneath the Crosstab matrix. Levels that share, or are connected by, the same letter do not differ statistically. Levels that are not connected by a common letter do differ statistically.

In Figure 3.16, levels 17, 12, 16, 13, and 15 are connected by the letter A. The connection indicates that these levels do not differ at the 0.05 significance level. Also, levels 16, 13, 15, and 14 are connected by the letter B, indicating that they do not differ statistically. However, ages 17 and 14, and ages 12 and 14, are not connected by a common letter, indicating that these two pairs of levels are statistically different.

Tip: Right-click in the connecting letters report and select Columns to add columns containing connecting letters (Letters), standard errors (Std Error), and confidence interval limits (Lower X% and Upper X%). In the Letters column, the connecting letters are concatenated into a single column. The significance and confidence levels are determined by the significance level that you specify in the Fit Model launch window using the Set Alpha Option.

LSMeans Student's t and LSMeans Tukey HSD Options

The red triangle options that appear in each report window show or hide optional reports. All of the options below are available for LSMeans Student's t. The first four options are available for LSMeans Tukey HSD. For both LSMeans Student's t and LSMeans Tukey HSD, the Crosstab Report and the Connecting Letters Report are shown by default.

Crosstab Report Shows a two-way table that provides, for each pair of levels, the difference in means, the standard error of the difference, and confidence limits for the difference. The contents of cells containing significant differences are highlighted in red.

Connecting Letters Report Illustrates significant and non-significant comparisons with connecting letters. Levels not connected by the same letter are significantly different. Levels connected by the same letter are not significantly different.

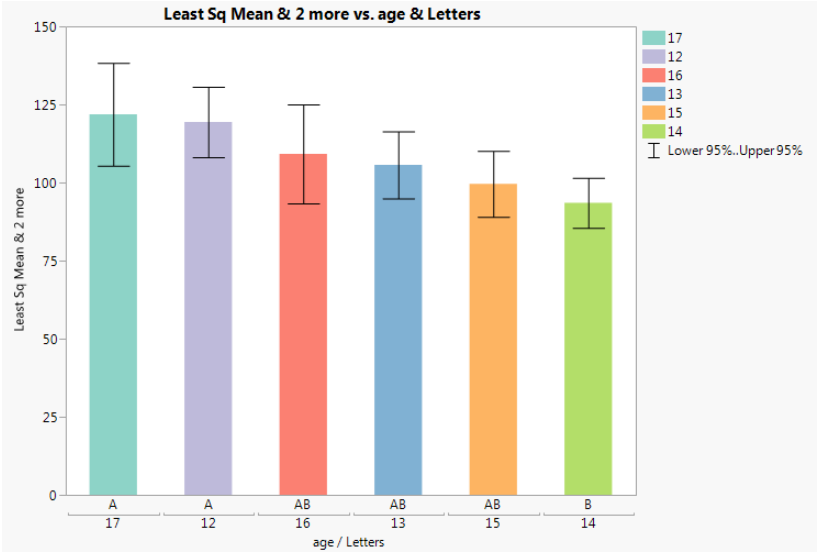
Save Connecting Letters Table Creates a data table whose columns give the levels of the effect, the connecting letters, the least squares means, their standard errors, and confidence intervals. The table contains a script called Bar Chart that produces a colored bar chart of the least squares means with their confidence intervals superimposed. The levels are arranged in decreasing order of least squares means.

Figure 3.17 shows the bar chart for an example based on Big Class.jmp. Run the **Fit Model** data table script, click the age red triangle, and select **LSMeans Tukey HSD**. Select **Save Connecting Letters Table** from the LSMeans Differences Tukey HSD report. Run the **Bar Chart** script in the data table that appears.

Ordered Differences Report Ranks the differences from largest to smallest, giving standard errors, confidence limits, and *p*-values. Also plots the differences on a bar chart with overlaid confidence intervals.

Detailed Comparisons Gives individual detailed reports for each comparison. For a given comparison, the report shows the estimated difference, standard error, confidence interval, *t* ratio, degrees of freedom, and *p*-values for one- and two-sided tests. Also shown is a plot of the *t* distribution, which illustrates the significance test for the comparison. The area of the shaded portion is the *p*-value for a two-sided test.

Figure 3.17 Bar Chart from LSMeans Differences HSD Connecting Letters Table



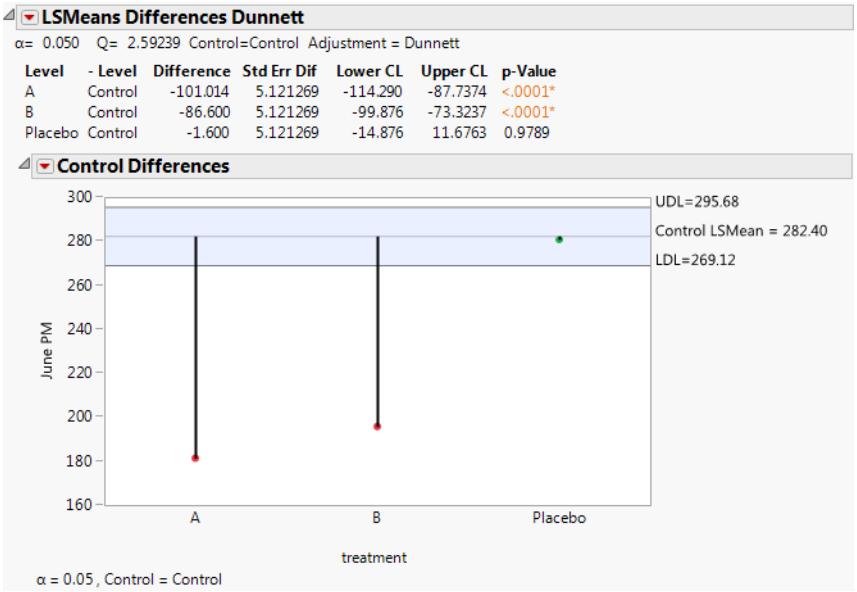
LSMeans Dunnett

Dunnett's test (Dunnett 1955) compares a set of means against the mean of a control group. The error rate applies to the collection of pairwise comparisons. The LSMeans Dunnett option conducts Dunnett's test for the levels of the given effect. Hsu's factor analytical approximation is used for the calculation of *p*-values and confidence intervals (Hsu 1992).

When you select LSMeans Dunnett, you are prompted to enter a control level for the effect. The LS Means Differences Hsu-Dunnett report shows the significance level, the value of the test statistic (Q), and the control level.

A report for the LSMeans Dunnett option for effect treatment in the Cholesterol.jmp sample data table is shown in Figure 3.18. Here, the response is June PM and the level of treatment called Control is specified as the control level.

Figure 3.18 LSMeans Dunnett Report



The report has two options:

Control Differences Report The Control Differences report is shown by default. For each level of the effect, a table shows the following information: the level being compared to the control level, the estimated difference, the standard error of the difference, a confidence interval, and the *p*-value for the comparison.

Control Differences Chart For each level other than the control, a point shows the difference between the LS Mean for that level and the LS Mean for the control level. Upper and lower decision limits (UDL, LDL) are plotted. The report has a Show Summary Report option and Display options. The Show Summary Report option gives the plot detail. The Display options enable you to modify the plot appearance.

Test Slices

The Test Slices option is enabled for interaction effects composed of nominal or ordinal columns. For each level of each nominal or ordinal column in the interaction, this option produces a report that jointly tests all pairwise comparisons of settings involving that level. The test is effectively a test of differences within the specified “slice” of the interaction.

Suppose that you are interested in an A*B*C interaction, where one of the levels of A is “Small”. The Test Slice report for the slice A = Small jointly tests all pairwise comparisons of the B*C levels when A = Small. It enables you to detect differences in levels within an interaction.

The Test Slice reports follow the same format as do the LSMeans Contrast reports. See "LSMeans Contrast".

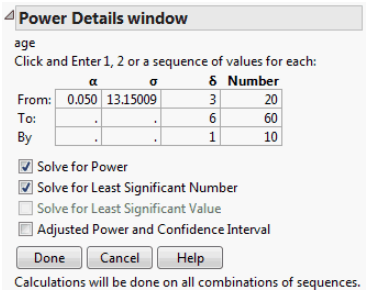
Power Analysis

Opens the Power Details window, where you can enter information to obtain retrospective or prospective details for the *F* test of a specific effect.

Note: To ensure that your study includes sufficiently many observations to detect the required differences, use information about power when you *design* your experiment. Such an analysis is called a *prospective* power analysis. Consider using the DOE platform to design your study. Both DOE > Sample Size Explorers and DOE > Design Diagnostics > Evaluate Design are useful for prospective power analysis. For an example of a prospective power analysis using standard least squares, see "Prospective Power Analysis".

Figure 3.19 shows an example of the Power Details window for the Big Class.jmp sample data table. Using the Power Details window, you can explore power for values of alpha (α), sigma (σ), delta (δ), and Number (study size). Enter a single value (From only), two values (From and To), or the start (From), stop (To), and increment (By) for a sequence of values. Power calculations are reported for all possible combinations of the values that you specify.

Figure 3.19 Power Details Window



See "Statistical Details for Power Analysis".

The Power Details window report contains the following columns and options:

Alpha (α) The significance level of the test. This value is between 0 and 1, and is often 0.05, 0.01, or 0.10. The initial value for Alpha, shown in the first row, is 0.05, unless you have selected Set Alpha Level and set a different value in the Fit Model launch window.

Sigma (σ) An estimate of the residual error in the model. The initial value shown in the first row, provided for guidance, is the RMSE (the square root of the mean square error).

Delta (δ) The effect size of interest. See "Effect Size". The initial value, shown in the first row, is the square root of the sum of squares for the hypothesis divided by the square root of the number of observations in the study (that is, $\delta = \sqrt{SS/n}$).

Number (*n*) The sample size. The initial value, shown in the first row, is the number of observations in the current study.

Solve for Power Solves for the power as a function of α, σ, δ, and *n*. The power is the probability of detecting a difference of size δ by seeing a test result that is significant at level α, for the specified σ and *n*. See "Computations for the Power".

Solve for Least Significant Number Solves for the smallest number of observations required to obtain a test result that is significant at level α, for the specified δ and σ. See "Computations for the LSN".

Solve for Least Significant Value Solves for the smallest positive value of a parameter or linear function of the parameters that produces a *p*-value of α. The least significant value is a function of α, σ, and *n*. This option is available only for one-degree-of-freedom tests. See "Computations for the LSV".

Adjusted Power and Confidence Interval Retrospective power calculations use estimates of the standard error and the test parameters in estimating the *F* distribution's noncentrality parameter. *Adjusted power* is retrospective power calculation based on an estimate of the noncentrality parameter from which positive bias has been removed (Wright and O'Brien 1988).

The confidence interval for the adjusted power is based on the confidence interval for the noncentrality estimate.

The adjusted power deals with a sample estimate, so it and its confidence limits are computed only for the δ estimated in the current study. See "Computations for the Adjusted Power".

Lack of Fit

In the Fit Least Squares report, the Lack of Fit option gives details for a test that assesses whether the model fits the data well. The Lack of Fit report appears only when it is possible to conduct this test. The test relies on the ability to estimate the variance of the response using an estimate that is independent of the model. Constructing this estimate requires that response values are available at replicated values of the model effects. The test involves computing an estimate of *pure error*, based on a sum of squares, using these replicated observations.

In the following situations, the Lack of Fit report does not appear because the test statistic cannot be computed:

- There are no replicated points with respect to the *X* variables, so it is impossible to calculate a pure error sum of squares.

- The model is *saturated*, meaning that there are as many estimated parameters as there are observations. Such a model fits perfectly, so it is impossible to assess lack of fit.

The difference between the error sum of squares from the model and the pure error sum of squares is called the *lack of fit* sum of squares. The lack of fit variation can be significantly greater than pure error variation if the model is not adequate. For example, you might have the wrong functional form for a predictor, or you might not have enough, or the correct, interaction effects in your model.

The Lack of Fit report contains the following columns:

Source The three sources of variation: Lack of Fit, Pure Error, and Total Error.

DF The degrees of freedom (DF) for each source of error:

- The DF for Total Error is the same as the DF value found on the Error line of the Analysis of Variance table. Based on the sum of squares decomposition, the Total Error DF is partitioned into degrees of freedom for Lack of Fit and for Pure Error.
- The Pure Error DF is pooled from each replicated group of observations. In general, if there are g groups and if each group has identical settings for each effect, the pure error DF, denoted DF_{PE}, is defined as follows:

$$\mathrm{DF}_{PE} = \sum_{i=1}^{g} (n_i - 1)$$

  where n_i is the number of replicates in the i^{th} group.
- The Lack of Fit DF is the difference between the Total Error and Pure Error DFs.

Sum of Squares The associated sum of squares (SS) for each source of error:

- The Total Error SS is the sum of squares found on the Error line of the corresponding Analysis of Variance table.
- The Pure Error SS is the total of the sum of squares values for each replicated group of observations. The Pure Error SS divided by its DF estimates the variance of the response at a given predictor setting. This estimate is unaffected by the model. In general, if there are g groups and if each group has identical settings for each effect, the Pure Error SS, denoted SS_{PE}, is defined as follows:

$$\mathrm{SS}_{PE} = \sum_{i=1}^{g} \mathrm{SS}_i$$

  where SS_i is the sum of the squared differences between each observed response and the mean response for the i^{th} group.
- The Lack of Fit SS is the difference between the Total Error and Pure Error sum of squares.

Mean Square The mean square for the Source, which is the Sum of Squares divided by the DF. A Lack of Fit mean square that is large compared to the Pure Error mean square suggests that the model is not fitting well. The *F* ratio provides a formal test.

F Ratio The ratio of the Mean Square for Lack of Fit to the Mean Square for Pure Error. The *F* Ratio tests the hypothesis that the variances estimated by the Lack of Fit and Pure Error mean squares are equal, which is interpreted as representing "no lack of fit".

Prob > F The *p*-value for the Lack of Fit test. A small *p*-value indicates a significant lack of fit.

Max RSq The maximum RSquare that can be achieved by a model based only on these effects. The Pure Error Sum of Squares is invariant to the form of the model. So the largest amount of variation that a model with these replicated effects can explain equals:

$$\frac{SS(\text{C. Total}) - SS(\text{Pure Error})}{SS(\text{C. Total})} = 1 - \frac{SS(\text{Pure Error})}{SS(\text{C. Total})}$$

This formula defines the Max RSq.

Estimates

In the Fit Least Squares report, the Estimates options (accessed from the Response red triangle menu) provide additional detail about model parameters. To better understand estimates, you might want to review how JMP codes nominal and ordinal effects. See "Statistical Details for the Custom Test Example", "Nominal Factors", and "Ordinal Factors".

If your model contains random effects, then only the options below that are appropriate are available from the Estimates menu.

The Estimates menu provides the following options:

Show Prediction Expression Shows or hides the Prediction Expression report, which contains the equation for the estimated model. See "Show Prediction Expression" for an example.

Sorted Estimates Shows or hides the Sorted Parameter Estimates report, which can be useful in screening situations. If the design is not saturated, this report is the Parameter Estimates report with the terms, other than the Intercept, sorted in decreasing order of significance. If the design is saturated, then Pseudo *t* tests are provided. See "Sorted Estimates".

Expanded Estimates (Available only when at least one of the effects is not continuous.) Shows or hides the Expanded Estimates report, which expands the Parameter Estimates report by giving parameter estimates for all levels of nominal effects. See "Expanded Estimates".

Indicator Parameterization Estimates (Available only when there are nominal columns and an intercept among the model effects.) Shows or hides the Indicator Function Parameterization report, which contains parameter estimates with the nominal effects in the model parametrized using the classical indicator functions. See "Indicator Parameterization Estimates".

Sequential Tests Shows or hides the Sequential (Type 1) Tests report that contains the sums of squares as effects are added to the model sequentially. Conducts *F* tests based on the sequential sums of squares. See "Sequential Tests".

Custom Test Enables you to test a custom hypothesis. See "Custom Test".

Multiple Comparisons Enables you to specify comparisons among effect levels. These comparisons can involve a single effect or you can define flexible custom comparisons. You can compare to the overall mean, to a control mean, or you can obtain all pairwise comparisons using Tukey HSD or Student's *t*. When you specify the Student's *t* method, you can also perform equivalence tests to identify pairwise differences that are of practical importance. See "Multiple Comparisons".

Compare Slopes (Available only when there is one nominal term, one continuous term, and their interaction effect for the fixed effects.) Produces a report that enables you to compare the slopes of each level of the interaction effect in an analysis of covariance (ANCOVA) model. See "Compare Slopes".

Joint Factor Tests (Available only when the model contains interactions.) For each main effect in the model, shows or hides a joint test on all of the parameters involving that main effect. See "Joint Factor Tests".

Inverse Prediction Enables you to predict values of explanatory variables for one or more values of the response. See "Inverse Prediction".

Cox Mixtures (Available only when the model contains mixture effects.) Produces parameter estimates for the Cox mixture model. Using these to derive factor effects and estimate the response surface shape relative to a reference point in the design space. See "Cox Mixtures".

Parameter Power Adds columns to the Parameter Estimates report that give power and other details relating to the corresponding hypothesis tests. See "Parameter Power".

Correlation of Estimates Shows or hides a correlation matrix for all parameter estimates in the model. See "Correlation of Estimates".

Error Specification (Available only when there are no random effects.) Specifies the error variance and the error degrees of freedom that are used for standard errors and tests in the Fit Least Squares report. Note that the Studentized Residuals plot and the Box Cox Transformations report are not affected by changing the Error Specification. When the

Error Specification is Pure Error or Specified, an additional column appears in the Analysis of Variance report. See "Analysis of Variance".

Default Estimate Uses the standard root mean square error and error degrees of freedom from the model to calculate all tests and standard errors.

Pure Error Uses the Pure Error mean square and associated degrees of freedom from the Lack of Fit report to calculate all tests and standard errors. See "Lack of Fit".

Caution: If the pure error degrees of freedom is 1, a warning message is displayed indicating that tests are weak and confidence limits are large.

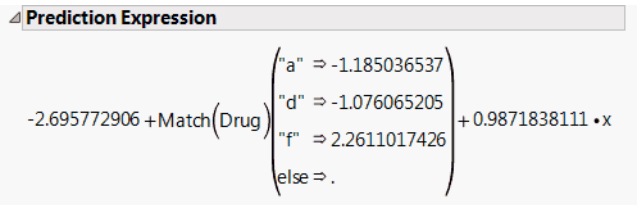
Specified Uses user-specified values for the error variance and error degrees of freedom to calculate all tests and standard errors.

Show Prediction Expression

In the Fit Least Squares report, the Show Prediction Expression option shows the equation used to predict the response. Figure 3.20 shows an example for the Drug.jmp sample data table. This expression is given as a typical JMP formula. For example, to predict the response for someone on Drug a with x = 10, you would calculate, with some rounding: –2.696 - 1.185 + 0.987(10) = 5.99.

Tip: To specify the number of digits in the prediction formula, go to **File > Preferences > Tables** and change the **Default Field Width** value.

Figure 3.20 Prediction Expression



To obtain the preceding report, follow these steps:

Example of a Prediction Expression

1. Select **Help > Sample Data Folder** and open Drug.jmp.
2. Select **Analyze > Fit Model**.
3. Select y and click **Y**.
4. Select Drug and x, and then click **Add**.

5. Click **Run**.
6. Click the Response y red triangle and select **Estimates > Show Prediction Expression**. The report in Figure 3.20 appears.

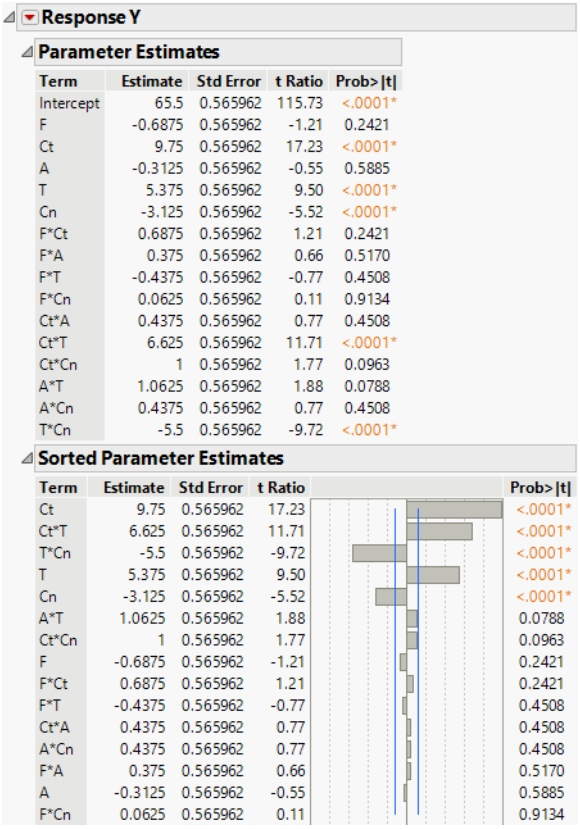
Sorted Estimates

In the Fit Least Squares report, the Sorted Estimates option produces a version of the Parameter Estimates report that is useful in screening situations. If the design is not saturated, the Sorted Estimates report gives the information found in the Parameter Estimates report, but with the terms, other than the Intercept, sorted in decreasing order of significance (second report in Figure 3.21). If the design is saturated, then Pseudo *t* tests are provided. These are based on Lenth's *pseudo standard error* (Lenth 1989). See "Lenth's PSE".

Example of a Sorted Estimates Report

1. Select **Help > Sample Data Folder** and open Reactor.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Make sure that 2 appears in the **Degree** box near the bottom of the window.
5. Select F, Ct, A, T, and Cn and click **Macros > Factorial to Degree**.
6. Click **Run**.
7. Click the Response Y red triangle and select **Estimates > Sorted Estimates**.

Figure 3.21 Sorted Parameter Estimates



Response Y

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	65.5	0.565962	115.73	<.0001*
F	-0.6875	0.565962	-1.21	0.2421
Ct	9.75	0.565962	17.23	<.0001*
A	-0.3125	0.565962	-0.55	0.5885
T	5.375	0.565962	9.50	<.0001*
Cn	-3.125	0.565962	-5.52	<.0001*
F*Ct	0.6875	0.565962	1.21	0.2421
F*A	0.375	0.565962	0.66	0.5170
F*T	-0.4375	0.565962	-0.77	0.4508
F*Cn	0.0625	0.565962	0.11	0.9134
Ct*A	0.4375	0.565962	0.77	0.4508
Ct*T	6.625	0.565962	11.71	<.0001*
Ct*Cn	1	0.565962	1.77	0.0963
A*T	1.0625	0.565962	1.88	0.0788
A*Cn	0.4375	0.565962	0.77	0.4508
T*Cn	-5.5	0.565962	-9.72	<.0001*

Sorted Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Ct	9.75	0.565962	17.23	<.0001*
Ct*T	6.625	0.565962	11.71	<.0001*
T*Cn	-5.5	0.565962	-9.72	<.0001*
T	5.375	0.565962	9.50	<.0001*
Cn	-3.125	0.565962	-5.52	<.0001*
A*T	1.0625	0.565962	1.88	0.0788
Ct*Cn	1	0.565962	1.77	0.0963
F	-0.6875	0.565962	-1.21	0.2421
F*Ct	0.6875	0.565962	1.21	0.2421
F*T	-0.4375	0.565962	-0.77	0.4508
Ct*A	0.4375	0.565962	0.77	0.4508
A*Cn	0.4375	0.565962	0.77	0.4508
F*A	0.375	0.565962	0.66	0.5170
A	-0.3125	0.565962	-0.55	0.5885
F*Cn	0.0625	0.565962	0.11	0.9134

The Sorted Parameter Estimates report also appears automatically if the Emphasis is set to Effect Screening and all of the effects have only one parameter.

Note the following differences between the Parameter Estimates report and the Sorted Parameter Estimates report (both shown in Figure 3.21):

- The Sorted Parameter Estimates report does not show the intercept.
- The effects are sorted by the absolute value of the *t* ratio, showing the most significant effects at the top.
- A bar chart shows the *t* ratio with vertical lines showing critical values for the 0.05 significance level.

Sorted Estimates Report for Saturated Models

Screening experiments often involve fully saturated models, where there are not enough degrees of freedom to estimate error. In these cases, the Sorted Estimates report (Figure 3.21) gives relative standard errors and constructs *t* ratios and *p*-values using Lenth's pseudo standard error (PSE). These quantities are labeled with *Pseudo* in their names. See "Lenth's PSE" and "Pseudo t-Ratios". A note explains the change and shows the PSE.

The report contains the following columns:

Term The model term whose coefficient is of interest.

Estimate The parameter estimates are presented in sorted order, with smallest *p*-values listed first.

Relative Standard Error If there are no degrees of freedom for residual error, the report gives relative standard errors. The relative standard error is computed by setting the root mean square error equal to 1.

Pseudo t-Ratio A *t* ratio for the estimate, computed using pseudo standard error. The value of Lenth PSE is shown in a note at the bottom of the report.

Pseudo p-Value A *p*-value computed using an error degrees of freedom value (DFE) of $m/3$, where m is the number of parameters other than the intercept. The value of DFE is shown in a note at the bottom of the report.

Lenth's PSE

Lenth's *pseudo standard error* (PSE) is an estimate of residual error due to Lenth (1989). It is based on the principle of effect sparsity: in a screening experiment, relatively few effects are active. The inactive effects represent random noise and form the basis for Lenth's estimate.

The value is computed as follows:

1. Consider the absolute values of all non-intercept parameters.
2. Remove all parameter estimates whose absolute values exceed 3.75 times the median absolute estimate.
3. Multiply the median of the remaining absolute values of parameter estimates by 1.5.

Pseudo t-Ratios

When relative standard errors are equal, Lenth's PSE is shown in a note at the bottom of the report. The Pseudo *t*-Ratio is calculated as follows:

$$\text{Pseudo } t\text{-Ratio} = \frac{\text{Estimate}}{\text{PSE}}$$

When relative standard errors are not equal, the TScale Lenth PSE is computed. This value is the PSE of the estimates divided by their relative standard errors. The Pseudo *t*-Ratio is calculated as follows:

$$\text{Pseudo } t\text{-Ratio} = \frac{\text{Estimate}}{\text{TScale Lenth PSE} \times \text{Relative Std Error}}$$

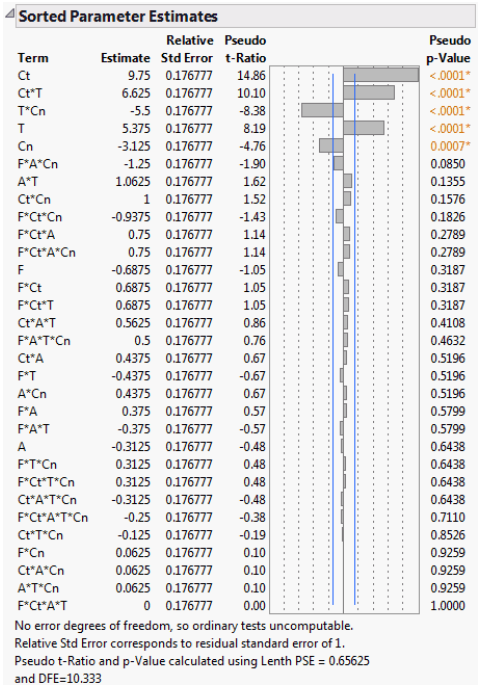
Note that, to estimate the standard error for a given estimate, TScale Lenth PSE is adjusted by multiplying it by the estimate's relative standard error.

Example of a Saturated Model

1. Select **Help > Sample Data Folder** and open Reactor.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Select the following five columns: F, Ct, A, T, and Cn.
5. Click the **Macros** button and select **Full Factorial**.
6. Click **Run**.
7. Click the Response Y red triangle and select **Estimates > Sorted Estimates**.

   Note that Lenth's PSE and the degrees of freedom used are given at the bottom of the report. The report indicates that, based on their Pseudo p-Values, the effects Ct, Ct*T, T*Cn, T, and Cn are highly significant.

Figure 3.22 Sorted Parameter Estimates Report for Saturated Model



Sorted Parameter Estimates

Term	Estimate	Relative Std Error	Pseudo t-Ratio	Pseudo p-Value
Ct	9.75	0.176777	14.86	<.0001*
Ct*T	6.625	0.176777	10.10	<.0001*
T*Cn	-5.5	0.176777	-8.38	<.0001*
T	5.375	0.176777	8.19	<.0001*
Cn	-3.125	0.176777	-4.76	0.0007*
F*A*Cn	-1.25	0.176777	-1.90	0.0850
A*T	1.0625	0.176777	1.62	0.1355
Ct*Cn	1	0.176777	1.52	0.1576
F*Ct*Cn	-0.9375	0.176777	-1.43	0.1826
F*Ct*A	0.75	0.176777	1.14	0.2789
F*Ct*A*Cn	0.75	0.176777	1.14	0.2789
F	-0.6875	0.176777	-1.05	0.3187
F*Ct	0.6875	0.176777	1.05	0.3187
F*Ct*T	0.6875	0.176777	1.05	0.3187
Ct*A*T	0.5625	0.176777	0.86	0.4108
F*A*T*Cn	0.5	0.176777	0.76	0.4632
Ct*A	0.4375	0.176777	0.67	0.5196
F*T	-0.4375	0.176777	-0.67	0.5196
A*Cn	0.4375	0.176777	0.67	0.5196
F*A	0.375	0.176777	0.57	0.5799
F*A*T	-0.375	0.176777	-0.57	0.5799
A	-0.3125	0.176777	-0.48	0.6438
F*T*Cn	0.3125	0.176777	0.48	0.6438
F*Ct*T*Cn	0.3125	0.176777	0.48	0.6438
Ct*A*T*Cn	-0.3125	0.176777	-0.48	0.6438
F*Ct*A*T*Cn	-0.25	0.176777	-0.38	0.7110
Ct*T*Cn	-0.125	0.176777	-0.19	0.8526
F*Cn	0.0625	0.176777	0.10	0.9259
Ct*A*Cn	0.0625	0.176777	0.10	0.9259
A*T*Cn	0.0625	0.176777	0.10	0.9259
F*Ct*A*T	0	0.176777	0.00	1.0000

No error degrees of freedom, so ordinary tests uncomputable.
Relative Std Error corresponds to residual standard error of 1.
Pseudo t-Ratio and p-Value calculated using Lenth PSE = 0.65625 and DFE=10.333

Expanded Estimates

In the Fit Least Squares report, use the Expanded Estimates option when there are nominal terms in the model and you want to see details for the full set of estimates. The Expanded Estimates option provides the estimates, their standard errors, *t* ratios, and *p*-values.

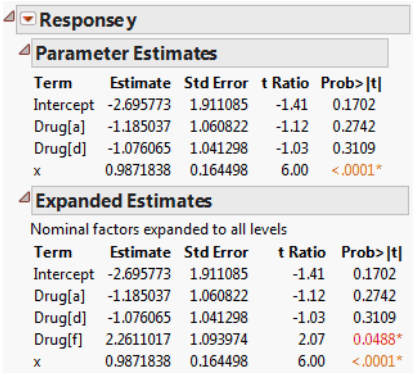
In dealing with parameter estimates, you must understand how JMP codes nominal and ordinal columns. For more information about how nominal columns are coded, see "Statistical Details for the Custom Test Example". For more information about how ordinal columns are coded and modeled, see "Nominal Factors" and "Ordinal Factors".

Example of an Expanded Estimates Report

1. Select **Help > Sample Data Folder** and open Drug.jmp.
2. Select **Analyze > Fit Model**.
3. Select y and click **Y**.
4. Select Drug and x, and then click **Add**.
5. Click **Run**.
6. Click the Response y red triangle and select **Estimates > Expanded Estimates**.

The Expanded Estimates report, along with the Parameter Estimates report, is shown in Figure 3.23. Note that an estimate for the term Drug[f] appears in the Expanded Estimates report. The null hypothesis for the test is that the mean for the Drug f group does not differ from the overall mean. The test for Drug[f] is significant at the 0.05 level, suggesting that the mean response for the Drug f group differs from the overall response. See "Interpretation of Tests for Expanded Estimates".

Figure 3.23 Comparison of Parameter Estimates and Expanded Estimates



Response y

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-2.695773	1.911085	-1.41	0.1702
Drug[a]	-1.185037	1.060822	-1.12	0.2742
Drug[d]	-1.076065	1.041298	-1.03	0.3109
x	0.9871838	0.164498	6.00	<.0001*

Expanded Estimates

Nominal factors expanded to all levels

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-2.695773	1.911085	-1.41	0.1702
Drug[a]	-1.185037	1.060822	-1.12	0.2742
Drug[d]	-1.076065	1.041298	-1.03	0.3109
Drug[f]	2.2611017	1.093974	2.07	0.0488*
x	0.9871838	0.164498	6.00	<.0001*

Interpretation of Tests for Expanded Estimates

Suppose that your model consists of a single nominal factor that has *n* levels. That factor is represented by *n-1* indicator variables, one for each of *n-1* levels. The parameter estimate corresponding to any one of these *n-1* indicator variables is the difference between the mean response for that level and the average response across all levels. This representation is due to how JMP codes nominal variables. See "Statistical Details for the Custom Test Example". The parameter estimate is often interpreted as the *effect* of that level.

For example, in the Cholesterol.jmp sample data table, consider the single factor treatment and the response June PM. The parameter estimate associated with the term, or indicator variable, treatment[A] is the difference between the mean of June PM for treatment A and the overall mean of June PM.

The effects across *all* levels of a nominal variable are constrained to sum to zero. Consider the effect of the last level in the level ordering, namely, the level that is coded with –1s. The effect of this level is the negative of the sum of the effects across the other *n-1* levels. It follows that the effect of the last level is the negative of the sum of the parameter estimates across the other *n-1* levels.

The Expanded Estimates option in the Estimates menu calculates missing estimates, tests for all effects that involve nominal columns, and shows them in a text report. You can verify that the mean (or sum) of the estimates across the levels of any such effect is zero. In particular, this relationship indicates that these estimates, and their associated tests, are not independent of each other.

In the Drug.jmp report shown in Figure 3.23, the estimates for the terms associated with Drug are based on a model that includes the covariate x.

Notes:

- The estimate for Drug[a] is the difference between the least squares mean for Drug a and the overall mean of y.
- The estimate for Drug[f], given in the Expanded Estimates report, is the negative of the sum of the estimates for Drug[a] and Drug[d].
- The *t* test for Drug [f] presented in the Expanded Estimates report tests whether the response for the Drug f group differs from the overall mean response.
- If nominal factors are involved in high-degree interactions, the Expanded Estimates report can be lengthy. For example, a five-way interaction of two-level nominal factors produces only one parameter estimate but has $2^5 = 32$ expanded effects, which are all identical up to sign changes.

Indicator Parameterization Estimates

In the Fit Least Squares report, the Indicator Parameterization Estimates option displays the Indicator Function Parameterization report, which gives parameter estimates for the model where nominal columns are coded using indicator (SAS GLM) parameterization and are treated as continuous. Ordinal columns remain coded using the usual JMP coding scheme. The SAS GLM and JMP coding schemes are described in "The Factor Models".

In the JMP coding scheme, the estimate that corresponds to the indicator for a level of a nominal variable is an estimate of the difference between the mean response at that level and the mean response over all the levels. To see the JMP coding, select **Save Columns > Save Coding Table** from the Standard Least Squares red triangle menu.

In the indicator coding scheme, the estimate that corresponds to the indicator for a level of a nominal variable is an estimate of the difference between the mean response at that level and the mean response at the last level. The last level is the level with the highest value order coding; it is the level whose indicator function is not included in the model.

Caution: Standard errors and t-ratios given in the Indicator Function Parameterization report differ from those in the Parameter Estimates report. This is because the estimates are estimating different parameters.

To create the report in Figure 3.24, follow the steps in "Example of an Expanded Estimates Report". But instead of selecting Expanded Estimates, select **Indicator Parameterization Estimates**.

Figure 3.24 Indicator Parameterization Estimates

Response y

Indicator Function Parameterization

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|
Intercept	-0.434671	2.471354	26.00	-0.18	0.8617
Drug[a]	-3.446138	1.886781	26.00	-1.83	0.0793
Drug[d]	-3.337167	1.853866	26.00	-1.80	0.0835
x	0.9871838	0.164498	26.00	6.00	<.0001*

The JMP coding scheme is used for nominal variables throughout JMP with the exception of the Generalized Regression personality of Fit Model. For more information about the coding scheme for nominal variables in the Generalized Regression personality, see "Launch the Generalized Regression Personality".

Note that there might be differences in models derived using the JMP versus the SAS GLM parameterization. Some models are equivalent. Other models (such as no-intercept models, models with missing cells, models with nominal or ordinal effects, and mixture models) might show differences.

Sequential Tests

In the Fit Least Squares report, the Sequential Tests option shows sums of squares and tests as effects are added to the model sequentially. The order of entry is defined by the order of effects as they appear in the Fit Model launch window's Construct Model Effects list. The report in Figure 3.25 is for the Drug.jmp sample data table.

Figure 3.25 Sequential Tests Report

Response y

Sequential (Type 1) Tests

Source	Nparm	DF	Seq SS	F Ratio	Prob > F
Drug	2	2	293.60000	9.1486	0.0010*
x	1	1	577.89740	36.0145	<.0001*

The sums of squares that form the basis for sequential tests are also called *Type I Sums of Squares*. They are computed by fitting models in steps following the specified entry order of effects. Consider a specific effect. Compute the model sum of squares for a model containing all effects entered *prior* to that effect. Then compute the model sum of squares for a model containing those effects *and* the specified effect. The sequential sum of squares for the specified effect is the increase in the model sum of squares.

Refer to Figure 3.25, showing sequential sums of squares for the Drug.jmp sample data table. In the Fit Model launch window, Drug was entered first, followed by x. A model consisting only of Drug has model sum of squares equal to 293.6. When x is added to the model, the model sum of squares becomes 871.4974. The increase of 577.8974 is the sequential sum of squares for x.

The tests shown in the Sequential (Type 1) Tests report are *F* tests based on sequential sums of squares, also called *Type I Tests*. The *F* Ratio tests the specified effect, where the model contains only that effect and the effects listed above it in the Source column.

The sequential sums of squares sum to the model sum of squares. Another nice feature is that, under the usual model assumptions, the values are statistically independent of each other. However, they do depend on the order of terms in the model and, as such, are not appropriate in many situations.

Sequential tests are considered appropriate in the following situations:

- balanced analysis of variance models specified in proper sequence (that is, two-way interactions follow main effects in the effects list, and so on)
- purely nested models specified in the proper sequence
- polynomial regression models specified in the proper sequence.

The tests given in the Parameter Estimates and Effect Tests reports are based on *Type III Sums of Squares*. Here the sum of squares for an effect is the extra sum of squares explained by the effect after all other effects have been entered in the model.

Custom Test

In the Fit Least Squares report, the Custom Test option enables you to test one or more custom hypotheses involving any model parameters. In the Custom Test window, you can specify one or more linear functions, or *contrasts*, of the model parameters.

The results include individual tests for each contrast and a joint test for all contrasts. See Figure 3.26. The report for the individual contrasts gives the estimated value of the specified linear function of the parameters and its standard error. A *t* ratio, its *p*-value, and the associated sum of squares are also provided. Below the individual contrast results, the joint test for all contrasts gives the sum of squares, the numerator degrees of freedom, the *F* ratio, and its *p*-value.

Caution: These tests are conducted using residual error. If you have random effects in your model and if you use EMS instead of REML, then these tests might not be appropriate.

Note: If you are testing for effects that are involved in higher-order effects, consider using a test for least squares means, rather than a custom test. Least squares means are adjusted for other model effects. You can test least squares means contrasts under Effect Details.

Custom Test Report Components

The Custom Test specification window has the following components:

Editable text box The space beneath the Custom Test title bar is an editable area for entering a test name.

Parameter Lists the model terms. To the right of the list of terms are columns of zeros corresponding to the corresponding parameters. Enter values in these cells to specify the linear functions for your tests.

The "=" sign The last line in the Parameter list is labeled =. Enter a constant into this cell to complete the specification for each contrast.

Add Column Adds columns of zeros so that you can jointly test several linear functions of the parameters.

Done Click the Done button to perform the tests. The report changes to show the test statistic value, the standard error, and other statistics for each test column. The joint *F* test for all columns is given in a box at the bottom of the report.

Custom Test Report Options

The Custom Test red triangle menu contains the following options:

Power Analysis Provides a power analysis for the joint test. This option is available only after the test has been conducted. See "Parameter Power".

Remove Removes the Custom Test report.

Note: Select **Estimates > Custom Test** repeatedly to conduct several joint custom tests.

Figure 3.26 shows an example of the specification window with three contrasts, using the Cholesterol.jmp sample data table. Note that the constant is set to zero for all three tests. The report for these tests is shown in Figure 3.27.

Example of a Custom Test

The Cholesterol.jmp sample data table gives repeated measures on 20 patients at six time periods. Four treatment groups are studied. Typically, this data should be properly analyzed using all repeated measures as responses. This example considers only the response for June PM.

Suppose that you want to test three contrasts. You want to compare the mean responses for the following:

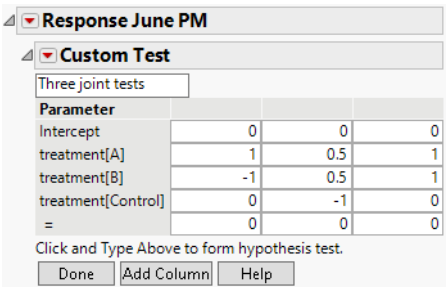
- treatment A to treatment B
- treatments A and B to the control group
- treatments A and B to the control and placebo groups

To test these contrasts using Custom Test:

1. Select **Help > Sample Data Folder** and open Cholesterol.jmp.
2. Select **Analyze > Fit Model**.
3. Select June PM and click **Y**.
4. Select treatment and click **Add**.
5. Click **Run**.
6. Click the red triangle next to Response June PM and select **Estimates > Custom Test**.
7. In the Custom Test specification report, click **Add Column** twice to create two additional columns.
8. Fill in the editable area with a test name and enter values in the three columns as shown in Figure 3.26.

   To see how to obtain these values, particularly those in the third column, see "Interpretation of Parameters".

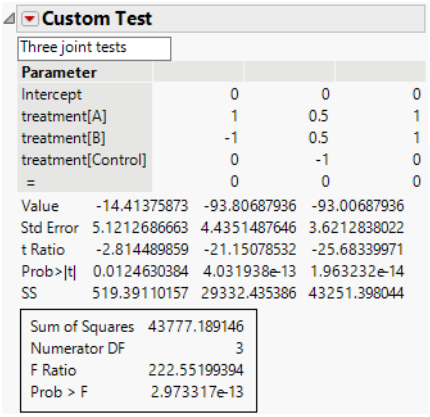
Figure 3.26 Custom Test Specification Report for Three Contrasts



9. Click **Done**.

The results shown in Figure 3.27 indicate that all three hypotheses are individually, as well as jointly, significant.

Figure 3.27 Custom Test Report Showing Tests for Three Contrasts



Custom Test

Three joint tests

Parameter			
Intercept	0	0	0
treatment[A]	1	0.5	1
treatment[B]	-1	0.5	1
treatment[Control]	0	-1	0
=	0	0	0
Value	-14.41375873	-93.80687936	-93.00687936
Std Error	5.1212686663	4.4351487646	3.6212838022
t Ratio	-2.814489859	-21.15078532	-25.68339971
Prob>|t|	0.0124630384	4.031938e-13	1.963232e-14
SS	519.39110157	29332.435386	43251.398044

Sum of Squares	43777.189146
Numerator DF	3
F Ratio	222.55199394
Prob > F	2.973317e-13

Compare Slopes

In the Fit Least Squares report, the Compare Slopes option appears when there is one nominal term, one continuous term, and their interaction effect for the fixed effects. This option produces a report that enables you to compare the slopes in an analysis of covariance (ANCOVA) model. The report compares the slopes of each level of the interaction effect to the overall slope. The comparison uses analysis of means (ANOM) with the overall average. For more information about the analysis of means (ANOM) report, see "Comparisons with Overall Average".

The overall average slope is a weighted average of the slopes, where the weights are inversely proportional to the variances of the slope estimates. These variances are the squared values of the Std Error column in the Differences from Overall Average Slope table.

Joint Factor Tests

In the Fit Least Squares report, the Joint Factor Test option appears when interaction effects are present. For each main effect in the model, JMP produces a joint test of whether all the coefficients for terms involving that main effect are zero. This test is conditional on all other effects being in the model. Specifically, the joint test is a general linear hypothesis test of a restricted model. In that model, all parameters that correspond to the specified effect and the interactions that contain it are set to zero.

Example of a Joint Factor Tests Report

1. Select **Help > Sample Data Folder** and open Big Class.jmp.
2. Select **Analyze > Fit Model**.
3. Select weight and click **Y**.

4. Verify that 2 appears in the **Degree** box.
5. Select age, sex, and height and click **Macros > Factorial to Degree**.
6. Click **Run**.
7. Click the Response weight red triangle and select **Estimates > Joint Factor Tests**.

Figure 3.28 Joint Factor Tests Report

Joint Factor Tests

Term	DF	Sum of Squares	F Ratio	Prob > F
age	15	6116.2127	2.8488	0.0139*
sex	7	2113.1080	2.1091	0.0879
height	7	9217.8156	9.2002	<.0001*

Note that the test for age has 15 degrees of freedom. This test involves the five parameters for age, the five parameters for age*sex, and the five parameters for height*age. The null hypothesis for this test is that all 15 parameters are zero.

Inverse Prediction

In the Fit Least Squares report, the Inverse Prediction option enables you to use a statistical model to infer the value of an explanatory variable, given a value of the response variable. Inverse prediction is sometimes referred to as *calibration*.

The Inverse Prediction option in the Estimates menu in Standard Least Squares also enables you to specify values for other explanatory variables in the model. The inverse prediction computation provides confidence limits for values of the explanatory variable that correspond to the specified response value. You can specify the response value to be the mean response or simply an individual response. For an example, see "Example of Inverse Prediction".

Analyzing Multiple Explanatory Variables

When the model includes multiple explanatory variables, you can predict the value of X for the specified values of the other variables. You might want to predict the amount of running time that results in an oxygen uptake of 50 when one's resting pulse rate is 60. You might want separate inverse predictions for both males and females. Specify these requirements using the inverse prediction option.

The inverse prediction window shows the list of explanatory variables to the left. (See Figure 3.32 for an example.) Each continuous variable is initially set to its mean. Each nominal or ordinal variable is set to its lowest level (in terms of value ordering). You must remove the value for the variable that you want to predict, setting it to missing. Also, you must specify the values of the other variables for which you want your inverse prediction to hold (if these differ from the default settings). In the list to the right in the window, you can supply one or more response values of interest. For an example, see "Example of Predicting a Single X Value with Multiple Model Effects".

Note: The confidence limits for inverse prediction can sometimes result in a one-sided or even an infinite interval. For technical details, see "Inverse Prediction with Confidence Limits".

Example of Inverse Prediction

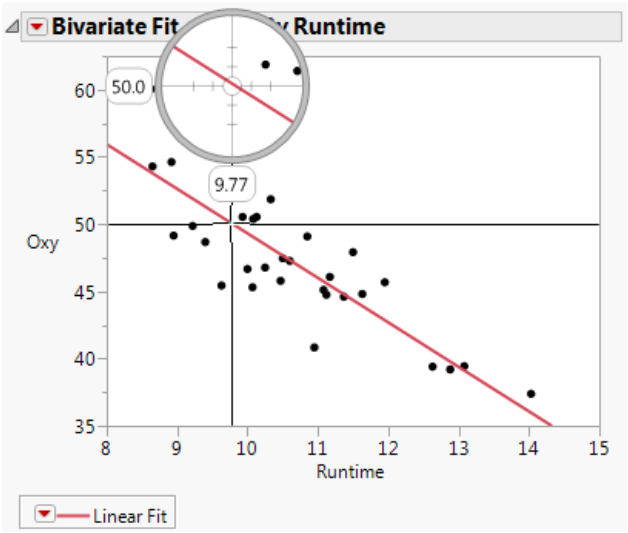
In this example, you fit a regression model that predicts oxygen uptake from Runtime. Then you estimate the Runtime values that result in specified oxygen uptake values. There is only a single *X*, Runtime, so you start by using the Fit Y by X platform to obtain a visual approximation of the inverse prediction values.

1. Select **Help > Sample Data Folder** and open Fitness.jmp.
2. Select **Analyze > Fit Y by X**.
3. Select Oxy and click **Y, Response**.
4. Select Runtime and click **X, Factor**.
5. Click **OK**.
6. Click the Bivariate Fit of Oxy By Runtime red triangle and select **Fit Line**.

   Use the crosshairs tool as described below to approximate the Runtime value that results in a mean Oxy value of 50.
7. Select **Tools > Crosshairs**.
8. Click on the prediction line and then drag the crosshairs tool to find the inverse prediction for Oxy = 50.

Figure 3.29 shows that a Runtime of about 9.77 gives an inverse prediction of about 50 for Oxy.

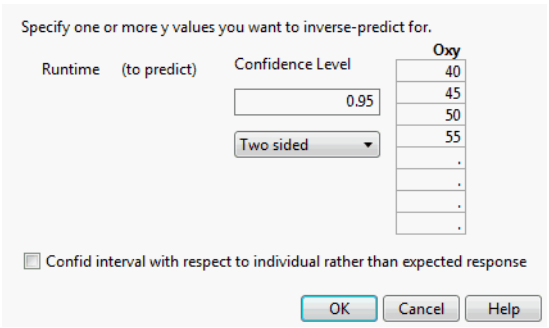
Figure 3.29 Bivariate Fit for Fitness.jmp



To obtain an exact prediction for Runtime, along with a confidence interval, use the Fit Model launch window:

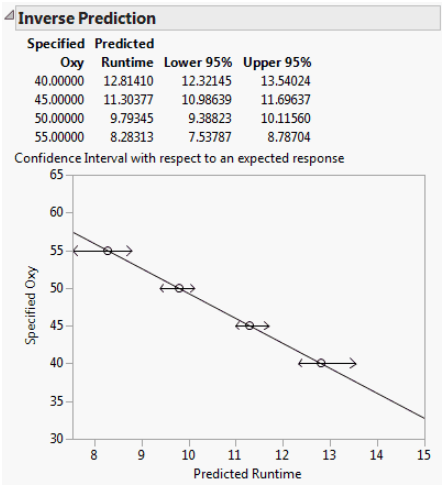
1. From the Fitness.jmp sample data table, select **Analyze > Fit Model**.
2. Select Oxy and click **Y**.
3. Select Runtime and then click **Add**.
4. Click **Run**.
5. Click the Response Oxy red triangle and select **Estimates > Inverse Prediction**.
6. Enter four values for Oxy as shown in Figure 3.30.
7. Click **OK**.

Figure 3.30 Completed Inverse Prediction Specification Window



The Inverse Prediction report gives predicted Runtime values that correspond to each specified Oxy value. The report also shows upper and lower 95% confidence limits for these Runtime values, relative to obtaining the mean response.

Figure 3.31 Inverse Prediction Report



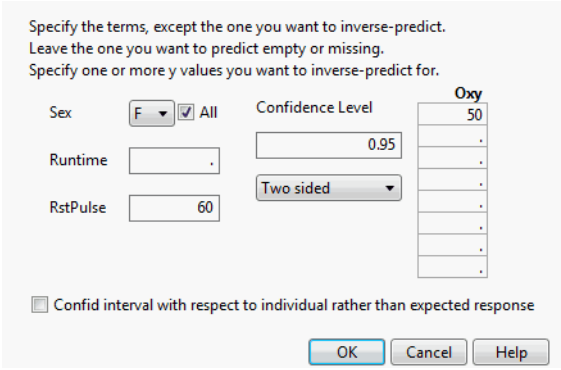
The exact predicted Runtime resulting in an Oxy value of 50 is 9.7935. This value is close to the approximate Runtime value of 9.77 found in the Bivariate Fit report shown in Figure 3.29. The Inverse Prediction report also gives a plot showing the linear relationship between Oxy and Runtime and the confidence intervals.

Example of Predicting a Single X Value with Multiple Model Effects

This example predicts the Runtime that results in oxygen uptake of 50 when RstPulse is 60. The Runtime is predicted for both males and females.

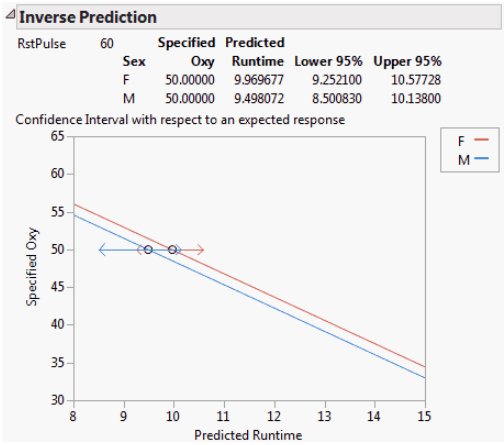
1. From the Fitness.jmp sample data table, select **Analyze > Fit Model**.
2. Select Oxy and click **Y**.
3. Select Sex, Runtime, and RstPulse and then select **Add**.
4. Click **Run**.
5. Click the Response Oxy red triangle and select **Estimates > Inverse Prediction**.
6. Delete the value for Runtime, because you want to predict that value.
7. Select the **All** box next to Sex to estimate Runtime for all levels of Sex.
8. Replace the mean for RstPulse with 60.
9. Enter the value 50 for Oxy as shown in Figure 3.32.
10. Click **OK**.

Figure 3.32 Inverse Prediction Specification for a Multiple Regression Model



The report, shown in Figure 3.33, gives the predicted values of Runtime for both females and males. The report also includes 95% confidence intervals for Runtime values that give a mean response of 50.

Figure 3.33 Inverse Prediction Report for a Multiple Regression Model



The plot shows the linear fits for females and males, given that RstPulse is 60. The two confidence intervals are shown in red and blue, respectively.

Cox Mixtures

In the Fit Least Squares report, the Cox Mixture option appears only for mixture models. The Standard Least Squares personality of the Fit Model platform fits mixture models using the parameterization suggested in Scheffé (1958). The parameters for this model cannot easily be used to judge the effects of the mixture components. The Cox Mixture model is a reparameterized and constrained version of the Scheffé model. Using its parameter estimates, you can derive factor effects and the response surface shape relative to a reference point in the design space. See Cornell (1990) for a complete discussion.

The Cox Mixture option opens a window where you enter the reference mixture. If you enter components for the reference mixture that do not sum to one, then the components are proportionately scaled so that they do sum to one. The rescaled mixture is shown in the report as the Reference Mixture. The component effects also appear in the report. A Cox component effect is the difference in the predicted response as the factor goes from its minimum to maximum values along the Cox effect direction.

Example of Cox Mixtures

1. Select **Help > Sample Data Folder** and open Five Factor Mixture.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y1 and click **Y**.
4. Select X1 through X5.
5. Select **Macros > Mixture Response Surface**.
6. Click **Run**.
7. Click the Response Y1 red triangle and select **Estimates > Cox Mixtures**.
8. In the Reference Mixture specification window, enter the following values:
   - Next to X1, type 0.15.
   - Next to X2, type 0.25.
   - Next to X3, type 0.4.
   - Next to X4, type 0.1.
   - Next to X5, type 0.1.
9. Click **OK**.

Figure 3.34 Cox Mixtures

Cox Reference Mixture Model

Cox Parameter Estimates

Parameter	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	2.071	1.2541	1.651	0.1035
X1	3.120	7.7538	0.402	0.6887
X2	-5.828	7.0104	-0.831	0.4089
X3	-0.696	3.5822	-0.194	0.8465
X4	3.935	12.6619	0.311	0.7570
X5	8.740	12.5405	0.697	0.4883
X1^2	-15.897	16.3627	-0.972	0.3349
X2^2	17.877	13.5932	1.315	0.1931
X3^2	5.234	8.2498	0.634	0.5280
X4^2	-91.239	159.9347	-0.570	0.5703
X5^2	-145.693	158.3991	-0.920	0.3611
X1*X2	-2.938	24.9012	-0.118	0.9065
X1*X3	-7.617	19.1697	-0.397	0.6924
X2*X3	-16.250	18.7489	-0.867	0.3893
X1*X4	53.690	50.5711	1.062	0.2923
X2*X4	-17.702	49.9082	-0.355	0.7240
X3*X4	-7.808	44.0229	-0.177	0.8598
X1*X5	31.813	50.6186	0.628	0.5319
X2*X5	-2.279	49.3761	-0.046	0.9633
X3*X5	17.983	43.5145	0.413	0.6808
X4*X5	177.431	97.3862	1.822	0.0731

Cox Reference Mixture

Factor	Reference Mixture
X1	0.1500000
X2	0.2500000
X3	0.4000000
X4	0.1000000
X5	0.1000000

Component Effects

Component	Cox Effect	Component Range
X1	0.80746	0.2200000
X2	-2.33103	0.3000000
X3	-0.48747	0.4200000
X4	0.65585	0.1500000
X5	1.45663	0.1500000

The report shows the parameter estimates for the Cox mixture model, along with standard errors and hypothesis tests. The reference mixture appears on the right. The component effects appear below, along with the component ranges.

Parameter Power

In the Fit Least Squares report, the Parameter Power option addresses *retrospective* power analysis. The *power* of a statistical test is the probability that the test will be significant, if a difference actually exists. The power of the test indicates how likely your study is to declare a true effect to be significant.

Note: To ensure that your study includes sufficiently many observations to detect the required differences, use information about power when you *design* your experiment. This type of analysis is called *prospective* power analysis. Consider using the DOE platform to design your study. Both DOE > Sample Size and Power and DOE > Evaluate Design are useful for prospective power analysis. For an example of a prospective power analysis using standard least squares, see "Prospective Power Analysis".

The power of a test to detect a difference is affected by the following factors:

- the sample size
- the unknown residual error variance
- the significance level of the test
- the size of the effect to be detected

Suppose that you have already conducted your study, analyzed your data, and found that an effect of interest is not significant. You might be interested in the size of the difference that you might have been likely to detect or the power of the test that you conducted. Or you might want to know the number of observations that you would have needed to detect a difference of a given size with high probability.

The Parameter Power option inserts three columns of values relating to retrospective power analysis in the Parameter Estimates report. The least significant value (LSV0.05), the least significant number (LSN0.05), and a power calculation (AdjPower0.05) are provided.

The Parameter Power calculations apply to a new sample that has the same variability profile as the observed sample.

Caution: The results provided by the LSV0.05, LSN, and AdjPower0.05 should not be used in prospective power analysis. They do not reflect the uncertainty inherent in a future study.

- LSV0.05 is the *least significant value*. This number is the smallest absolute value of the estimate that would make this test significant at significance level 0.05. To be more specific, suppose that the number of observations, the mean square error and that the sum of squares and cross-products matrix for the design remain unchanged. Then, if the absolute value of the estimate had been less than LSV0.05, the Prob>|t| value would have exceeded 0.05. See "The Least Significant Value (LSV)".
- LSN is the *least significant number*. This number is the number of observations that would make this test significant at significance level 0.05. Specifically, suppose that the estimate of the parameter, the mean square error, and the sum of squares and cross-products matrix for the design remain unchanged. Then, if the number of observations had been less than the LSN, the Prob>|t| value would have exceeded 0.05. See "The Least Significant Number (LSN)".
- AdjPower0.05 is the *adjusted power* value. This number is an estimate of the probability that this test will be significant. Sample values from the current study are substituted for the parameter values typically used in a power calculation. The *adjusted* power calculation adjusts for bias that results from direct substitution of sample estimates into the formula for the non-centrality parameter (Wright and O'Brien 1988). See "The Adjusted Power and Confidence Intervals".

The LSV, LSN, and adjusted power are useful in assessing a test's sensitivity. These retrospective calculations also provide an enlightening instructional tool. However, you must be cautious in interpreting these values (Hoenig and Heisey 2001).

For more information about LSV, LSN, and adjusted power, see "Statistical Details for Power Analysis". For an example of a retrospective analysis, see "Example of Retrospective Power Analysis".

Correlation of Estimates

In the Fit Least Squares report, the Correlation of Estimates option computes the correlation matrix for the parameter estimates. These correlations indicate whether collinearity is present.

For insight on the construction of this matrix, consider the typical least squares regression formulation. Here, the response (Y) is a linear function of predictors (x's) plus error (ε):

$$Y = \beta_0 + \beta_1 x_1 + \ldots + \beta_p x_p + \varepsilon$$

Each row of the data table contains a response value and values for the p predictors. For each observation, the predictor values are considered fixed. However, the response value is considered to be a realization of a random variable.

Considering the values of the predictors fixed, for any set of Y values, the coefficients, β_0, β_1, …, β_p, can be estimated. In general, different sets of Y values lead to different estimates of the coefficients. The Correlation of Estimates option calculates the theoretical correlation of these parameter estimates. For technical details, see "Statistical Details for the Custom Test Example".

The correlations of the parameter estimates depend solely on the predictor values and a term representing the intercept. The correlation between two parameter estimates is not affected by the values of the response.

A high positive correlation between two estimates suggests that a collinear relationship might exist between the two corresponding predictors. Note, though, that you need to interpret these correlations with caution (Belsley et al. 1980, p. 185, 92–94). Also, a rescaling of a predictor that shifts its mean changes the correlation of its parameter estimate with the intercept's value.

Example of Correlation of Estimates

1. Select **Help > Sample Data Folder** and open Socioeconomic.jmp.
2. Select **Analyze > Fit Model**.
3. Select Median House Value and click **Y**.

4. Select Total Population, Median School Years, Total Employment, and Professional Services and click **Add**.
5. In the Emphasis list, select **Minimal Report**.
6. Click **Run**.
7. Click the red triangle next to Response Median House Value and select **Estimates > Correlation of Estimates**.

Figure 3.35 Correlation of Estimates Report

Correlation of Estimates

Corr

	Intercept	Total Population	Median School Years	Total Employment	Professional Services
Intercept	1.0000	-0.5743	-0.9818	0.4719	0.6903
Total Population	-0.5743	1.0000	0.5871	-0.9746	-0.2398
Median School Years	-0.9818	0.5871	1.0000	-0.5132	-0.7085
Total Employment	0.4719	-0.9746	-0.5132	1.0000	0.1094
Professional Services	0.6903	-0.2398	-0.7085	0.1094	1.0000

The report shows high negative correlations between the parameter estimates for the Intercept and Median School Years (–0.9818). High negative correlations also exist between Total Population and Total Employment (–0.9746).

Effect Screening

In the Fit Least Squares report, the Effect Screening options (accessed from the Response red triangle menu) are useful when classical tests for effects are not available. This happens with screening designs, which often provide no degrees of freedom for error.

For these designs, most inferences about effect sizes assume that the estimates for non-intercept parameters are uncorrelated and have equal variances. These assumptions hold for the models associated with many classical experimental designs. However, there are situations where these assumptions do not hold. In both of these situations, the Effect Screening option guides you in determining which effects are significant.

The Effect Screening option uses the principle of *effect sparsity* (Box and Meyer 1986). This principle asserts that relatively few of the effects that you study in a screening design are active. Most are inactive, meaning that their true effects are zero and that their estimates are random error.

The following Effect Screening options are available:

Scaled Estimates Gives parameter estimates corresponding to factors that are scaled to have a mean of zero and a range of two. See "Scaled Estimates and the Coding of Continuous Terms".

Normal Plot Identifies parameter estimates that deviate from normality, helping you determine which effects are active. See "Normal Plot Report".

Bayes Plot Computes posterior probabilities for all model terms using a Bayesian approach. See "Bayes Plot Report".

Pareto Plot Plots the absolute values of the orthogonalized and standardized parameter estimates, relating these to the sum of their absolute values. See "Pareto Plot Report".

Scaled Estimates and the Coding of Continuous Terms

In the Fit Least Squares report, the Scaled Estimates option provides coefficients that correspond to factors that are scaled to have a mean of zero and a range of two. A parameter estimate is highly dependent on the scale of its corresponding factor. When you convert a factor from grams to kilograms, its parameter estimate changes by a multiple of a thousand. When you apply the same change to a squared (quadratic) term, its parameter estimate changes by a multiple of a million.

To better understand and compare effect sizes, you should examine parameter estimates in a scale-invariant fashion. It makes sense to use a scale that relates the size of a parameter estimate to the size of the effect of its corresponding term on the response. There are many approaches to doing this.

If the sample values for the factor are such that the maximum and minimum values are equidistant from the sample mean, then the scaled factor ranges from –1 to 1. This scaling corresponds to the traditional coding used in the design of experiments. In the case of regression with a single factor, the scaled parameter estimate is half of the predicted response change as the factor travels its whole range.

Scaled estimates are important in assessing the impact of model terms when the data involve uncoded values. For orthogonal designs, the scaled estimates are identical to the estimates for the uncoded data.

Note: The Coding column property scales factor values linearly so that their coded values range from –1 to 1. Parameter estimates are given in terms of these coded values, so that scaled estimates are not required in this situation. (Unlike the transformation used to obtain scaled estimates, the coded values might not have mean zero.)

Example of Scaled Estimates

1. Select **Help > Sample Data Folder** and open Drug.jmp.
2. Select **Analyze > Fit Model**.
3. Select y and click **Y**.
4. Select Drug and x and add these to the **Construct Model Effects** list.

5. Change the Emphasis to **Minimal Report**.
6. Click **Run**.
7. Click the Response y red triangle and select **Effect Screening > Scaled Estimates**.

The report indicates that the continuous factor, x, is centered by its mean and scaled by its half-range.

Figure 3.36 Scaled Estimates Report

Scaled Estimates

Nominal factors expanded to all levels
Continuous factors centered by mean, scaled by range/2

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
Intercept	7.9	0.731352	10.80	<.0001*
Drug[a]	-1.185037	1.060822	-1.12	0.2742
Drug[d]	-1.076065	1.041298	-1.03	0.3109
Drug[f]	2.2611017	1.093974	2.07	0.0488*
x	8.8846543	1.480478	6.00	<.0001*

The model fits three parallel lines, one for each Drug group. The x values range from 3 to 21. The Scaled Estimate for x, 8.8846543, is half the difference between the predicted value for x = 21 and the predicted value for x = 3 for any one of the Drug groups. You can verify this fact by selecting **Save Columns > Prediction Formula** from the report's red triangle menu. Then add rows to obtain predicted values for each of the Drug groups at x = 21 and x = 3.

So, over the range of x values in this particular data set, the impact of x is to vary the response over a range of about 17.8 units. Note that the parameter estimate for x based on the raw data is 0.9871838; it does not permit direct interpretation in terms of the response.

Effect Screening Plot Options

In the Fit Least Squares report, the Normal, Bayes, and Pareto Plot options provide reports that appear as part of the Effect Screening report. These reports can be constructed so that they correct for unequal variances and correlations among the estimates.

Note: The Normal, Bayes, and Pareto Plot options require that your model involves at least four parameters. One of these parameters can be the intercept.

Transformations

When you select any of the plot options, the following information appears directly beneath the Effect Screening report title:

- If the estimates have equal variances and are uncorrelated, these two notes appear:
  - The parameter estimates have equal variances.
  - The parameter estimates are not correlated.

- If the estimates have unequal variances or are correlated, then an option list replaces the relevant note. The list items selected by default show that JMP has transformed the estimates. If you want to undo either or both transformations, select the appropriate list items.

Lenth PSE Values

A Lenth PSE (*pseudo standard error*) table appears directly beneath the notes or option lists. For a description of the PSE, see "Lenth's PSE". The statistics that appear in the Lenth table depend on the variances and correlation of the parameter estimates.

When the parameter estimates have equal variances and are uncorrelated, the Lenth table provides the following statistic:

Lenth PSE The Lenth pseudo standard error for the estimates.

When the parameter estimates have unequal variances or are correlated or both, the Lenth table provides the following statistics:

t-Test Scale Lenth PSE The Lenth pseudo standard error computed for the transformed parameter estimates divided by their standard errors in the transformed scale.

Coded Scale Lenth PSE The Lenth pseudo standard error for the transformed parameter estimates.

Parameter Estimate Population Report

The Parameter Estimate Population report gives tests for the parameter estimates. The tests are based on transformations as specified in the option lists.

- The option **Using estimates standardized to have equal variances** applies a normalizing transformation to standardize the variances. This option is selected by default when the variances are unequal.
- The option **Using estimates orthogonalized to be uncorrelated** applies a transformation to remove correlation. This option is selected by default when the estimates are correlated. The transformation that is applied is identical to the transformation that is used to calculate sequential sums of squares. The estimates measure the additional contribution of the variable after all previous variables have been entered into the model.
- If the notes indicate that the estimates have equal variances and are not correlated, no transformation is applied.

The columns that appear in the table depend on the selections initially described in the notes or option lists. The report highlights any row corresponding to an estimate with a *p*-value of 0.20 or less. All versions of the report give Term, Estimate, and either *t*-Ratio and Prob>|t| or Pseudo *t*-Ratio and Pseudo *p*-Value.

Term Gives the model term whose parameter estimate is of interest.

Estimate Gives the estimate for the parameter. Estimate sizes can be compared only when the model effects have identical scaling.

t-Ratio Appears if there are degrees of freedom for error. This value is the parameter estimate divided by its standard error.

Prob>|t| Gives the *p*-value for the test. If a transformation is applied, this option gives the *p*-value for a test using the transformed data.

Pseudo t-Ratio Appears when there are no degrees of freedom for error. If the relative standard errors of the parameters are equal, Pseudo t-Ratio is the parameter estimate divided by Lenth's PSE. If the relative standard errors vary, it is calculated as shown in "Pseudo t-Ratios".

Pseudo p-Value Appears when there are no degrees of freedom for error. The *p*-value is derived using a *t* distribution. The degrees of freedom are *m*/3, rounded down to the nearest integer, where *m* is the number of parameters.

If **Using estimates standardized to have equal variances** is selected and the note indicating that the parameter estimates are not correlated appears, the report shows a column called Standardized Estimate. This column provides estimates of the parameters resulting from the transformation used to transform the estimates to have equal variances.

If both **Using estimates standardized to have equal variances** and **Using estimates orthogonalized to be uncorrelated** are selected, the report gives a column called Orthog Coded. The following information is provided:

Orthog Coded Gives the estimate of the parameter resulting from the transformation that is used to orthogonalize the estimates.

Orthog t-Ratio Appears if there are degrees of freedom for error. Gives the *t* ratio for the transformed estimates.

Pseudo Orthog t-Ratio Appears if there are no degrees of freedom for error. It is a *t* ratio computed by dividing the orthogonalized estimate, Orthog Coded, by Coded Scale Lenth PSE.

Effect Screening Report

Figure 3.37 shows the Effect Screening report that you create by running the Fit Model script in the Bicycle.jmp sample data table. Note that you would select **Effect Screening > Normal Plot** in order to obtain this form of the report. The notes directly beneath the report title indicate that no transformation is required. Consequently, the Lenth PSE is displayed. Because there are no degrees of freedom for error, no estimate of residual error can be constructed. For this reason, Lenth's PSE is used as an estimate of residual error to obtain pseudo *t* ratios. Pseudo *p*-values are given for these *t* ratios. Rows for non-intercept terms corresponding to the three estimates with *p*-values of 0.20 or less are highlighted.

Figure 3.37 Effect Screening Report for Equal Variance and Uncorrelated Estimates

Effect Screening

The parameter estimates have equal variances.
The parameter estimates are not correlated.

Lenth PSE
0.75

Parameter Estimate Population

Term	Estimate	Pseudo t-Ratio	Pseudo p-Value
Intercept	66.5000	88.6667	<.0001*
HBars[down]	0.5000	0.6667	0.5649
Dynamo[off]	-6.0000	-8.0000	0.0096*
Seat[down]	1.7500	2.3333	0.1266
Tires[hard]	-1.2500	-1.6667	0.2195
Gear[low]	-11.2500	-15.0000	0.0023*
Raincoat[off]	0.2500	0.3333	0.7665
Brkfast[no]	0.5000	0.6667	0.5649

Orthog t Test used Pseudo Standard Error

Normal Plot

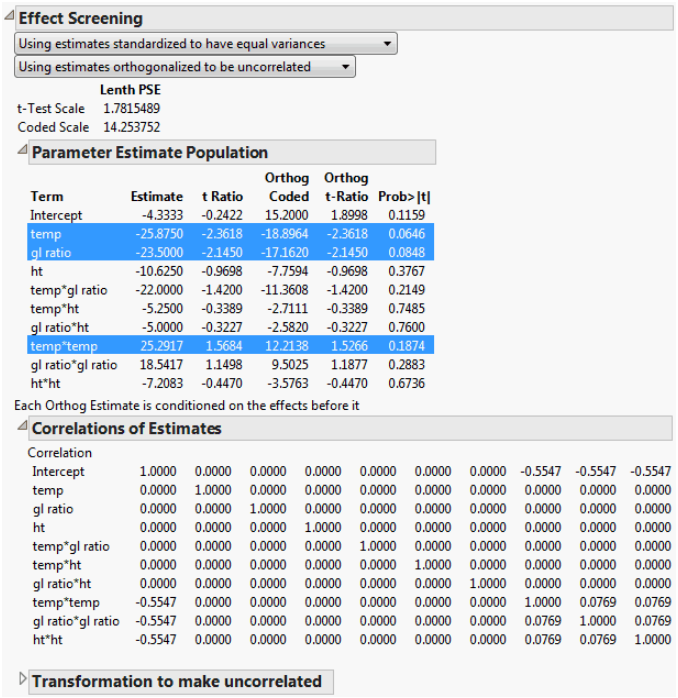
Effect Screening Report for Unequal Variances and Correlated Estimates

In the Odor.jmp sample data table, run the Model script and click **Run**. To create the report shown in Figure 3.38, click the Response Y red triangle and select **Effect Screening > Normal Plot**. You can also create the report by clicking the Response Y red triangle and selecting the Bayes Plot or Pareto Plot options.

The notes directly beneath the report title indicate that transformations were required both to standardize and orthogonalize the estimates. The correlation matrix is shown in the Correlation of Estimates report.

The report shows the *t*-Test Scale and Coded Scale Lenth PSEs. But, because there are degrees of freedom for error, the tests in the Parameter Estimate Population report do not use the Lenth PSEs. Rows for non-intercept terms corresponding to the three estimates with *p*-values of 0.20 or less are highlighted. A note at the bottom of the Parameter Estimate Population report indicates that orthogonalized estimates depend on their order of entry into the model.

Figure 3.38 Effect Screening Report for Unequal Variances and Correlated Estimates



Effect Screening

Using estimates standardized to have equal variances

Using estimates orthogonalized to be uncorrelated

	Lenth PSE
t-Test Scale	1.7815489
Coded Scale	14.253752

Parameter Estimate Population

Term	Estimate	t Ratio	Orthog Coded	Orthog t-Ratio	Prob>|t|
Intercept	-4.3333	-0.2422	15.2000	1.8998	0.1159
temp	-25.8750	-2.3618	-18.8964	-2.3618	0.0646
gl ratio	-23.5000	-2.1450	-17.1620	-2.1450	0.0848
ht	-10.6250	-0.9698	-7.7594	-0.9698	0.3767
temp*gl ratio	-22.0000	-1.4200	-11.3608	-1.4200	0.2149
temp*ht	-5.2500	-0.3389	-2.7111	-0.3389	0.7485
gl ratio*ht	-5.0000	-0.3227	-2.5820	-0.3227	0.7600
temp*temp	25.2917	1.5684	12.2138	1.5266	0.1874
gl ratio*gl ratio	18.5417	1.1498	9.5025	1.1877	0.2883
ht*ht	-7.2083	-0.4470	-3.5763	-0.4470	0.6736

Each Orthog Estimate is conditioned on the effects before it

Correlations of Estimates

Correlation										
Intercept	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.5547	-0.5547	-0.5547
temp	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
gl ratio	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ht	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
temp*gl ratio	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
temp*ht	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
gl ratio*ht	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
temp*temp	-0.5547	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0769	0.0769
gl ratio*gl ratio	-0.5547	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0769	1.0000	0.0769
ht*ht	-0.5547	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0769	0.0769	1.0000

Transformation to make uncorrelated

Correlations of Estimates Report

The Correlations of Estimates report appears only if the estimates are correlated (Figure 3.38). The report provides the correlation matrix for the parameter estimates. This matrix is similar to the one that you obtain by selecting the Estimates > Correlation of Estimates red triangle option. However, to provide a more compact representation, the report does not show column headings. See "Correlation of Estimates".

"Transformation to make uncorrelated" Report

The "Transformation to make uncorrelated" report appears only if the estimates are correlated. The report gives the matrix used to transform the design matrix to produce uncorrelated parameter estimates. The transformed, or *orthogonally coded*, estimates are obtained by pre-multiplying the original estimates with this matrix and multiplying the result by a scale factor. The scale factor is a function of the root mean square error (RMSE) and the number of rows. It is defined as follows:

RMSE / sqrt(NRows)

The transformation matrix can be obtained using the Cholesky decomposition. Express X′X as LL′, where L is the lower triangular matrix in the Cholesky decomposition. Then the transformation matrix is given by L′.

This transformation orthonormalizes each regressor with respect to the regressors that precede it in the ordering of model terms. The transformation produces a diagonal covariance matrix with equal diagonal elements. The coded estimates are a result of this iterative process.

Note: The orthogonally coded estimates depend on the order of terms in the model. Each effect's contribution is estimated only after it is made orthogonal to previously entered effects. Consider entering main effects first, followed by two-way interactions, then three-way interactions, and so on.

Normal Plot Report

Below the Normal Plot report title in the Fit Least Squares report, select either a normal plot or a half-normal plot (Daniel 1959). Both plots are predicated on the principle of effect sparsity, namely, the idea that relatively few effects are active. Those effects that are inactive represent random noise. The plots are based on a pseudo standard error (PSE). The PSE is an approximation of variance (σ^2) using the order statistics of the parameter estimates that are smallest in magnitude. JMP's choice of the active effects comes from a simulation of the distribution of the PSE for a given data set using 10,000 bootstrapped samples. On a normal probability plot, estimates representing inactive effects fall close to a line with slope σ.

Notes:

- If many effects are active, a Normal plot can miss more than one active effect.
- If there is one very large effect, other effects seem small by comparison.
- The theory of a Normal plot assumes a full or regular factorial design. Other designs can have ambiguous results.
- A Pareto plot of the effects provides an alternative graphical method of identifying the largest effects.

Normal Plot

If no transformation is required, the vertical coordinate of the normal plot represents the value of the estimate and the horizontal coordinate represents its normal quantile. Points that represent inactive effects should follow a line with slope of σ. Lenth's PSE is used to estimate σ and a blue line with this slope is shown on the plot.

If a transformation to orthogonality has been applied, the vertical axis represents the Normalized Estimates. These are the Orthog *t*-Ratio values found in the Parameter Estimate Population report. (The Orthog *t*-Ratio values are the Orthog Coded estimates divided by the Coded Scale Lenth PSE.)

Because the estimates are normalized by an estimate of σ, the points corresponding to inactive effects should fall along a line of slope 1. A red line with slope 1 is shown on the plot, as well as a blue line with slope equal to the t-Test Scale Lenth PSE.

In all cases, estimates that deviate from normality at the 0.20 level, based on the *p*-values in the Parameter Estimate Population report, are labeled on the plot.

Half-Normal Plot

The half normal plot shows the absolute values of effects. The construction of the axes and the lines displayed mirror those aspects of normal plot.

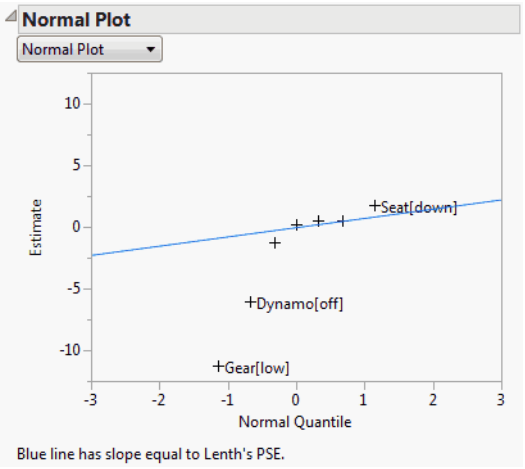
Figure 3.39 shows the Normal Plot report for the Bicycle.jmp sample data table. No transformation is needed for this model, so the plot shows the raw estimates plotted against their normal quantiles. A line with slope equal to Lenth's PSE is shown on the plot. The plot suggests that Gear, Dynamo, and Seat are active factors.

Example of a Normal Plot

1. Select **Help > Sample Data Folder** and open Bicycle.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Select HBars through Brkfast and click **Add**.
5. Click **Run**.
6. Click the Response Y red triangle and select **Effect Screening > Normal Plot**.

   The following Normal Plot appears.

Figure 3.39 Normal Plot



Bayes Plot Report

The Bayes Plot report in the Fit Least Squares report provides another approach to determining which effects are active. This report helps you compute posterior probabilities using a Bayesian approach. This method, due to Box and Meyer (1986), assumes that the estimates are a mixture from two distributions. The majority of the estimates, corresponding to inactive effects, are assumed to be pure random normal noise with variance σ^2. The remaining estimates, the active ones, are assumed to come from a *contaminating* distribution that has a variance K times larger than σ^2.

Term Gives the model term corresponding to the parameter estimate.

Estimate Gives the parameter estimate. The Bayes plot is constructed with respect to estimates that have estimated standard deviation equal to 1. If the estimates are not correlated, the *t*-Ratio is used. If the estimates are correlated, the Orthog *t*-Ratio is used.

Prior Prob Enables you to specify a probability that the estimate is nonzero (equivalently, that the estimate is in the contaminating distribution). Prior probabilities for estimates are usually set to equal values. The commonly recommended value of 0.2 appears initially, though you can change it.

K Contam The value of the contamination coefficient, representing the ratio of the contaminating distribution variance to the error variance. K is commonly set to 10, which is the default value.

Std Err Scale If there are degrees of freedom for an estimate of standard error, this value is set to 1. JMP uses this value because the estimates used in the report are transformed and scaled to unit variance. The value is set to 0 for a saturated model with no estimate of error. If you specify a different value, think of it in terms of a scale factor of the RMSE estimate.

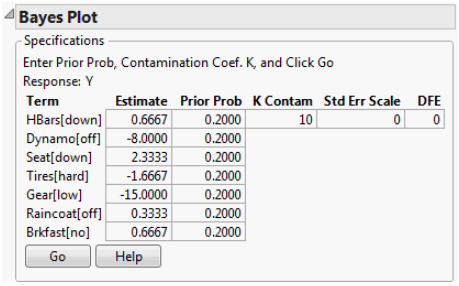
DFE Gives the error degrees of freedom.

The specifications window, showing default settings for a Bayes Plot for the Bicycle.jmp sample data table, is shown in Figure 3.40. Clicking Go in this window updates the report to show Posterior probabilities for each of the terms and a bar chart (Figure 3.41).

Example of a Bayes Plot

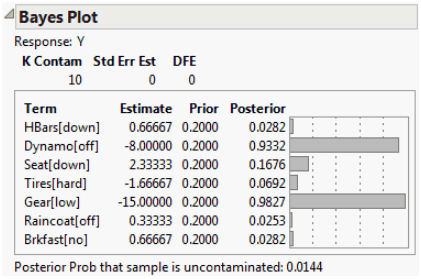
1. Select **Help > Sample Data Folder** and open Bicycle.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Select HBars through Brkfast and click **Add**.
5. Click **Run**.
6. Click the Response Y red triangle and select **Effect Screening > Bayes Plot**.

Figure 3.40 Bayes Plot Specifications



7. Click **Go** to calculate the posterior probabilities.

Figure 3.41 Bayes Plot Report



The note beneath the plot in the Bayes Plot report gives the Posterior Prob that the sample is uncontaminated. Posterior Prob is the probability, based on the priors and the data, that there are no active effects whatsoever. The probability is small, 0.0144, indicating that it is likely that there are active effects. The Posterior probability column suggests that at least Dynamo and Gear are active effects.

Pareto Plot Report

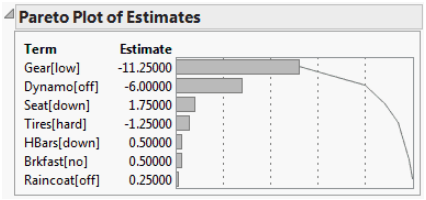
The Pareto Plot report in the Fit Least Squares report presents a Pareto chart of the absolute values of the estimates. Figure 3.42 shows a Pareto Plot report for the Bicycle.jmp sample data table.

- If the original estimates have equal variances and are not correlated, the original estimates are plotted.
- If the original estimates have unequal variances and are not correlated, the *t* ratios are plotted.
- If the original estimate have unequal variances and are correlated, the Orthog Coded estimates are plotted.

The cumulative sum line in the plot sums the absolute values of the estimates. Keep in mind that the orthogonal estimates depend on the order of entry of terms into the model.

Note: The estimates that appear in the plot are standardized to have equal variances and are transformed to be orthogonal. You have the option of undoing these transformations. See "Transformations".

Figure 3.42 Pareto Plot



Factor Profiling

In the Fit Least Squares report, the Factor Profiling options (accessed from the Response red triangle menu) help you explore and visualize your estimated model. You can explore the shape of the response surface, find optimum settings, simulate response data based on your specified noise assumptions, and transform the response if needed.

The Profiler, Contour Profiler, Mixture Profiler, and Surface Profiler are extremely versatile tools whose use extends beyond standard least squares models. For more information about their interpretation and use, see *Profilers*.

If the response column in your model contains an invertible transformation of one variable, the profilers and Interaction Plots show the response on the untransformed scale.

The following Factor Profiling options are available:

Note: If your model contains an expression or vector as an effect, most of these options are not available.

Profiler Shows prediction traces for each X variable. Enables you to find optimum settings for one or more responses and to explore response distributions using simulation. See "Profiler" and *Profilers*.

Interaction Plots Shows a matrix of interaction plots, when there are interaction effects in the model. See "Interaction Plots".

Contour Profiler Provides an interactive contour profiler, which is useful for optimizing response surfaces graphically. See "Contour Profiler" and *Profilers*.

Mixture Profiler Shows response contours of mixture experiment models on a ternary plot. See "Mixture Profiler" and *Profilers*.

> **Note:** This option appears only if you apply the Mixture Effect attribute to three or more effects or the Mixture property to three or more columns.

Cube Plots Shows predicted values for the extremes of the factor ranges laid out on the vertices of cubes. See "Cube Plots".

Box Cox Y Transformation Finds a Box-Cox power transformation of the response that is best in terms of satisfying the normality and homogeneity of variance assumptions. See "Box Cox Y Transformation".

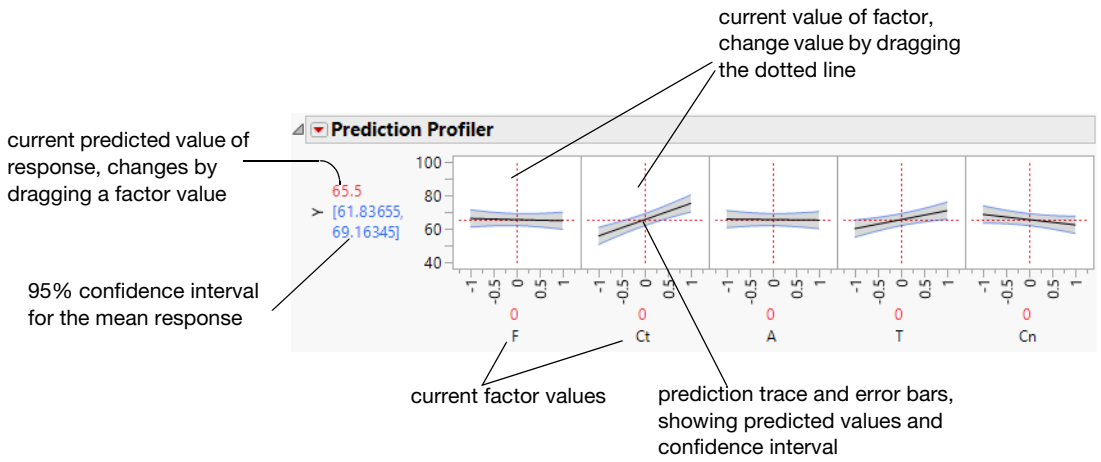
Surface Profiler Shows a three-dimensional surface plot of the response surface. See "Surface Profiler" and *Profilers*.

Profiler

In the Fit Least Squares report, the Profiler option shows prediction traces for each *X* variable. For more information about the profiler, see *Profilers*.

Figure 3.43 illustrates part of the profiler for the Reactor.jmp sample data table. The vertical dotted line for each *X* variable shows its *current value* or *current setting*. Use the Profiler to change one variable at a time in order to examine the effect on the predicted response.

Figure 3.43 Illustration of Prediction Traces



The factors F and Ct in Figure 3.43 are continuous. If the factor is nominal, the levels are displayed on the horizontal axis.

For each X variable, the value above the factor name is its current value. You change the current value by clicking in the graph or by dragging the dotted line where you want the new current value to be.

- The horizontal dotted line shows the *current predicted value* of each Y variable for the current values of the X variables.
- The black lines within the plots show how the predicted value changes when you change the current value of an individual X variable. The 95% confidence interval for the predicted values is shown by a dotted curve surrounding the prediction trace (for continuous variables) or an error bar (for categorical variables).

Interaction Plots

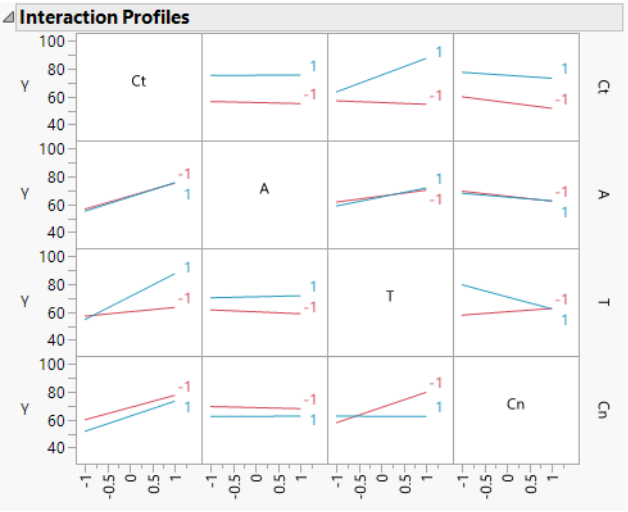
When there are interaction effects in the model, the Interaction Plots option in the Fit Least Squares report shows a matrix of interaction plots. Each cell of the matrix contains a plot whose horizontal axis is scaled for the effect displayed in the column in which the plot appears. Line segments are plotted for the interaction of that effect with the effect displayed in the corresponding row. So, an interaction plot shows the interaction of the row effect with the column effect.

A line segment is plotted for each level of the row effect. Response values predicted by the model are joined by line segments. Non-parallel line segments give visual evidence of possible interactions. However, the *p*-value for such a suggested interaction should be checked before concluding that it exists. Figure 3.44 gives an interaction plot matrix for the Reactor.jmp sample data table.

Example of Interaction Plots

1. Select **Help > Sample Data Folder** and open Reactor.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Make sure that the **Degree** box has a 2 in it.
5. Select Ct, A, T, and Cn and click **Macros > Factorial to Degree**.
6. Click **Run**.
7. Click the Response Y red triangle and select **Factor Profiling > Interaction Plots**.

Figure 3.44 Interaction Plots



The plot corresponding to the T*Cn interaction is the third plot in the bottom row of plots or equivalently, the third plot in the last column of plots. Either plot shows that the effect of Cn on Y is fairly constant at the low level of T, whether Cn is set at a high or low level. However, at the high level of T, the effect of Cn on Y differs based on its level. Cn at –1 leads to a higher predicted Y than Cn at 1. Note that this interaction is significant with a p-value < 0.0001.

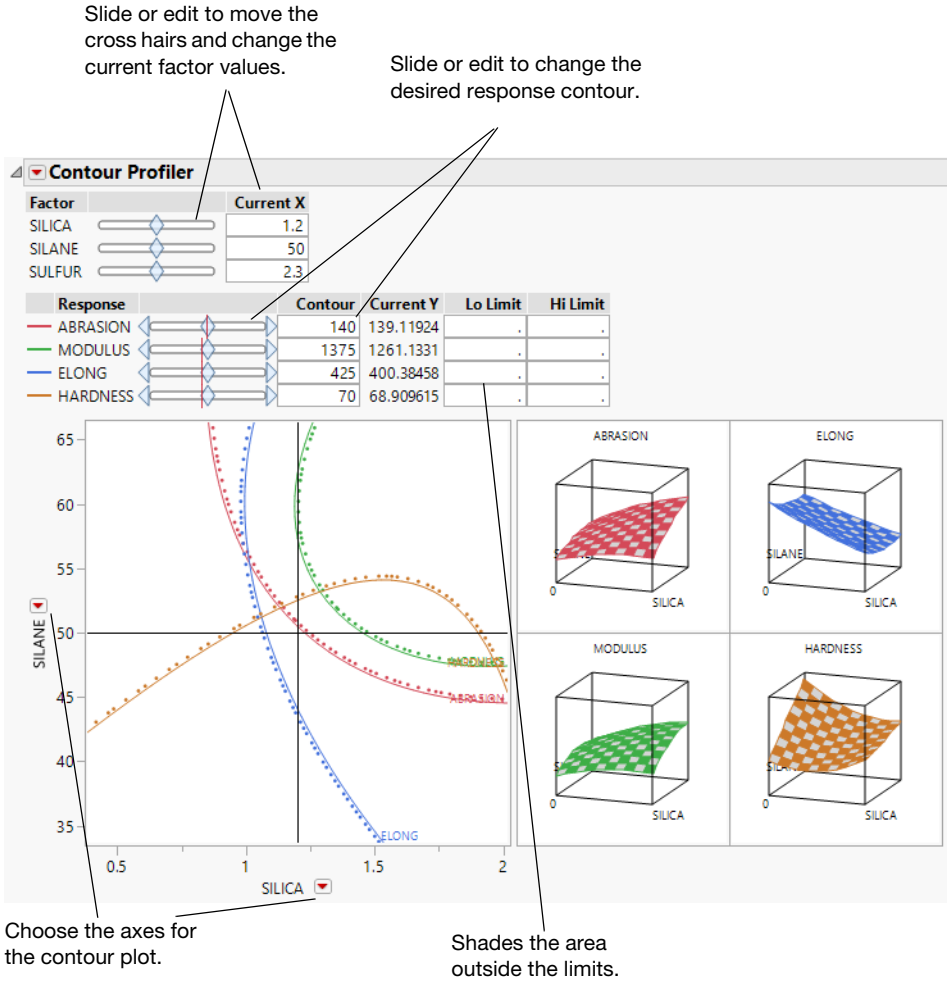
In certain designs, two-way interactions are aliased with other two-way interactions. When this aliasing occurs, cells in the Interaction Profiles plot corresponding to these interactions are dimmed.

Contour Profiler

In the Fit Least Squares report, use the interactive Contour Profiler option for optimizing response surfaces graphically. The Contour Profiler shows contours for the fitted model for two factors at a time. The report also includes a surface plot. For more information about the contour profiler, see *Profilers*.

Figure 3.45 shows a contour profiler view for the Tiretread.jmp sample data table. Run the data table script **RSM for 4 Responses** and select **Profilers > Contour Profiler** from the Least Squares Fit report menu.

Figure 3.45 Contour Profiler



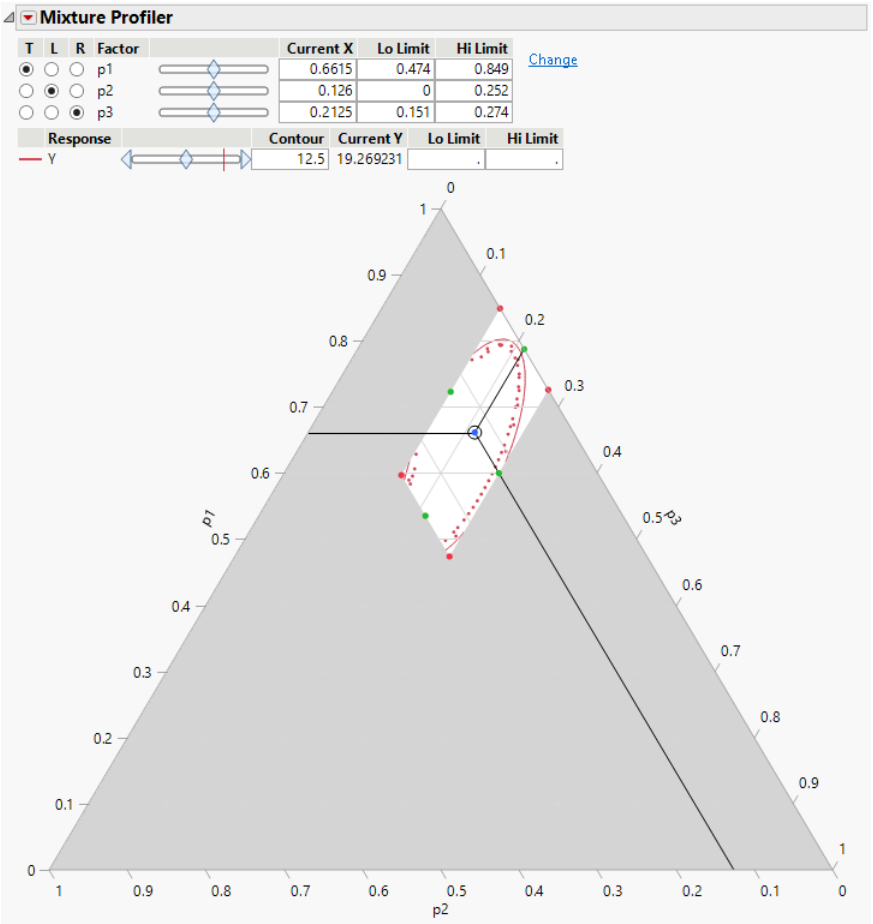
Mixture Profiler

In the Fit Least Squares report, the Mixture Profiler option shows response contours of mixture experiment models on a ternary plot. Use the Mixture Profiler when three or more factors in your experiment are components in a mixture. The Mixture Profiler helps you visualize and optimize the response surfaces of your experiment.

Note: This option appears only if you specify the **Macros > Mixture Response Surface** option for an effect. For more information about the mixture profiler, see *Profilers*.

Figure 3.46 shows the Mixture Profiler for the model in the Plasticizer.jmp sample data table. Run the **Model** data table script and then select **Factor Profiling > Mixture Profiler** from the report's red triangle menu. You modify plot axes for the factors by selecting different radio buttons at the top left of the plot. The Lo and Hi Limit columns at the upper right of the plot let you enter constraints for both the factors and the response.

Figure 3.46 Mixture Profiler



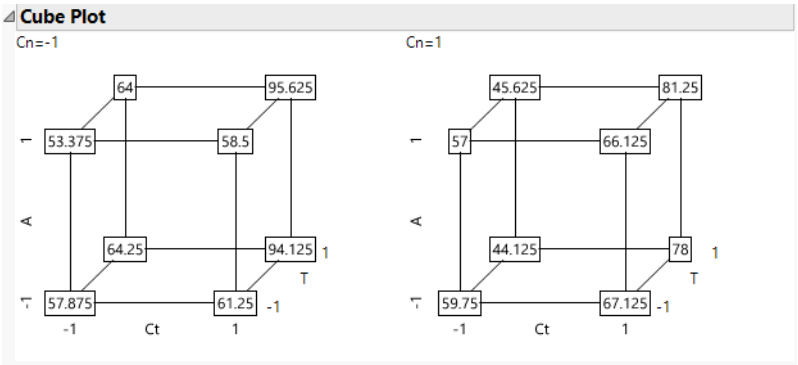
Cube Plots

In the Fit Least Squares report, the Cube Plots option shows predicted values for the extremes of the factor ranges. These values appear on the vertices of cubes (Figure 3.47). The vertices are defined by the smallest and largest observed values of the factor. When you have multiple responses, the multiple responses are shown stacked at each vertex.

Example of Cube Plots

1. Select **Help > Sample Data Folder** and open Reactor.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Make sure that the **Degree** box has a 2 in it.
5. Select Ct, A, T, and Cn and click **Macros > Factorial to Degree**.
6. Click **Run**.
7. Click the Response Y red triangle and select **Factor Profiling > Cube Plots**.

Figure 3.47 Cube Plots



Note that there is one cube for Cn = –1 and one for Cn = 1. To change the layout so that the factors are mapped to different cube coordinates, click a factor name in the first cube. Drag it to cover the factor name for the desired axis. For example, in Figure 3.47, if you click T and drag it over Ct, then T and Ct (and their corresponding coordinates) exchange places. To see the levels of Cn in a single cube, exchange it with another factor in the first cube by dragging it over that factor.

Box Cox Y Transformation

In the Fit Least Squares report, you can choose the Box-Cox Y Transformation option to transform the response so that the usual regression assumptions of normality and homogeneity of variance are more closely satisfied. The transformed response can then be fit using a regression model. However, you can also use the Box-Cox power transformation to transform a variable for other reasons. This transformation is appropriate only when the response, Y, is strictly positive.

A commonly used transformation raises the response to some power. Box and Cox (1964) formalized and described this family of power transformations. The formula for the transformation is constructed to provide a continuous definition in terms of the parameter λ, and so that the error sums of squares are comparable. Specifically, the following equation provides the family of transformations:

$$Y_\lambda = \begin{cases} \dfrac{y^\lambda - 1}{\lambda \dot{y}^{\lambda-1}} & \text{if } \lambda \neq 0 \\ \dot{y}\ln(y) & \text{if } \lambda = 0 \end{cases}$$

Here, $\dot{y}$ denotes the geometric mean.

The Box Cox Y Transformation option fits transformations from λ = –2 to 2 in increments of 0.2. To choose a value of λ, the likelihood function for each of these transformations is computed. They are computed under the assumption that the errors are independent and normal with mean zero and variance σ^2. The value of λ that maximizes the likelihood is selected. This value also minimizes the SSE over the values of λ. The value of λ that minimizes the SSE is found using a quadratic interpolation between the two incremental grid points surrounding the grid point with the smallest SSE. If this interpolation results in a negative SSE value, then the grid value of λ that minimizes the SSE is reported as the best λ.

The Box-Cox Transformations report displays a plot showing the sum of squared errors (SSE) values against the values of λ. The horizontal red line on the plot represents a one-sided 95% confidence interval for λ. This confidence interval is based on the confidence region defined in Box and Cox (1964, p. 216). The confidence region is defined by the following inequality:

$SSE(\lambda) < SSE(\lambda_{best}) * \exp(ChiSquareQuantile(0.95,1) / dfe)$

where

$SSE(\lambda_{best})$ is the SSE calculated using the reported Best λ

ChiSquareQuantile(0.95,1) is the 0.95^{th} quantile of a χ^2 distribution with 1 degree of freedom

dfe is the error degrees of freedom in the Analysis of Variance table for the regression model

The Box Cox Transformations report provides the following options:

Refit with Transform Enables you to specify a value for lambda to define a transformed Y variable and then provides a least squares fit to the transformed variable.

Replace with Transform Enables you to specify a value for lambda to define a transformed Y variable and then replaces the existing least squares fit with a fit to the transformed

variable. If you have multiple responses, Replace with Transform replaces only the report for the response that you are transforming.

Save Best Transformation Creates a new column in the data table and saves the formula for the best transformation.

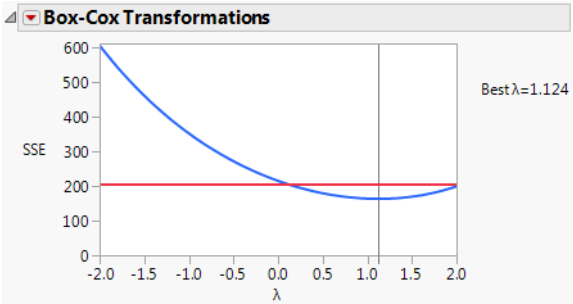
Save Specific Transformation Enables you to specify a value for lambda and creates a column in the data table with the formula for your specified transformation.

Table of Estimates Creates a new data table containing parameter estimates and SSE values for all λ from –2 to 2, in increments of 0.2.

Example of Box-Cox Y Transformation

1. Select **Help > Sample Data Folder** and open Reactor.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Make sure that the **Degree** box has a 2 in it.
5. Select F, Ct, A, T, and Cn and click **Macros > Factorial to Degree**.
6. Click **Run**.
7. Click the Response Y red triangle and select **Factor Profiling > Box Cox Y Transformation**.

Figure 3.48 Box Cox Y Transformation



The plot shows that the best values of λ are between 0.1 and 2.0. The value that JMP selects, using interpolation between the best two values in the 0.2-unit grid of λ values, is 1.124.

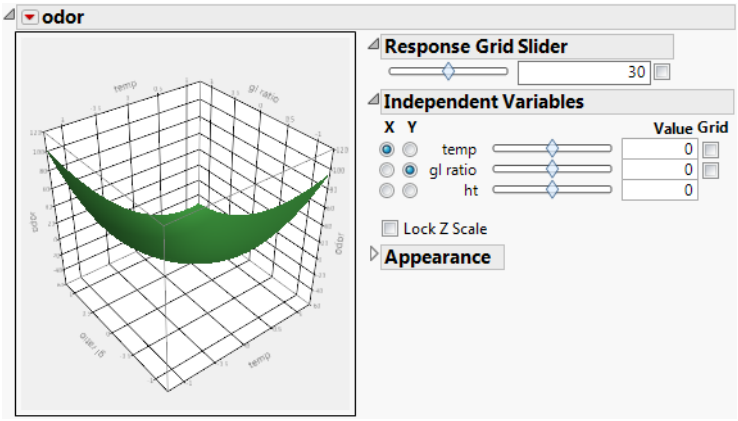
8. (Optional) To see the SSE values used to construct the graph, select **Table of Estimates**.

Surface Profiler

In the Fit Least Squares report, the Surface Profiler option shows a three-dimensional surface plot of the response surface. For more information about the surface profiler, see *Profilers*.

Figure 3.49 shows the Surface Profiler for the model in the Odor.jmp sample data table. Run the **Model** data table script and then select **Factor Profiling > Surface Profiler** from the report's red triangle menu. You can change the variables on the axes using the radio buttons under Independent Variables. Also, you can plot points by clicking Actual under Appearance.

Figure 3.49 Surface Plot



Row Diagnostics

In the Fit Least Squares report, the Row Diagnostics options (accessed from the Response red triangle menu) address issues specific to rows or observations.

Plot Regression Shows a Regression Plot report, displaying a scatterplot of the data and regression lines for each level of the categorical effect.

This option appears only if there is exactly one continuous effect and no more than one categorical effect in the model. In that case, the Regression Plot report is provided by default.

Plot Actual by Predicted Shows an Actual by Predicted plot, which plots the observed values of Y against the predicted values of Y. This plot is the leverage plot for the whole model. See "Effect Leverage Plots".

Note: The Actual by Predicted Plot is shown by default when Effect Leverage or Effect Screening is selected as the Emphasis in the Fit Model launch window and the RSquare value is less than 0.999.

Plot Effect Leverage Shows a Leverage Plot report for each effect in the model. The plot shows how observations influence the test for that effect and gives insight on multicollinearity. See "Effect Leverage Plots".

Note: Effect Leverage Plots are shown by default when Effect Leverage is selected as the Emphasis in the Fit Model launch window and the RSquare value is less than 0.999. They appear to the right of the Whole Model report. When another Emphasis is selected, the Effect Leverage Plots appear in the Effect Details report. In all cases, the option Regression Reports > Effect Details must be selected in order for Effect Leverage plots to display.

Plot Residual by Predicted Shows a Residual by Predicted Plot report. The plot shows the residuals plotted against the predicted values of Y. You typically want to see the residual values scattered randomly about zero.

Note: The Residual by Predicted Plot is shown by default when Effect Leverage or Effect Screening is selected as the Emphasis in the Fit Model launch window and the RSquare value is less than 0.999.

Plot Residual by Row Shows a Residual by Row Plot report. The residual values are plotted against the row numbers. This plot can help you detect patterns that result from the row ordering of the observations.

Plot Studentized Residuals Shows a Studentized Residuals plot. Each point on the plot is computed using an estimate of its standard deviation obtained with the current observation deleted. These residuals are also called *RStudent* or *externally Studentized* residuals.

The plot contains two sets of limits:

- The outer limits that appear in red on the plot are 95% Bonferroni limits. These limits are placed at ± tQuantile(0.025/n, n–p–1), where n is the number of observations and p is the number of predictors.
- The inner limits that appear in green on the plot are 95% individual t distribution limits. These limits are placed at ± tQuantile(0.025, n–p–1), where n is the number of observations and p is the number of predictors.

Points that fall outside the red limits should be treated as probable outliers. Points that fall outside the green limits but within the red limits should be treated as possible outliers, but with less certainty. The confidence level of 95% for these limits is not affected by your selection in the Set Alpha Level option in the Model Specification window.

Caution: The residuals saved using Save Columns > Studentized Residuals are *not* externally Studentized.

Note: If the model contains random effects and REML is the specified Method in the launch window, the Studentized Residuals plot does not contain limits and the points that are plotted are *not* externally Studentized.

Plot Residual by Normal Quantiles (Not available when REML is the specified Method in the launch window.) Shows a Residual Normal Quantile Plot. The residual values are plotted against quantiles of the normal distribution. This plot can help you assess the assumption of normality of the residuals.

Press Shows a Press Report giving the Press statistic and its root mean square error (RMSE). The Press statistic is useful when comparing multiple models. Models with lower Press statistics are favored. See "Press".

Durbin-Watson Test (Not available when you specify a Frequency column.) Shows the Durbin-Watson report, which gives a statistic to test whether the residuals have first-order autocorrelation. The report also displays the autocorrelation of the residuals and Prob<DW, which is the exact probability associated with the statistic. This option is appropriate only for time series data and assumes that your observations are in time order.

Effect Leverage Plots

In the Fit Least Squares report, the Plot Effect Leverage option is useful in the following ways:

- You can see which points might be exerting influence on the hypothesis test for X.
- You can spot unusual patterns and violations of the model assumptions.
- You can spot multicollinearity issues.

Construction

A leverage plot for an effect shows the impact of adding this effect to the model, given the other effects already in the model. For illustration, consider the construction of an effect leverage plot for a single continuous effect X. See "Horizontal Axis Scaling" for information about the scaling of the horizontal axis in more general situations.

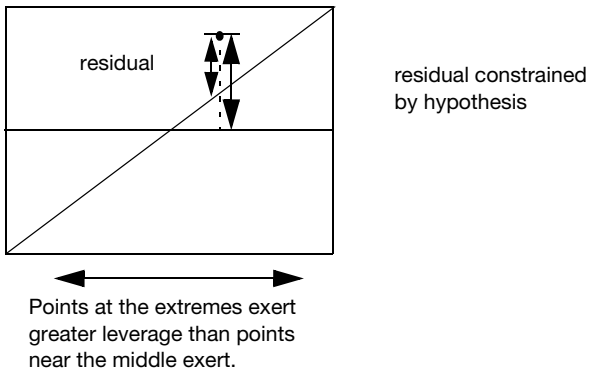
The response Y is regressed on all the predictors except X, and the residuals are obtained. Call these residuals the Y-residuals. Then X is regressed on all the other predictors in the model and the residuals are computed. Call these residuals the X-residuals. The X-residuals might contain information beyond what is present in the Y-residuals, which were obtained without X in the model.

The effect leverage plot for X is essentially a scatterplot of the X-residuals against the Y-residuals (Figure 3.52). To help interpretation and comparison with other plots that you might construct, JMP adds the mean of Y to the Y-residuals and the mean of X to the X-residuals. The translated Y-residuals are called the Y Leverage Residuals and the translated X-residuals are called X Leverage values. The points on the Effect Leverage plots are these X Leverage and Y Leverage Residual pairs.

JMP fits a least squares line to these points as well as confidence bands for the mean; the line of fit is solid red and the confidence bands are shaded red. The slope of the least squares line is precisely the estimate of the coefficient on X in the model where Y is regressed on X and the other predictors. The dashed horizontal blue line is set at the mean of the Y Leverage Residuals. This line describes a situation where the X residuals are not linearly related to the Y residuals. If the line of fit has nonzero slope, then adding X to the model can be useful in terms of explaining variation.

Figure 3.50 shows how residuals are depicted in the leverage plot. The distance from a point to the line of fit is the residual for a model that includes the effect. The distance from the point to the horizontal line is what the residual error would be without the effect in the model. In other words, the mean line in the leverage plot represents the model where the hypothesized value of the parameter (effect) is constrained to zero.

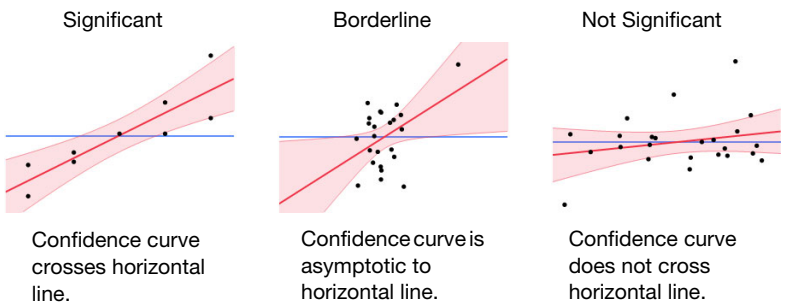
Figure 3.50 Illustration of a Generic Leverage Plot



Confidence Curves

Confidence curves for the line of fit are shown on leverage plots. These curves provide a visual indication of whether the test of interest is significant at the 5% level (or at the Set Alpha Level that you specified in the Fit Model launch window). If the confidence region between the curves contains the horizontal line representing the hypothesis, then the effect is not significant. If the curves cross the line, the effect is significant. See the examples in Figure 3.51.

Figure 3.51 Comparison of Significance Shown in Leverage Plots



Horizontal Axis Scaling

If the modeling type of a predictor X is continuous, then the horizontal axis is scaled in terms of the units of the X. The horizontal axis range mirrors the range of X values. The slope of the line of fit in the leverage plot is the parameter estimate for X. See the left illustration in Figure 3.52.

If the effect is nominal or ordinal, or if the effect is a complex effect such as an interaction, then the horizontal axis cannot represent the values of the effect directly. In this case the horizontal axis is scaled in units of the response, and the line of fit is a diagonal with a slope of 1. The Whole Model leverage plot, where the hypothesis of interest is that all parameter values are zero, uses this scaling. See "Statistical Details for Leverage Plots". For this plot, the horizontal axis is scaled in terms of predicted response values for the whole model, as illustrated by the right-hand plot in Figure 3.52.

The leverage plot for the linear effect in a simple regression is the same as the traditional plot of actual response values against the predictor.

Leverage

The term *leverage* is used because these plots help you visualize the influence of points on the test for including the effect in the model. A point that is horizontally distant from the center of the plot exerts more influence on the effect test than does a point that is close to the center. Recall that the test for an effect involves comparing the sum of squared residuals to the sum of squared residuals of the model with that effect removed. At the extremes, the differences of the residuals before and after being constrained by the hypothesis tend to be comparatively larger. Therefore, these residuals tend to have larger contributions to the sums of squares for that effect's hypothesis test.

Multicollinearity

Multicollinearity is a condition where two or more predictors are highly related, or more technically, involved in a nearly linear dependent relationship. When multicollinearity is present, standard errors can be inflated and parameters estimates can be unstable. If an effect is collinear with other predictors, the horizontal values of the points tend to cluster toward the middle of the plot. This situation indicates that the slope of the line of fit is unstable.

The Whole Model Actual by Predicted Plot

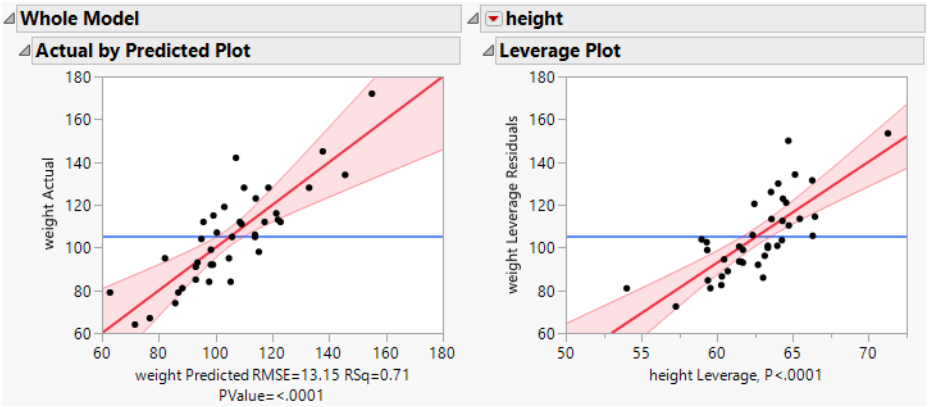
The Plot Effect Leverage option produces a leverage plot for each effect in the model. In addition, the Actual by Predicted plot can be considered to be a leverage plot. This plot enables you to visualize the test that all the parameters in the model (except the intercept) are zero. The same test is conducted analytically in the Analysis of Variance report. See "Statistical Details for Leverage Plots" for more information about this plot.

Example of a Leverage Plot for a Linear Effect

1. Select **Help > Sample Data Folder** and open Big Class.jmp.
2. Select **Analyze > Fit Model**.
3. Select weight and click **Y**.
4. Select height, age, and sex, and click **Add**.
5. Click **Run**.

The Whole Model Actual by Predicted Plot and the effect Leverage Plot for height are shown in Figure 3.52. The Whole Model plot, on the left, tests for all effects. You can infer that the model is significant because the confidence curves cross the horizontal line at the mean of the response, weight. The Leverage Plot for height, on the right, also shows that height is significant, even with age and sex in the model. Neither plot suggests concerns relative to influential points or multicollinearity.

Figure 3.52 Whole Model and Effect Leverage Plots



Press

In the Fit Least Squares report, the Press option represents the *prediction error sum of squares* statistic, which is an estimate of prediction error computed using leave-one-out cross validation. In leave-one-out cross validation, each observation, in turn, is removed. Consider a specific observation. The model is fit with that observation withheld and then a predicted value is obtained for that observation. The residual for that observation is computed. This procedure is applied to all observations and the residuals are squared and summed to give the Press value.

Specifically, the Press sum of squares for error (SSE) statistic is given by the following:

$$\text{Press} = \sum_{i=1}^{n} (\hat{y}_{(i)} - y_i)^2$$

where n is the number of observations, y_i is the observed response value for the i^{th} observation, and $\hat{y}_{(i)}$ is the predicted response value for the i^{th} observation. These values are based on a model fit without including that observation.

The Press RMSE is defined as $\sqrt{Press/n}$.

The Press RSquare is defined as 1 - $Press/SS_{\text{Total}}$.

The Press report also contains the SSE, RMSE, and RSquare values for the overall model. The statistics for the overall model are in the row labeled Ordinary.

Save Columns

In the Fit Least Squares report, the Save Columns options (accessed from the Response red triangle menu) add one or more new columns to the current data table. Additional Save Columns options appear when the fitting method is REML. These are detailed in "REML Save Columns Options".

Note the following:

- When formulas are created, they are entered as Formula column properties.
- For many of the new columns, a Notes column property is added describing the column and indicating that Fit Least Squares created it.
- For the Predicted Formula and Predicted Values options, a Predicting column property is added. This property is used internally by JMP in conducting model comparisons (Analyze > Predictive Modeling > Model Comparison). When you fit many models, it is also useful to you because it documents the origin of the column.

The following Save Columns options are available:

Prediction Formula Creates a new column called Pred Formula <colname> that contains both the formula and the predicted values. A Predicting column property is added, noting the source of the prediction.

Note: Pred Formula <colname> inherits certain properties from <colname>. These include Response Limits, Spec Limits, and Control Limits. If you change these properties for <colname> after saving Pred Formula <colname>, they will not update in Pred Formula <colname>.

See "Prediction Formula".

Predicted Values Creates a new column called Predicted <colname> that contains the predicted values computed by the specified model. Both a Notes and a Predicting column property are added, noting the source of the prediction.

Note: Predicted <colname> inherits certain properties from <colname>. These include Response Limits, Spec Limits, and Control Limits. If you change these properties for <colname> after saving Predicted <colname>, they will not update in Predicted <colname>.

Residuals Creates a new column called Residual <colname> that contains the observed response values minus their predicted values.

Mean Confidence Interval Creates two new columns called Lower 95% Mean <colname> and Upper 95% Mean <colname>. These columns contain the lower and upper 95% confidence limits for the mean response.

Note: If you press Shift while selecting the option, you are prompted to enter an α level for the computations.

Indiv Confidence Interval Creates two new columns called Lower 95% Indiv <colname> and Upper 95% Indiv <colname>. These columns contain lower and upper 95% confidence limits for individual response values.

Note: If you press Shift while selecting the option, you are prompted to enter an α level for the computations.

Studentized Residuals Creates a new column called Studentized Resid <colname> that contains the residuals divided by their standard errors.

Externally Studentized Residuals (Not available when the fitting method is REML.) Creates a new column called Externally Studentized Residuals <colname> that contains the residuals divided by standard error estimates that exclude the current row. See "Plot Studentized Residuals".

Hats Creates a new column called h <colname>. The column values are the diagonal values of the matrix $X(X'X)^{-1}X'$, sometimes called hat values.

Std Error of Predicted Creates a new column called StdErr Pred <colname> that contains the standard errors of the predicted mean response.

Std Error of Residual Creates a new column called StdErr Resid <colname> that contains the standard errors of the residual values.

Std Error of Individual Creates a new column called StdErr Indiv <colname> that contains the standard errors of the individual predicted response values.

Effect Leverage Pairs Creates a set of new columns that contain the X Leverage values and Y Leverage Residuals for each leverage plot. For each effect in the model, two columns are added. If the response column name is R and the effect is X, the new column names are:

- X Leverage of X for R
- Y Leverage of X for R

In the columns panel, these columns are organized in a columns group called Leverage.

Cook's D Influence Creates a new column called Cook's D Influence <colname>, which contains values of the Cook's *D* influence statistic.

StdErr Pred Formula Creates a new column called PredSE <colname> that contains both the formula and the values for the standard error of the predicted values.

Note: The saved formula can be large. If you do not need the formula, use the Std Error of Predicted option.

Mean Confidence Limit Formula Creates two new columns called Lower 95% Mean <colname> and Upper 95% Mean <colname>. These columns contain both the formulas and the values for lower and upper 95% confidence limits for the mean response.

Note: If you press Shift while selecting the option, you are prompted to enter an α level for the computations.

Indiv Confidence Limit Formula Creates two new columns called Lower 95% Indiv <colname> and Upper 95% Indiv <colname>. These columns contain both the formulas and the values for lower and upper 95% confidence limits for individual response values.

Note: If you press Shift while selecting the option, you are prompted to enter an α level for the computations.

Save Coding Table Creates a new data table whose first columns show the JMP coding for all model parameters. The last column gives the values of the response. If you entered more than one response column, all of these columns appear as the last columns in the coding table.

Note: The coding data table contains a table variable called Original Data that gives the name of the data table that was used for the analysis. In the case where a By variable is specified, the Original Data table variable gives the By variable and its level.

JMP PRO **Publish Prediction Formula** Creates a prediction formula and saves it as a formula column script in the Formula Depot platform. If a Formula Depot report is not open, this option creates a Formula Depot report. See *Predictive and Specialized Modeling*.

JMP PRO **Publish Standard Error Formula** Creates a standard error formula and saves it as a formula column script in the Formula Depot platform. If a Formula Depot report is not open, this option creates a Formula Depot report. See *Predictive and Specialized Modeling*.

JMP PRO **Publish Mean Confid Limit Formula** Creates confidence limit formulas for the mean response and saves them as formula column scripts in the Formula Depot platform. If a Formula Depot report is not open, this option creates a Formula Depot report. See *Predictive and Specialized Modeling*.

JMP PRO **Publish Indiv Confid Formula** Creates confidence formulas for individual response values and saves them as formula column scripts in the Formula Depot platform. If a

Formula Depot report is not open, this option creates a Formula Depot report. See *Predictive and Specialized Modeling*.

Prediction Formula

In the Fit Least Squares report, the Prediction Formula option is useful for predicting values in new rows or for use with the profilers. This option saves a column called Pred Formula <colname> to the data table. Pred Formula <colname> differs from Predicted <colname> in that it contains the prediction formula. Right-click in the Pred Formula <colname> column heading and select **Formula** to see the prediction formula. The prediction formula can require considerable space if the model is large. If you do not need the formula with the column of predicted values, use the **Save Columns > Predicted Values** option. For information about formulas, see *Using JMP*.

Use Prediction Formulas with Profilers

Profilers are available from the Response red triangle menu under Factor Profiling. However, when your data table includes formula columns, you can also use the profilers provided in the **Graph** menu. When you are analyzing multiple responses, accessing the profilers from the **Graph** menu can be useful.

Note: If you select **Graph > Profiler** to access the profilers, first save the formula columns to the data table using Prediction Formula and StdErr Pred Formula. Then place both of these formulas into the Y, Prediction Formula role in the Profiler window. After you click **OK**, specify whether you want to use PredSE <colname> to construct confidence intervals for Pred Formula <colname>. Otherwise, JMP creates a separate profiler plot for PredSE <colname>.

Multiple Comparisons

In the Fit Least Squares report, use the Multiple Comparisons option to obtain tests and confidence levels that compare means defined by levels of your model effects. The goal of multiple comparisons methods is to determine whether group means differ, while controlling the probability of reaching an incorrect conclusion. The Multiple Comparisons option enables you to compare group means with the overall average (analysis of means) and with a control group mean. You can also conduct pairwise comparisons using either Tukey HSD or Student's *t*. You can also perform equivalence tests to identify pairwise differences that are of practical importance.

The Student's *t* and equivalence testing methods control only the error rate for an individual comparison. As such, they are not true multiple comparison procedures. All other methods provided control the overall error rate for all comparisons of interest. Each of these methods uses a multiple comparison *adjustment* in calculating *p*-values and confidence limits.

If your model contains nominal and ordinal effects, you can conduct comparisons using Least Squares Means estimates, or you can define specific comparisons using User-Defined Estimates. If your model contains only continuous effects, you can compare means using User-Defined Estimates.

Tip: Suppose that a continuous effect consists of relatively few levels. If you are interested in comparisons using Least Squares Means Estimates, consider assigning that effect an ordinal (or nominal) modeling type.

This section contains information about the following topics:

- "Launch the Multiple Comparisons Option"
- "Comparisons with Overall Average"
- "Comparisons with Control"
- "All Pairwise Comparisons"
- "Equivalence Tests"

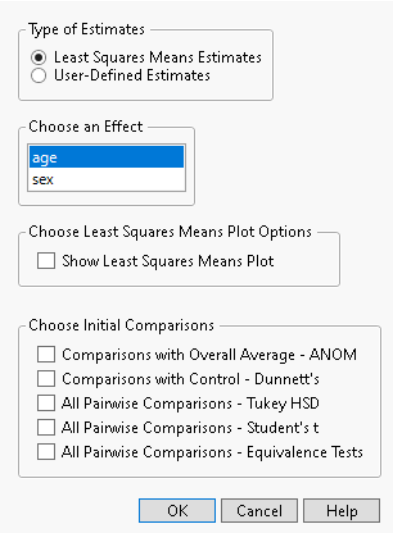
Launch the Multiple Comparisons Option

In the Fit Least Squares report, click the Response red triangle menu and select Multiple Comparisons. An example of the control window for the Multiple Comparisons option in Standard Least Squares is shown in Figure 3.53. This example is based on the Big Class.jmp data table, with weight as Y and age, sex, and height as model effects. Two classes of estimates are available for comparisons: Least Squares Means Estimates and User-Defined Estimates.

Least Squares Means Estimates

This option compares least squares means and is available only if there are nominal or ordinal effects in the model. Recall that least squares means are means computed at some neutral value of the other effects in the model. (For a definition of least squares means, see "LSMeans Table".) You must select the effect of interest. In Figure 3.53, Least Squares Means Estimates for age are specified. There is an option to show the least squares means plot. See "Least Squares Means Plot Options".

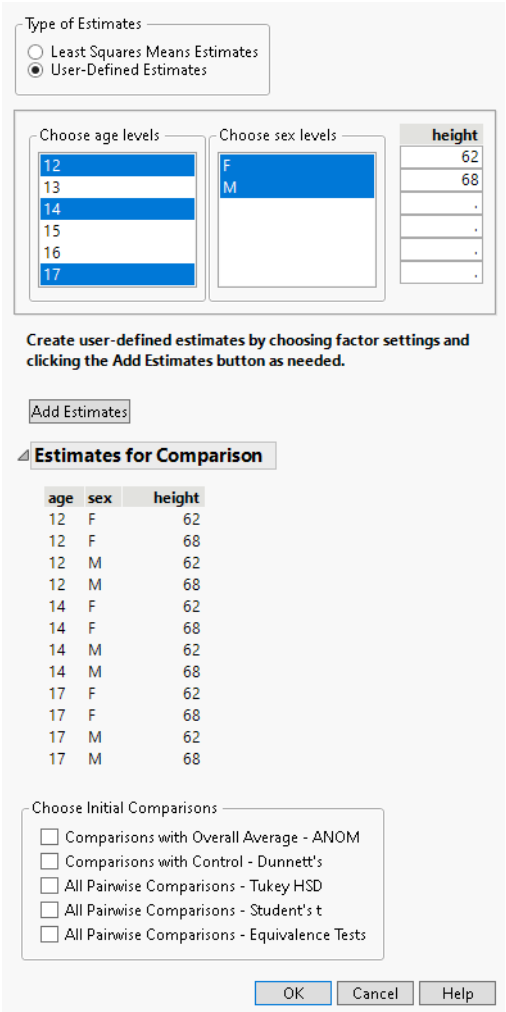
Figure 3.53 Launch Window for Least Squares Means Estimates



User-Defined Estimates

The specification of User-Defined Estimates is illustrated in Figure 3.54. Three levels of age and both levels of sex have been selected. Also, two values of height have been manually entered. The Add Estimates button has been clicked, which results in the listing of all possible combinations of the specified levels. At this point, you can specify more estimates and click the Add Estimates button again to add them to the Estimates for Comparison table.

Figure 3.54 Launch Window for User-Defined Estimates



When you use User-Defined Estimates, effects that have no specified levels are set as follows, according to their modeling type:

- Continuous effects are set to the mean of the effect.
- Nominal and ordinal effects are set to the first level in the value ordering.

Note: In this section, the term *mean* is used to refer to either estimates of least squares means or user-defined estimates.

Choose Least Squares Means Plot Options

Select the Show Least Squares Means Plot option to obtain a least square means plot. If your effect is an interaction term, then you have the option to create an interaction plot. You select the term for the overlay. If you do not select the interaction plot, then the least squares plot will nest the effect terms. See "Least Squares Means Plot Options".

Choose Initial Comparisons

Once you have specified estimates, you can choose the types of comparisons that you would like to see in your initial report by making selections under Choose Initial Comparisons. Or click OK without making any selections.

Comparisons with Overall Average - ANOM Compares each effect least squares mean with the overall least squares mean. (Analysis of Means).

Comparisons with Control - Dunnett's Compares each effect least squares mean with the least squares mean of a control level.

Tip: You can add a Control Level column property to the factor column to avoid specifying the control group each time you select Comparison with Control - Dunnett's. See *Using JMP*.

All Pairwise Comparisons - Tukey HSD Tests all pairwise comparisons of the effect least squares means using the Tukey HSD adjustment for multiplicity.

All Pairwise Comparisons - Student's t Tests all pairwise comparisons of the effect least squares means with no multiplicity adjustment.

All Pairwise Comparisons - Equivalence Tests Tests all pairwise comparisons of the effect least squares means against a specified difference that is deemed practically equivalent.

Each of these selections opens a report with an area at the top that shows details specific to the report. This information includes the quantile, or critical value. For the true multiple comparisons procedures, the method used for the multiple comparison adjustment is shown. For the equivalence tests, the structure of the tests is shown. If you have specified User-Defined Estimates, the report contains a list of effects that do not vary relative to the specified estimates and the levels at which these effects are set. Unless you have specified otherwise, any continuous effect is set to its mean. Any nominal or ordinal effect is set to the first level in its value ordering.

If you click OK without selecting from the Choose Initial Comparisons list, the Multiple Comparisons report opens, showing the Least Squares Means Estimates table or the User-Defined Estimates table. From the Multiple Comparison red triangle menu, all of the options listed above are available. The available reports and options are described below.

Least Squares Means or User-Defined Estimates Report

By default, the Multiple Comparisons option displays a Least Squares Means Estimates report or a User-Defined Estimates report, depending on the type of estimates that you selected in the Multiple Comparisons launch window. For each combination of levels of interest, this table gives an estimate of the mean, as well as a test and confidence interval. Specifically, this table contains the following columns:

Levels of the Categorical Effects The first columns in the report identify the effect or effects of interest. The values in the columns specify the groups being analyzed.

Estimate An estimate of the mean for each group.

Std Error The standard error of the mean for each group.

DF The degrees of freedom for a test of whether the mean is 0.

Lower 95% The lower confidence limit for the mean. You can change the confidence level by selecting Set Alpha Level in the Fit Model window.

Upper 95% The upper confidence limit for the mean.

t Ratio (Appears only if you right-click in the report and select Columns > t Ratio.) The *t* ratio for the significance test.

Prob>|t| (Appears only if you right-click in the report and select Columns > Prob>|t|.) The *p*-value for the significance test.

Arithmetic Mean Estimate (Appears only in the Least Squares Means Estimates report.) An estimate of the arithmetic mean for each group.

N (Appears only in the Least Squares Means Estimates report.) The number of observations used to calculate the mean for each group.

Comparisons with Overall Average

When you select the Multiple Comparisons option, you can choose the initial comparison to be with the overall average. This option compares the means for the specified levels specified to the overall mean for these levels. It displays a table showing confidence intervals for differences from the overall mean and a chart showing decision limits. The method used to make the comparisons is called analysis of means (ANOM) (Nelson et al. 2005). ANOM is a multiple comparison procedure that controls the joint error rate for all pairwise comparisons to the overall mean. See Figure 3.55 for a report based on the Lipid Data.jmp sample data table.

ANOM might appear similar to analysis of variance. However, it is fundamentally different in that it identifies levels with means that differ from the overall mean for all levels. In contrast, analysis of variance tests for differences in the means themselves.

At the top of the Comparisons with Overall Average report, you find:

Quantile The value of Nelson's h statistic used in constructing the decision limits.

Adjusted DF The degrees of freedom used in constructing the decision limits.

Avg The average mean. For least squares estimates, the average mean is a weighted average of the group least squares means. This weighted average represents the overall mean at the neutral settings where the group least squares means are calculated.

Specifically, the average least squares mean is a weighted average with weights inversely proportional to the diagonal entries of the matrix $L(X'X)^{-1}L'$. Here L is the matrix of coefficients used to compute the group least squares means. For a technical definition of least squares means, see the GLM Procedure chapter in SAS Institute Inc. (2022b).

For user-defined estimates, the average mean is defined similarly. However, in this case, L is the matrix of coefficients used to define the estimates.

Adjustment Describes the method used to obtain the critical value:

Nelson Provides exact critical values and p-values. Used whenever possible, in particular, when the estimates are uncorrelated.

Nelson-Hsu Provides approximate critical values and p-values based on Hsu's factor analytical approximation is used (Hsu 1992). Used when exact values cannot be obtained.

Sidak Used when both Nelson and Nelson-Hsu fail.

For technical details, see the GLM Procedure chapter in SAS Institute Inc. (2022b).

Three options are available from the Comparisons with Overall Average report menu:

Differences from Overall Average

For each comparison of a group's mean to the overall mean, this report provides the following details:

- The levels being compared
- Difference - the estimated difference
- Std Error - the standard error of the difference
- Lower and Upper limits for the confidence interval
- t Ratio - the ratio of the Difference and Std Error columns

Comparisons with Overall Average Decision Chart

This decision chart plots a point at the mean for each group. A horizontal line is plotted at the average mean. Upper and lower decision limits are plotted. Suppose that a point corresponding to a group mean falls outside these limits. This occurrence indicates that the group mean differs from the overall mean, based on the analysis of means test at the specified significance level. The significance level is shown below the chart.

The Comparisons with Overall Average Decision Chart report menu has these options:

Show Summary Report Produces a table showing the estimate, decision limits, and the limit exceeded for each group

Display Options Gives several options for controlling the display of the chart.

Calculate Adjusted P-Values

Adds a column that contains *p*-values (Prob>|t|) to the Comparisons with Overall Average report. Note that computing exact critical values and *p*-values for unbalanced designs requires complex integration and can be computationally challenging. When calculations for such a quantile fail, the Sidak quantile is computed but *p*-values are not available.

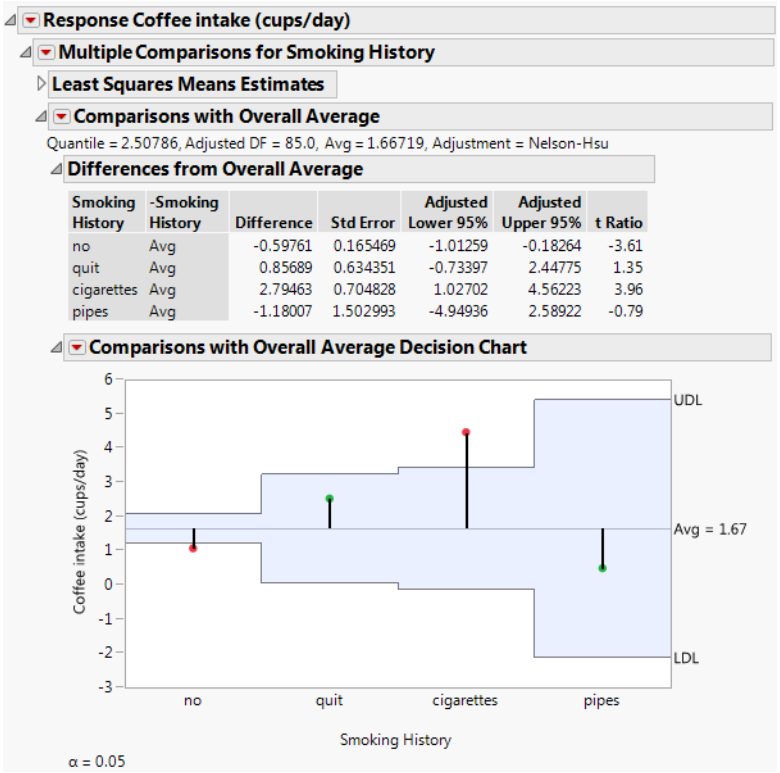
Example of Comparisons with Overall Average

Consider the Lipid Data.jmp sample data table. You are interested in whether any of the four Smoking History categories are unusual in that their mean Coffee intake (cups/day) differ from the overall average coffee intake while controlling for alcohol use and heart history. You specify a model with Coffee intake (cups/day) as the response and Smoking History, Alcohol Use, and Heart History as model effects.

1. Select **Help > Sample Data Folder** and open Lipid Data.jmp.
2. Select **Analyze > Fit Model**.
3. Select Coffee intake (cups/day) and click **Y**.
4. Select Smoking History, Alcohol Use, and Heart History, and click **Add**.
5. Click **Run**.
6. Click the red triangle next to Response Coffee intake (cups/day) and select **Estimates > Multiple Comparisons**.
7. From the Choose an Effect list, select Smoking History.
8. In the Choose Initial Comparisons list, select **Comparisons with Overall Average - ANOM**.
9. Click **OK**.

The results shown in Figure 3.55 indicate that the least squares means for non-smokers and cigarette smokers differ significantly from the overall average in terms of coffee intake.

Figure 3.55 Comparisons with Overall Average for Ratings



Comparisons with Control

When you select the Multiple Comparisons option, you can choose the initial comparison to be with a control group. If you select Comparisons with Control - Dunnett's, a window opens, asking you to specify a control group. If you selected Least Squares Means Estimates, the list consists of all levels of the effect you that you selected. If you selected User-Defined Estimates, the list consists of the combinations of effect levels that you specified.

Tip: You can add a Control Level column property to the factor column to avoid specifying the control group each time you select Comparisons with Control - Dunnett's. See *Using JMP*.

After you choose a control group and click OK, the Comparisons with Control report appears in your Fit Least Squares report. This option compares the means for the specified settings to the control group mean. It displays a table showing confidence intervals for differences from the control group and a chart showing decision limits. Dunnett's method is used to make the comparisons. Dunnett's method is a multiple comparison procedure that controls the error rate over all comparisons (Hsu 1996; Westfall et al. 2011).

When exact calculation of *p*-values and confidence intervals is not possible, Hsu's factor analytical approximation is used (Hsu 1992). Note that computing exact critical values and *p*-values for unbalanced designs requires complex integration and can be computationally intensive. When calculations for such a quantile fail, the Sidak quantile is computed.

In addition to the list of effects that do not vary for the specified estimates, at the top of the Comparisons with Control report you also find:

Quantile The critical value for Dunnett's test.

Adjusted DF The degrees of freedom used in constructing the confidence intervals.

Control The setting that defines the control group. This is a single level if you have selected a single effect; it is a combination of levels if you specified a user-defined combination of more than one effect.

Adjustment The method used to obtain the critical value:

Dunnett Provides exact critical values and *p*-values. Used whenever possible, in particular, when the estimates are uncorrelated.

Dunnett-Hsu Provides approximate critical values and *p*-values based on Hsu's factor analytical approximation (Hsu 1992). Used when exact values cannot be obtained.

Sidak Used when both Dunnett and Dunnett-Hsu fail.

For technical details, see the GLM Procedure chapter in SAS Institute Inc. (2022b).

Three options are available from the Comparisons with Control report menu:

Differences from Control

For each comparison of a group mean to the control mean, this report provides the following details:

- The levels being compared
- Difference - the estimated difference
- Std Error - the standard error of the difference
- Lower and Upper limits for the confidence interval
- t Ratio - the ratio of the Difference and Std Error columns

Comparisons with Control Decision Chart

This decision chart plots a point at the mean for each group being compared to the control group. A horizontal line shows the mean for the control group. Upper and lower decision limits are plotted. When a point falls outside these limits, it corresponds to a group whose mean differs from the control group mean based on Dunnett's test at the specified significance level. That level is shown beneath the chart.

The Comparisons with Control Decision Chart report menu has these options:

Show Summary Report Produces a table showing the estimate, decision limits, and the limit exceeded for each group

Display Options Gives several options for controlling the display of the chart.

Calculate Adjusted P-Values

Adds a column that contains *p*-values (Prob>|t|) to the Comparisons with Control report. Note that computing exact critical values and *p*-values for unbalanced designs requires complex integration and can be computationally challenging. When calculations for such a quantile fail, the Sidak quantile is computed but *p*-values are not available.

All Pairwise Comparisons

When you select the Multiple Comparisons option, you can choose the initial comparison to be with all pairwise comparisons. The All Pairwise Comparisons option shows either a Tukey HSD All Pairwise Comparisons or Student's *t* All Pairwise Comparisons report (Hsu 1996; Westfall et al. 2011). Tukey HSD comparisons are constructed so that the significance level applies jointly to all pairwise comparisons. In contrast, for Student's *t* comparisons, the significance level applies to each individual comparison. When making several pairwise comparisons using Student's *t* tests, the risk that one of the comparisons incorrectly signals a difference can well exceed the stated significance level.

At the top of the Tukey HSD All Pairwise Comparisons report you find:

Quantile The critical value for the test. Note that, for Tukey HSD, the quantile is $q/(\sqrt{2})$, where q is the appropriate percentage point of the Studentized range statistic.

Adjusted DF The degrees of freedom used in constructing the confidence intervals.

Adjustment Describes the method used to obtain the critical value:

Tukey Provides exact critical values and *p*-values. Used when the means are uncorrelated and have equal variances, or when the design is variance-balanced.

Tukey-Kramer Provides approximate critical values and *p*-values. Used when exact values cannot be obtained.

For technical details, see the GLM Procedure chapter in SAS Institute Inc. (2022b).

The top of the Student's t All Pairwise Comparisons report shows the Quantile, or critical value, for the *t* test and DF, the degrees of freedom used for the *t* test.

All Pairwise Differences Report

Both Tukey HSD and Student's *t* compare all pairs of levels. For each pairwise comparison, the All Pairwise Differences report shows:

- The levels being compared
- Difference - the estimated difference between the means
- Std Error - the standard error of the difference
- t Ratio - the *t* ratio for the test of whether the difference is zero
- Prob > |t| - the *p*-value for the test
- Lower and Upper limits for a confidence interval for the difference in means

This report also contains a plot column that shows a visual representation of the confidence interval for each difference between means. Colors indicate which differences are significant.

All Pairwise Comparisons Scatterplot

This plot, sometimes called a *diffogram* or a *mean-mean scatterplot*, displays the confidence intervals for all means pairwise differences. (See Figure 3.57 for an example.) Colors indicate which differences are significant.

The plot shows a reference line as an upwardly sloping line on the diagonal. This line represents points where the two means are equal. Each line segment corresponds to a confidence interval for a pairwise comparison. The coordinates of the point displayed on the line segment are the means for the corresponding groups. Hover over one of these points to show a tooltip that identifies the groups being compared and shows the estimated difference. If a line segment crosses the line on the diagonal, then the means can be equal and the comparison is not significant.

The Pairwise Comparisons Scatterplot has the following option:

Show Reference Lines Displays reference grid lines for the points on the scatterplot. This is not recommended if there are many points in the scatterplot. If there are many points, it is better to hover over the points to view the tooltip labels.

All Pairwise Differences Connecting Letters

Use this option to display a report that illustrates significant and non-significant comparisons with connecting letters. Levels not connected by the same letter are significantly different. Levels connected by the same letter are not significantly different.

Save All Pairwise Differences Connecting Letters Table

This option creates a data table whose columns contain the levels of the effect, the connecting letters, the least squares means, their standard errors, and confidence intervals. The data table contains a script called Bar Chart that produces a colored bar chart of the least squares means with their confidence intervals superimposed. The levels are arranged in decreasing order of least squares means.

Example of Tukey HSD All Pairwise Comparisons

Consider the Lipid Data.jmp sample data table. You are interested in Cholesterol differences for gender and non-smokers versus former smokers (Smoking History equal to no and quit, respectively) across two ages (25 and 35) and average height.

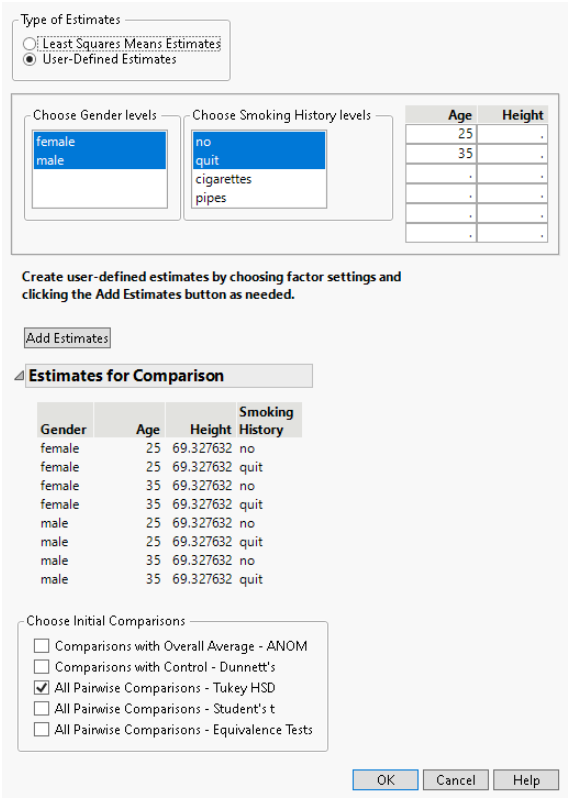
1. Select **Help > Sample Data Folder** and open Lipid Data.jmp.
2. Select **Analyze > Fit Model**.
3. Select Cholesterol and click **Y**.
4. Select Gender, Age, Height, and Smoking History, and click **Add**.
5. Click **Run**.
6. Click the red triangle next to Response Cholesterol and select **Estimates > Multiple Comparisons**.
7. From the Type of Estimates list, click **User-Defined Estimates**.
8. From the Choose Gender levels list, select female (it should already be selected by default) and male.
9. From the Choose Smoking History levels list, select no and quit.
10. In the Age list, enter the ages 25 and 35 in the first two rows.

    Do not enter any values in the list entitled Height. Because no values for Height are specified, the mean value of the Height column is used in the multiple comparisons report.
11. Click **Add Estimates**.

    Note that all possible combinations of the levels that you specified appear in the Estimates for Comparison report.
12. In the Choose Initial Comparisons list, select **All Pairwise Comparisons - Tukey HSD**.

    Check that your window is completed as shown in Figure 3.56.

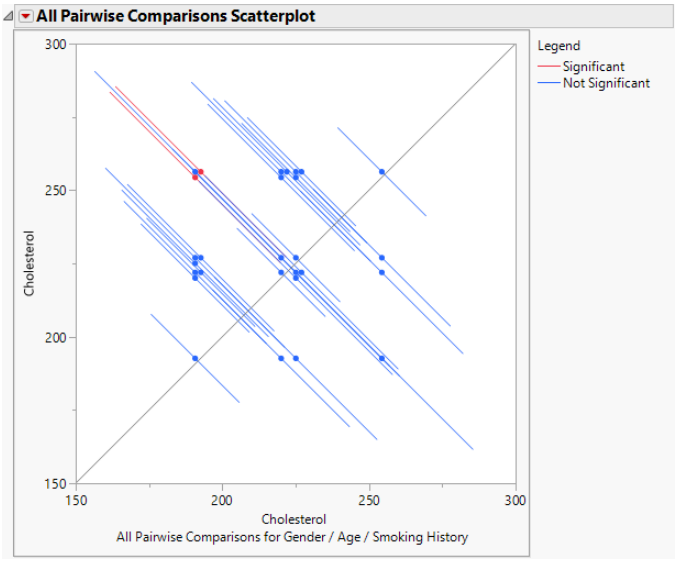
Figure 3.56 Populated Used-Defined Estimates Window



13. Click **OK**.

    The All Pairwise Differences report indicates that two of the 28 pairwise comparisons are significant. The All Pairwise Comparisons Scatterplot shows the confidence intervals for these comparisons in red. You can hover over any of the points to determine which pairwise comparison the point represents. The tooltips also contain the difference between the two levels in the comparison. The two red points represent the points comparing 35-year-old former smokers to 25-year-old non-smokers, for both females and males.

Figure 3.57 All Pairwise Comparisons Scatterplot for User-Defined Comparisons



Equivalence Tests

When you select the Multiple Comparisons option, you can choose the initial comparison to use equivalence testing to test for practical differences. Use this option to conduct one or more equivalence tests. Equivalence tests are useful when you want to detect differences that are of *practical* interest. You must specify a threshold difference for group means for which smaller differences are considered practically equivalent. In other words, if two group means differ by this amount or less, you are willing to consider them equivalent.

Once you have specified this value, the Equivalence Tests report appears. The bounds that you have specified are given at the top of the report. The report consists of a table giving the equivalence tests and a scatterplot that displays them. The equivalence tests and confidence intervals are based on Student's *t* critical values.

Equivalence Test Report

The Two One-Sided Tests (TOST) method is used to test for a practical difference between the means (Schuirmann 1987). Two one-sided pooled-variance *t* tests are constructed for the null hypotheses that the true difference exceeds the threshold values. If both tests reject, the difference in the means does not statistically exceed either threshold value. Therefore, the groups are considered practically equivalent. If only one or neither test rejects, then the groups might not be practically equivalent.

For each comparison, the TOST Tests report contains the following information:

Difference The estimated difference in the means.

Lower Bound t Ratio, Upper Bound t Ratio The lower and upper bound *t* ratios for the two one-sided pooled-variance significance tests.

Lower Bound p-Value, Upper Bound p-Value The significance probabilities (*p*-values) that correspond to the lower and upper bound *t* ratios.

Max p-Value The maximum of the lower and upper bound *p*-values.

Lower 90%, Upper 90% Limits for a 1–2α confidence interval for the difference in the means.

Assessment An assessment of the hypothesis test for the specified alpha level.

Equivalence Tests Scatterplot

Using colors, this scatterplot indicates which means are practically equivalent and which are not practically equivalent as determined by the equivalence test. This plot is sometimes called a *diffogram* or a mean-mean scatterplot.

The plot shows a solid reference line on the diagonal as well as a shaded reference band. The width of the band is twice the practical difference. The coordinates of the point on the line segment are the means for the corresponding groups. There is an implied third axis on the diagonal where each line segment corresponds to a 1–2α confidence interval for a pairwise comparison. Hover over one of these points to show a tooltip that indicates the groups being compared and the estimated difference. When a line segment is entirely contained within the diagonal band, it follows that the means are practically equivalent.

The Equivalence Tests Scatterplot has the following option:

Show Reference Lines Displays reference lines for the points on the scatterplot. This is not recommended if there are many points in the scatterplot. If there are many points, it is better to hover over the points to view the tooltip labels.

Equivalence Tests Forest Plot

In the Forest Plot, the comparison confidence intervals are plotted versus the difference in means or ratio of standard deviations. The intervals are plotted on a difference of means or ratio of standard deviations scale. Shading indicates the equivalent regions.

Tip: Hover over a point to show the groups being compared and the estimated difference or ratio.

Remove

This option removes the Equivalence Tests report from the Multiple Comparisons report.

Effect Summary Report

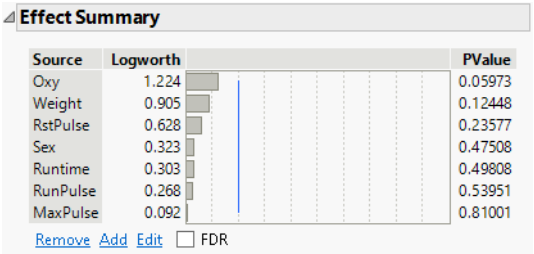
In the Fit Least Squares report, the Effect Summary option shows an interactive report. It gives a plot of the logworth (or FDR logworth) values for the effects in the model. The report also provides controls that enable you to add or remove effects from the model. The model fit report updates automatically based on the changes made in the Effects Summary report.

The Effect Summary report is available in the following personalities:

- Standard Least Squares
- Nominal Logistic
- Ordinal Logistic
- Proportional Hazard
- Parametric Survival
- Generalized Linear Model

Figure 3.58 shows the initial view of the Effect Summary report for the Fitness.jmp data table. The check box labeled **FDR** controls the columns that appear in the summary table.

Figure 3.58 Effect Summary Report



Effect Summary Table Columns

The Effect Summary table contains the following columns:

Source Lists the model effects, sorted by ascending *p*-values.

Logworth Shows the logworth for each model effect, defined as $-\log_{10}(p\text{-value})$. This transformation adjusts *p*-values to provide an appropriate scale for graphing. A value that exceeds 2 is significant at the 0.01 level (because $-\log_{10}(0.01) = 2$).

FDR Logworth Shows the false discovery rate logworth for each model effect, defined as $-\log_{10}$(FDR PValue). This is the best statistic for plotting and assessing significance. However, it is highly dependent on the ordering of the significances, is conservative for positively correlated tests, and does not give experimentwise protection at the alpha level. Select the **FDR** check box to replace the Logworth column with the FDR Logworth column.

Bar Graph Shows a bar graph of the logworth (or FDR logworth) values. The graph has dashed vertical lines at integer values and a blue reference line at 2.

PValue Shows the *p*-value for each model effect. This is generally the *p*-value corresponding to the significance test displayed in the Effect Tests table or Effect Likelihood Ratio Tests table of the model report.

FDR PValue Shows the false discovery rate *p*-value for each model effect calculated using the Benjamini-Hochberg technique. This technique adjusts the *p*-values to control the false discovery rate for multiple tests. Select the **FDR** check box to replace the **PValue** column with the **FDR PValue** column.

For more information about the FDR correction, see Benjamini and Hochberg (1995). For more information about the false discovery rate, see *Predictive and Specialized Modeling* or Westfall et al. (2011).

Effect Heredity Column Identifies lower-order effects that are components of more significant higher-order effects. The lower-order effects are identified with a caret. See "Effect Heredity".

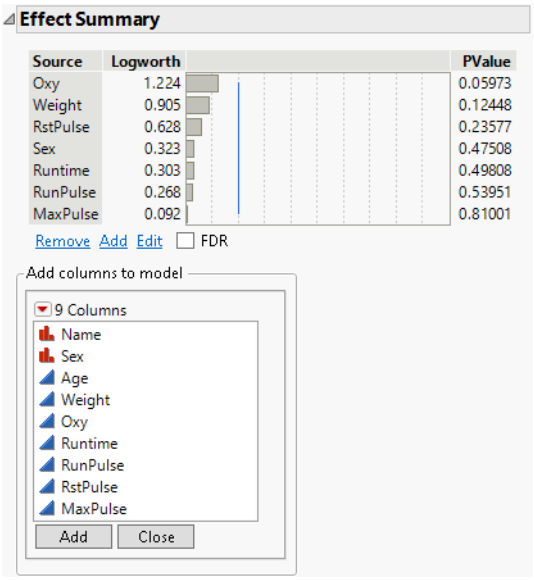
Effect Summary Table Options

The options below the summary table enable you to add and remove effects:

Remove Removes the selected effects from the model. To remove one or more effects, select the rows corresponding to the effects and click the **Remove** button.

Add Opens a panel that contains a list of all columns in the data table. Select columns that you want to add to the model, and then click **Add** below the column selection list to add the columns to the model. Click **Close** to close the panel. Figure 3.59 shows the Add Columns panel.

Figure 3.59 Effect Summary Add Columns Panel



Edit Opens the Edit Model panel, which contains a Select Columns list and an Effects specification panel. The Effects panel resembles the Construct Model Effects panel in the Fit Model launch window. The Edit Model panel enables you to add individual, crossed, nested, and transformed effects. You can also add multiple effects using the Macros menu. For more information about how to construct effects using Add, Cross, Nest, Macros, and Transform, see "Construct Model Effects".

The following options are available to the right of the Effects panel:

Commit applies your updates to the model.

Close closes the panel without making changes to the model.

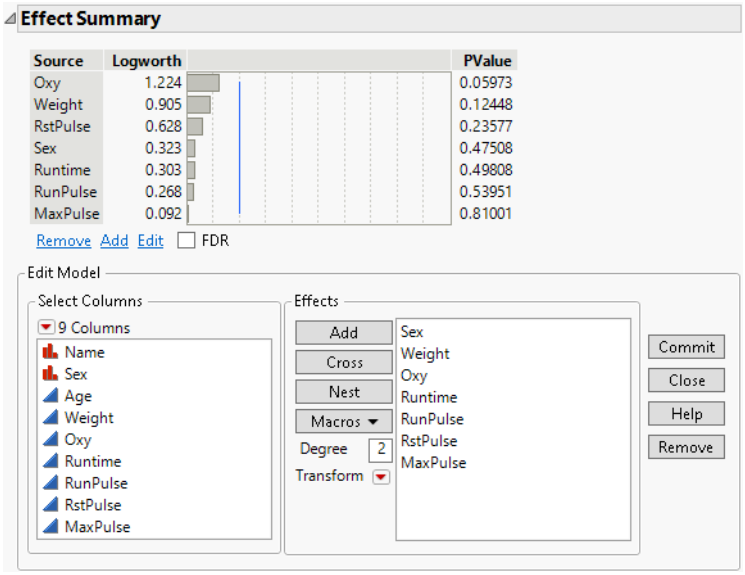
Remove removes one or more selected effects from the Effects list.

Figure 3.60 shows the Edit Model panel.

Tip: The Edit button gives you the greatest degree of control over updates to your model. It includes the functionality of the Remove and Add buttons and enables you to construct effects to add to your model.

Undo Enables you to undo changes to the effects in the model.

Figure 3.60 Effect Summary Edit Model Panel



Effect Heredity

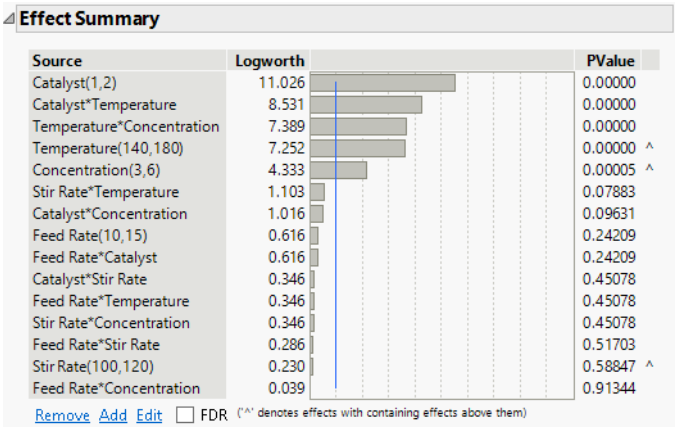
When a model contains significant higher-order effects, you might want to retain some or all of their lower-order components, even though these are not significant. The principle of *strong effect heredity* states that, if a higher-order effect is included in the model, all of its lower-order components should be included as well. The principle of *weak effect heredity* indicates that a chain of components should be included.

When a lower-order component of a higher-order effect appears in the Effect Summary table below the higher-order effect, a caret appears in the right-most column. The caret indicates that the containing higher-order effect is more significant than the lower-order effect. If all higher-order effects that contain a lower-order effect are less significant than the lower-order effect, no caret appears in the row for the lower-order effect.

If you remove an effect marked with a caret, you can choose one of two approaches for removing effects. Choose **Remove all selected effects** to remove all the selected effects, including the ones marked with a caret. Choose **Remove only non-contained effects** to remove only the selected effects that do not have a higher-order effect that still remains in the model.

Figure 3.61 show an example of an Effect Summary table where three lower-order effects appear below higher-order effects that contain the lower-order effects. For example, Stir Rate(100,120) appears below Stir Rate*Temperature.

Figure 3.61 Effect Summary Table with Effect Heredity for Reactor 32 Runs.jmp



Effect Summary

Source	Logworth	PValue
Catalyst(1,2)	11.026	0.00000
Catalyst*Temperature	8.531	0.00000
Temperature*Concentration	7.389	0.00000
Temperature(140,180)	7.252	0.00000 ^
Concentration(3,6)	4.333	0.00005 ^
Stir Rate*Temperature	1.103	0.07883
Catalyst*Concentration	1.016	0.09631
Feed Rate(10,15)	0.616	0.24209
Feed Rate*Catalyst	0.616	0.24209
Catalyst*Stir Rate	0.346	0.45078
Feed Rate*Temperature	0.346	0.45078
Stir Rate*Concentration	0.346	0.45078
Feed Rate*Stir Rate	0.286	0.51703
Stir Rate(100,120)	0.230	0.58847 ^
Feed Rate*Concentration	0.039	0.91344

Remove Add Edit FDR ("^" denotes effects with containing effects above them)

Multiple Responses

In the case of multiple responses, each effect appears in each response model, but only one Effect Summary report appears. For each effect, the table shows the minimum *p*-value among the *p*-values for that effect. Adding or removing an effect applies to the models for all of the responses.

Mixed and Random Effect Model Reports and Options

In the Fit Model launch window, you can specify mixed and random effect models. The Standard Least Squares personality fits the variance component covariance structure using the REML and EMS methods.

Note: JMP PRO JMP Pro users are encouraged to use the Mixed Model personality of the Fit Model window. The Mixed Model personality offers a broader set of covariance structures than does Standard Least Squares.

- "Mixed Models and Random Effect Models"
- "Restricted Maximum Likelihood (REML) Method"
- "EMS (Traditional) Model Fit Reports"

Mixed Models and Random Effect Models

A *random effect model* is a model all of whose factors represent random effects. See "Random Effects". Such models are also called *variance component models*. Random effect models are often hierarchical models. A model that contains both fixed and random effects is called a *mixed model*. Repeated measures and split-plot models are special cases of mixed models. Often the term *mixed model* is used to subsume random effect models.

To fit a mixed model, you must specify the random effects in the Fit Model launch window. However, if all of your model effects are random, you can also fit your model in the Variability / Attribute Gauge Chart platform. Only certain models can be fit in this platform. Note that the fitting methods used in the Variability / Attribute Gauge Chart platform do not allow variance component estimates to be negative. For more information about how the Variability / Attribute Gauge Chart platform fits variance components models, see *Quality and Process Methods*.

Random Effects

A random effect is a factor whose levels are considered a random sample from some population. Often, the precise levels of the random effect are not of interest, rather it is the variation reflected by the levels that is of interest (the *variance components*). However, there are also situations where you want to predict the response for a given level of the random effect. Technically, a random effect is considered to have a normal distribution with mean zero and nonzero variance.

Suppose that you are interested in whether two specific ovens differ in their effect on mold shrinkage. An oven can process only one batch of 50 molds at a time. You design a study where three randomly selected batches of 50 molds are consecutively placed in each of the two ovens. Once the batches are processed, shrinkage is measured for five parts randomly selected from each batch.

Note that Batch is a factor with six levels, once for each batch. So, in your model, you include two factors, Oven and Batch. Because you are specifically interested in comparing the effect of each oven on shrinkage, Oven is a fixed effect. But you are not interested in the effect of these specific six batches on the mean shrinkage. These batches are representative of a whole population of batches that could have been chosen for this experiment and to which the results of the analysis must generalize. Batch is considered a random effect. In this experiment, the Batch factor is of interest in terms of the variation in shrinkage among all possible batches. Your interest is in estimating the amount of variation in shrinkage that it explains. (Note that Batch is also nested within Oven, because only one batch can be processed once in one oven.)

Now suppose that you are interested in the weight of eggs for hens subjected to two feed regimes. Ten hens are randomly assigned to feed regimes: Five are given Feed regime A and five are given Feed regime B. However, these ten hens have some genetic differences that are not accounted for in the design of the study. In this case, you are interested the predicted weight of the eggs from certain specific hens as well as in the variance of the weights of eggs among hens.

The Classical Linear Mixed Model

JMP fits the classical linear mixed effects model:

$$\begin{aligned} Y &= X\beta + Z\gamma + \varepsilon \\ \gamma &\sim N(0, G) \\ \varepsilon &\sim N(0, \sigma^2 I_n) \end{aligned}$$

Here,

- Y is an n x 1 vector of responses
- $\mathbf{X}$ is the n x p design matrix for the fixed effects
- β is a p x 1 vector of unknown fixed effects with design matrix X
- $\mathbf{Z}$ is the n x s design matrix for the random effects
- γ is an s x 1 vector of unknown random effects with design matrix Z
- ε is an n x 1 vector of unknown random errors
- $\mathbf{G}$ is an s x s diagonal matrix with identical entries for each level of the random effect
- I_n is an n x n identity matrix
- γ and ε are independent

The diagonal elements of $\mathbf{G}$, as well as σ^2, are called *variance components*. These variance components, together with the vector of fixed effects β and the vector of random effects γ, are the model parameters that must be estimated.

The covariance structure for this model is sometimes called the *variance component* structure (SAS Institute Inc. 2022d). This covariance structure is the only one available in the Standard Least Squares personality.

JMP PRO The Mixed Model personality fits a variety of covariance structures, including Residual, First-order Autoregressive (or *AR(1)*), Unstructured, and Spatial. See "Repeated Structure Tab".

REML versus EMS for Fitting Models with Random Effects

JMP provides two methods for fitting models with random effects:

- REML, which stands for *restricted maximum likelihood* (always the recommended method)
- EMS, which stands for *expected mean square*s (use only for teaching from old textbooks)

The REML method is now the mainstream fitting methodology, replacing the traditional EMS method. REML is considerably more general in terms of applicability than the EMS method. The REML approach was pioneered by Patterson and Thompson (1974). See also Wolfinger et al. (1994) and Searle et al. (1992).

The EMS method, also called the *method of moments*, was developed before the availability of powerful computers. Researchers restricted themselves to balanced situations and used the EMS methodology, which provided computational shortcuts to compute estimates for random effect and mixed models. Because many textbooks still in use today use the EMS method to introduce models containing random effects, JMP provides an option for EMS. (See, for example, McCulloch et al., 2008; Poduri, 1997; Searle et al., 1992.)

The REML methodology performs maximum likelihood estimation of a restricted likelihood function that does not depend on the fixed-effect parameters. This yields estimates of the variance components that are then used to obtain estimates of the fixed effects. Estimates of precision are based on estimates of the covariance matrix for the parameters. Even when the data are unbalanced, REML provides useful estimates, tests, and confidence intervals.

The EMS methodology solves for estimates of the variance components by equating observed mean squares to expected mean squares. For balanced designs, a complex set of rules specifies how estimates are obtained. There are problems in applying this technique to unbalanced data.

For balanced data, REML estimates are identical to EMS estimates. But, unlike EMS, REML performs well with unbalanced data.

Specifying Random Effects and Fitting Method

Models with random effects are specified in the Fit Model launch window. To specify a random effect, highlight it in the Construct Model Effects list and select **Attributes > Random Effect**. This appends &Random to the effect name in the model effect list. For a definition of random effects, see "Random Effects". Random effects can also be specified in a separate effects tab. See "Construct Model Effects Tabs".

In the Fit Model launch window, once the &Random attribute has been appended to an effect, you are given a choice of fitting Method: REML (Recommended) or EMS (Traditional).

Caution: You must declare crossed and nested relationships explicitly. For example, a subject ID might also identify the group that contains the subject, as when each subject is in only one group. In such a situation, subject ID must still be declared as nested within group. Take care to be explicit in defining the design structure.

Unrestricted Parameterization for Variance Components

There are two different approaches to parameterizing the variance components: the *unrestricted* and the *restricted* approaches. The issue arises when there are mixed effects in the model, such as the interaction of a fixed effect with a random effect. Such an interaction term is considered to be a random effect.

In the restricted approach, for each level of the random effect, the sum of the interaction effects across the levels of the fixed effect is assumed to be zero. In the unrestricted approach, the mixed terms are simply assumed to be independent random realizations of a normal distribution with mean 0 and common variance. (This assumption is analogous to the assumption typically applied to residual error.)

JMP and SAS use the unrestricted approach. This distinction is important because many statistics textbooks use the restricted approach. Both approaches have been widely taught for 60 years. For a discussion of both approaches, see Cobb (1998, Section 13.3).

Negative Variances

Though variances are always positive, it is possible to have a situation where the unbiased estimate of the variance is negative. Negative estimates can occur in experiments when an effect is very weak or when there are very few levels corresponding to a variance component. By chance, the observed data can result in an estimate that is negative.

Unbounded Variance Components

JMP can produce negative estimates for both REML and EMS. For REML, there are two options in the Fit Model launch window: Unbounded Variance Components and Estimate Only Variance Components. The Unbounded Variance Components option is selected by default. Deselecting this option constrains variance component estimates to be nonnegative.

You should leave the Unbounded Variance Components option selected if you are interested in fixed effects. *Constraining the variance estimates to be nonnegative leads to bias in the tests for the fixed effects.*

Estimate Only Variance Components

Select this option if you want to see only the REML Variance Component Estimates report. If you are interested only in variance components, you might want to constrain variance components to be nonnegative. Deselecting the Unbounded Variance Components option and selecting the Estimate Only Variance Components option might be appropriate.

Restricted Maximum Likelihood (REML) Method

Based on the fitting method selected, the Fit Least Squares report provides different analysis results and provide additional menu options for Save Columns and Profiler. In particular, the analysis of variance report is not shown because variances and degrees of freedom do not partition in the usual way. You can obtain the residual variance estimate from the REML Variance Component Estimates report. See "REML Variance Component Estimates". The Effect Tests report is replaced by the Fixed Effect Tests report where fixed effects are tested. Additional reports give predicted values for the random effects and details about the variance components.

Figure 3.62 shows the report obtained for a fit to the Investment Castings.jmp sample data using the REML method. Run the script **Model - REML**, and then fit the model. Note that Casting is a random effect and is nested within Temperature.

Figure 3.62 Fit Least Squares Report for REML Method

Response Shrinkage

Parameter Estimates

Random Effect Predictions

Term	BLUP	Std Error	DFDen	t Ratio	Prob>|t|
Temperature[1]:Casting[1]	-0.353959	0.337993	7.939	-1.05	0.3258
Temperature[1]:Casting[3]	0.0524938	0.337993	7.939	0.16	0.8805
Temperature[1]:Casting[5]	0.5139011	0.337993	7.939	1.52	0.1672
Temperature[1]:Casting[7]	-0.212436	0.337993	7.939	-0.63	0.5473
Temperature[2]:Casting[2]	0.4367848	0.337993	7.939	1.29	0.2326
Temperature[2]:Casting[4]	-0.094099	0.337993	7.939	-0.28	0.7878
Temperature[2]:Casting[6]	-0.862231	0.337993	7.939	-2.55	0.0343*
Temperature[2]:Casting[8]	0.519546	0.337993	7.939	1.54	0.1631

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Wald p-Value	Pct of Total
Casting[Temperature]	1.511973	0.3205234	0.2161944	-0.10321	0.7442566	0.1382	60.191
Residual		0.2119901	0.0611963	0.1292489	0.4102654		39.809
Total		0.5325135	0.2204825	0.2716915	1.4771758		100.000

-2 LogLikelihood = 57.247690262

Note: Total is the sum of the positive variance components.

Total including negative estimates = 0.5325135

Covariance Matrix of Variance Component Estimates

Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Temperature	1	1	6	6.5130	0.0434*

Random Effect Predictions

For each term in the model, this report gives an empirical estimate of its *best linear unbiased predictor* (BLUP) and a test for whether the corresponding coefficient is zero.

Note: The Regression Reports > Parameter Estimates option must be selected for the Random Effect Predictions report to appear.

Term The terms in the model that correspond to random effects.

BLUP The empirical estimate of the *best linear unbiased predictor* (BLUP) for each random effect. See "Best Linear Unbiased Predictors".

Std Error The standard error of the BLUP.

DFDen The denominator degrees of freedom for a test that the effect is zero. In most cases, the degrees of freedom for the *t* test is fractional.

t Ratio The *t* ratio for testing that the effect is zero. The *t* ratio is obtained by dividing the BLUP by its standard error.

Prob>|t| The *p*-value for the test.

Lower 95% The lower 95% confidence limit for the BLUP. This column appears only if you have the Regression Reports > Show All Confidence Intervals option selected or if you right-click in the report and select Columns > Lower 95%.

Upper 95% The upper 95% confidence limit for the BLUP. This column appears only if you have the Regression Reports > Show All Confidence Intervals option selected or if you right-click in the report and select Columns > Upper 95%.

Best Linear Unbiased Predictors

The term *best linear unbiased predictor* (BLUP) refers to an estimator of a random effect. Specifically, it is an estimator that, among all unbiased estimators, minimizes mean square prediction error. The Random Effect Predictions report gives estimates of the BLUPs, or *empirical* BLUPs. These are empirical because the BLUPs depend on the values of the variance components, which are unknown. The estimated values of the variance components are substituted into the formulas for the BLUPs, resulting in the estimates shown in the report.

REML Variance Component Estimates

When REML is selected as the fitting method in the Fit Model launch window, the REML Variance Component Estimates report is provided. This report contains the following columns:

Random Effect The random effects in the model.

Var Ratio The ratio of the variance component for the effect to the variance component for the residual. It compares the effect's estimated variance to the model's estimated error variance.

Var Component The estimated variance component for the effect. Note that the variance component for the Total is the sum of the positive variance components only. The sum of all variance components is given beneath the table.

Std Error The standard error for the variance component estimate.

95% Lower The lower 95% confidence limit for the variance component. See "Confidence Intervals for Variance Components".

95% Upper The upper 95% confidence limit for the variance component. See "Confidence Intervals for Variance Components".

Wald p-Value The *p*-value for the test that the covariance parameter is equal to zero. This column appears only when you have selected Unbounded Variance Components in the Fit Model launch window.

Pct of Total The ratio of the variance component for the effect to the variance component for the total as a percentage.

Sqrt Variance Component The square root of the corresponding variance component. It is an estimate of the standard deviation for the effect. This column appears only if you right-click in the report and select Columns > Sqrt Variance Component.

CV The coefficient of variation for the variance component. It is 100 times the square root of the variance component, divided by the mean response. This column appears only if you right-click in the report and select Columns > CV.

Norm KHC The Kackar-Harville correction. See "Kackar-Harville Correction". This column appears only if you right-click in the report and select Columns > Norm KHC.

Confidence Intervals for Variance Components

The method used to calculate the confidence limits depends on whether you have selected Unbounded Variance Components in the Fit Model launch window. Note that Unbounded Variance Components is selected by default.

- If Unbounded Variance Components is selected, Wald-based confidence intervals are computed. These are valid asymptotically but note that they can be unreliable with small samples.
- If Unbounded Variance Components is not selected, meaning that parameters have a lower boundary constraint of zero, a Satterthwaite approximation is used (Satterthwaite 1946).

Kackar-Harville Correction

In the REML method, the standard errors of the fixed effects are estimated using estimates of the variance components. However, if variability in these estimates is not taken into account, the standard error is underestimated. To account for the increased variability, the covariance matrix of the fixed effects is adjusted using the Kackar-Harville correction (Kackar and Harville 1984; Kenward and Roger 1997). All calculations that involve the covariance matrix of the fixed effects use this correction. These include least squares means, fixed effect tests, confidence intervals, and prediction variances. For statistical details, see "Statistical Details for the Kackar-Harville Correction".

Norm KHC is the Frobenius (matrix) norm of the Kackar-Harville correction. In cases where the design is fairly well balanced, Norm KHC tends to be small.

Covariance Matrix of Variance Components Estimates

This report gives an estimate of the asymptotic covariance matrix for the variance components. It is the inverse of the observed Fisher information matrix.

Iterations

The estimates of σ^2 and the variance components in G are obtained by maximizing a residual log-likelihood function that depends on only these parameters. An iterative procedure attempts to maximize the residual log-likelihood function, or equivalently, to minimize twice the negative of the residual log-likelihood (–2LogLike). The Iterations report provides details about this procedure.

Iter Iteration number.

-2LogLike Twice the negative log-likelihood. It is the objective function. See "Likelihood, AICc, and BIC".

Norm Gradient The norm of the gradient (first derivative) of the objective function

Parameters The column labeled Parameters and the remaining columns each correspond to a random effect. The order of the columns follows the order in which random effects are listed in the REML Variance Component Estimates report. At each iteration, the value in the column is the estimate of the variance component at that point.

The convergence criterion is based on the gradient, with a default tolerance of 10^{-8}. You can change the criterion in the Fit Model launch window by selecting the option Convergence Settings > Convergence Limit and specifying the desired tolerance.

Fixed Effect Tests

When REML is used, the Effect Tests report provides tests for the fixed effects. This report contains the following columns:

Source The fixed effects in the model.

Nparm The number of parameters associated with the effect.

DF The degrees of freedom associated with the effect.

DFDen The denominator degrees of freedom. These are based on an approximation to the distribution of the statistic obtained when the covariance matrix is adjusted using the Kenward-Roger correction. See "Kackar-Harville Correction" and "Random Effects".

FRatio The computed *F* ratio.

Prob > F The *p*-value for the effect test.

REML Save Columns Options

When you use the REML method, six additional options appear in the Save Columns menu. These option names start with the adjective *Conditional*. This prefix indicates that the calculations for these columns use the predicted values for the terms associated with the random effects, rather than their expected values of zero.

Conditional Pred Formula Saves the prediction formula to a new column in the data table.

Conditional Pred Values Saves the predicted values to a new column in the data table.

Conditional Residuals Saves the residuals to a new column in the data table.

Conditional Mean CI Saves the confidence interval for the mean.

Conditional Indiv CI Saves the confidence interval for individuals.

JMP PRO **Publish Conditional Formula** Creates a conditional prediction formula and saves it as a formula column script in the Formula Depot platform. If a Formula Depot report is not open, this option creates a Formula Depot report. See *Predictive and Specialized Modeling*.

REML Profiler Option

When you use the REML method and select Factor Profiling > Profiler, a new option, Conditional Predictions, appears on the red triangle menu next to Prediction Profiler. Note that the conditional values use the predicted values for the random effects, rather than their zero expected values.

Note: The profiler displays conditional predicted values and conditional mean confidence intervals for all combinations of factors levels. Some of these combinations might not be meaningful due to nesting.

EMS (Traditional) Model Fit Reports

In the Fit Model launch window, if you select EMS as the fitting method, four new reports appear. The Effect Tests report is not shown, as tests for both fixed and random effects are conducted in the Tests wrt Random Effects report.

Caution: The use of EMS is not recommended. REML is the recommended method.

Expected Mean Squares

The expected mean square for a model effect is a linear combination of variance components and fixed effect values, including the residual error variance. This table gives the coefficients that define each model effect's expected mean square. The rows of the matrix correspond to the effects, listed on the left. The columns correspond to the variance components, identified across the top. Each expected mean square includes the residual variance with a coefficient of one. This information is given beneath the table.

Figure 3.63 shows the Expected Mean Squares report for the Investment Castings.jmp sample data table. Run the Model - EMS script and then run the model.

Figure 3.63 Expected Mean Squares Report

Expected Mean Squares

The Mean Square per row by the Variance Component per column

EMS

	Intercept	Temperature	Casting[Temperature]&Random
Intercept	0	0	0
Temperature	0	16	4
Casting[Temperature]&Random	0	0	4

plus 1.0 times Residual Error Variance

As indicated by the table, the expected mean square for Casting[Temperature] is

$$4\sigma^2_{Casting[Temperature]} + \sigma^2_{Error}$$

Variance Component Estimates

Estimates of the variance components are obtained by equating the expected mean squares to the corresponding observed mean squares and solving. The Variance Component Estimates report gives the estimated variance components.

Component Lists the random effects.

Var Comp Est Gives the estimate of the variance component.

Percent of Total Gives the ratio of the variance component to the sum of the variance components.

CV Gives the coefficient of variation for the variance component. It is 100 times the square root of the variance component, divided by the mean response.

Note: Appears only if you right-click in the report and select Columns > CV.

Test Denominator Synthesis

For each effect to be tested, an F statistic is constructed. The denominator for this statistic is the mean square whose expectation is that of the numerator mean square under the null hypothesis. This denominator is constructed, or *synthesized*, from variance components and values associated with fixed effects.

Source Shows the effect to be tested.

MS Den Gives the estimated mean square for the denominator of the *F* test.

DF Den Gives the degrees of freedom for the synthesized denominator. These are constructed using Satterthwaite's method (Satterthwaite 1946).

Denom MS Synthesis Gives the variance components used in the denominator synthesis. The residual error variance is always part of this synthesis.

Tests wrt Random Effects

Tests for fixed and random effects are presented in this report.

Source Lists the effects to be tested. These include fixed and random effects.

SS Gives the sum of squares for the effect.

MS Num Gives the numerator mean square.

DF Num Gives the numerator degrees of freedom.

F Ratio Gives the *F* ratio for the test. It is the ratio of the numerator mean square to the denominator mean square. The denominator mean square can be obtained from the Test Denominator Synthesis report.

Prob > F Gives the *p*-value for the effect test.

Caution: Standard errors for least squares means and denominators for contrast *F* tests use the synthesized denominator. In certain situations, such as tests involving crossed effects compared at common levels, these tests might not be appropriate. Custom tests are conducted using residual error, and leverage plots are constructed using the residual error, so these also might not be appropriate.

EMS Profiler

When you use the EMS method and select Factor Profiling > Profiler, the profiler gives predictions and conditional mean confidence intervals based on the fixed-effects model. These values are not based on the predicted values for the random effects.

Models with Linear Dependencies among Model Terms

When there are linear dependencies among the model predictors, the following sections of the Fit Least Squares reports are affected:

- "Singularity Details"
- "Parameter Estimates Report"
- "Effect Tests Report"
- "Examples of Models with Linear Dependencies"

Singularity Details

In the Fit Least Squares report, when there are linear dependencies among the columns of the matrix of predictors, the Singularity Details section reports the linear dependencies.

The linear regression model is formulated as $Y = X\beta + \varepsilon$. Here X is a matrix whose first column consists of *1*s, and whose remaining columns are the values of the non-intercept terms in the model. If the model consists of p terms, including the intercept, then X is an n by p matrix, where n is the number of observations. The parameter estimates, denoted by the vector b, are typically given by the formula:

$$b = (X'X)^{-1}X'Y$$

However, this formula presumes that $X'X^{-1}$ exists, in other words, that the $p \times p$ matrix $X'X$ is invertible, or equivalently, of full rank. Situations often arise when $X'X$ is not invertible because there are linear dependencies among the columns of X.

In such cases, the matrix $X'X$ is singular, and the Fit Least Squares report contains the Singularity Details report. This report contains a table of expressions that describe the linear dependencies. The terms involved in these linear dependencies are aliased (confounded).

Figure 3.64 shows reports for the Reactor 8 Runs.jmp sample data table. To obtain these reports, fit a model with Percent Reacted as Y. Enter Feed Rate, Catalyst, Stir Rate, Temperature, Concentration, Catalyst*Stir Rate, Catalyst*Concentration, and Feed Rate*Catalyst as model effects.

Figure 3.64 Singularity and Parameter Estimates Report for Model with Linear Dependencies

Singularity Details

Term	Details
Concentration(3,6)	=Catalyst*Stir Rate
Stir Rate(100,120)	=Catalyst*Concentration

Studentized Residuals

Not enough data.

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob>|t|
Intercept		65.875	6.125	10.76	0.0590
Feed Rate(10,15)		-4.875	6.125	-0.80	0.5720
Catalyst(1,2)		10.125	6.125	1.65	0.3463
Stir Rate(100,120)	Biased	1.875	6.125	0.31	0.8109
Temperature(140,180)		5.875	6.125	0.96	0.5133
Concentration(3,6)	Biased	-3.375	6.125	-0.55	0.6794
Catalyst*Stir Rate	Zeroed	0	0	.	.
Catalyst*Concentration	Zeroed	0	0	.	.
Feed Rate*Catalyst		0.375	6.125	0.06	0.9611

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F	
Feed Rate(10,15)	1	1	190.12500	0.6335	0.5720	
Catalyst(1,2)	1	1	820.12500	2.7326	0.3463	
Stir Rate(100,120)	1	0	0.00000	.	.	LostDFs
Temperature(140,180)	1	1	276.12500	0.9200	0.5133	
Concentration(3,6)	1	0	0.00000	.	.	LostDFs
Catalyst*Stir Rate	1	0	0.00000	.	.	LostDFs
Catalyst*Concentration	1	0	0.00000	.	.	LostDFs
Feed Rate*Catalyst	1	1	1.12500	0.0037	0.9611	

Parameter Estimates Report

In the Fit Least Squares report, when $X'X$ is singular, a generalized inverse is used to obtain estimates. This approach permits some, but not all, of the parameters involved in a linear dependency to be estimated. Parameters are estimated based on the order of entry of their associated terms into the model, so that the last terms entered are the ones whose parameters are not estimated. Estimates are given in the Parameter Estimates report, and parameters that cannot be estimated are given estimates of 0.

However, estimates of parameters for terms involved in linear dependencies are not unique. Because the associated terms are aliased, there are infinitely many vectors of estimates that satisfy the least squares criterion. In these cases, "Biased" appears to the left of these estimates in the Parameter Estimates report. "Zeroed" appears to the left of the estimates of 0 in the Parameter Estimates report for terms involved in a linear dependency whose parameters cannot be estimated. For an example, see Figure 3.64.

If there are degrees of freedom available for an estimate of error, *t* tests for parameters estimated using biased estimates are conducted. These tests should be interpreted with caution, though, given that the estimates are not unique.

Effect Tests Report

In a standard least squares fit, only as many parameters are estimable as there are model degrees of freedom. In conducting the tests in the Effect Tests report, each effect is considered to be the last effect entered into the model.

- If all the Model degrees of freedom are used by the other effects, an effect shows DF equal to 0. When DF equals 0, no sum of squares can be computed. Therefore, the effect cannot be tested.
- If not all Model degrees of freedom are used by the other effects, then that effect has nonzero DF. However, its DF might be less than its number of parameters (Nparm), indicating that only some of its associated parameters are testable.

An *F* test is conducted if the degrees of freedom for an effect are nonzero, assuming that there are degrees of freedom for error. Whenever DF is less than Nparm, the description LostDFs is displayed to the far right in the row corresponding to the effect (Figure 3.64). These effects have the opportunity to explain only model sums of squares that have not been attributed to the aliased effects that have absorbed their lost degrees of freedom. It follows that the sum of squares given in the Effect Tests report most likely under represents the "true" sum of squares associated with the effect. If the test is significant, its significance is meaningful. But lack of significance should be interpreted with caution.

For statistical details, see the section "Statistical Background" in the "Introduction to Statistical Modeling with SAS/STAT Software" chapter in SAS Institute Inc. (2022c).

Examples of Models with Linear Dependencies

This section contains examples of models with various linear dependencies. The Singularity.jmp sample data table contains a response Y, four predictors X1, X2, X3, and A, and five observations. The predictors are continuous except for A, which is nominal with four levels. Also note that there is a linear dependency among the continuous effects, namely, X3 = X1 + X2.

Non-Uniqueness of Estimates

To see that estimates are not unique when there are linear dependencies:

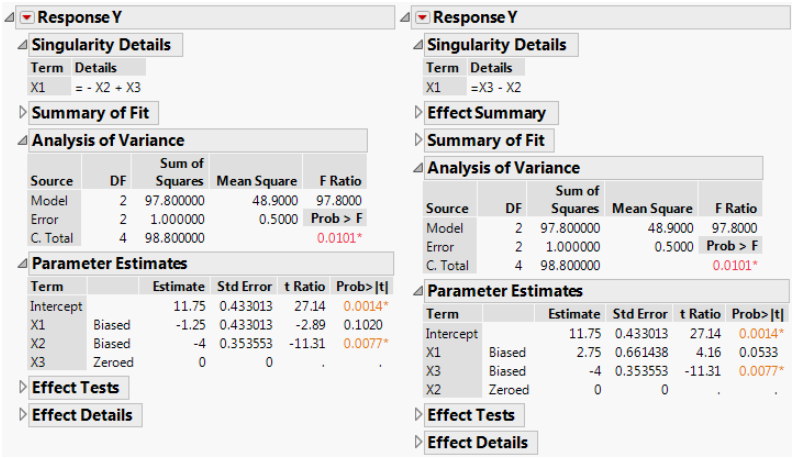
1. Select **Help > Sample Data Folder** and open Singularity.jmp.
2. Run the script **Model 1**. The script opens a Fit Model launch window where the effects are entered in the order X1, X2, X3.
3. Click **Run** and leave the report window open.
4. Run the script **Model 2**. The script opens a Fit Model launch window where the effects are entered in the order X1, X3, X2.
5. Click **Run** and leave the report window open.

Compare the two reports (Figure 3.65). The Singularity Details report at the top of both reports displays the linear dependency, indicating that X1 = X3 - X2.

Now compare the Parameter Estimates reports for both models. Note, for example, that the estimate for X1 for Model 1 is –1.25 while for Model 2 it is 2.75. In both models, only two of the terms associated with effects are estimated, because there are only two model degrees of freedom. See the Analysis of Variance report. The estimates of the two terms that are estimated are labeled Biased while the remaining estimate is set to 0 and labeled Zeroed.

The Effect Tests report shows that no tests are conducted. Each row is labeled LostDFs. The reason this happens is that the effect test for any one of these effects requires it to be entered into the model last. However, the other two effects entirely account for the model sum of squares associated with the two model degrees of freedom. So there are no degrees of freedom or associated sum of squares left for the effect of interest.

Figure 3.65 Fit Least Squares Reports for Model 1 (on left) and Model 2 (on right)



Response Y

Singularity Details

Term	Details
X1	= - X2 + X3

Summary of Fit

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	97.800000	48.9000	97.8000
Error	2	1.000000	0.5000	Prob > F
C. Total	4	98.800000		0.0101*

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob>|t|
Intercept		11.75	0.433013	27.14	0.0014*
X1	Biased	-1.25	0.433013	-2.89	0.1020
X2	Biased	-4	0.353553	-11.31	0.0077*
X3	Zeroed	0	0	.	.

Effect Tests

Effect Details

Response Y

Singularity Details

Term	Details
X1	=X3 - X2

Effect Summary

Summary of Fit

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	97.800000	48.9000	97.8000
Error	2	1.000000	0.5000	Prob > F
C. Total	4	98.800000		0.0101*

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob>|t|
Intercept		11.75	0.433013	27.14	0.0014*
X1	Biased	2.75	0.661438	4.16	0.0533
X3	Biased	-4	0.353553	-11.31	0.0077*
X2	Zeroed	0	0	.	.

Effect Tests

Effect Details

LostDFs

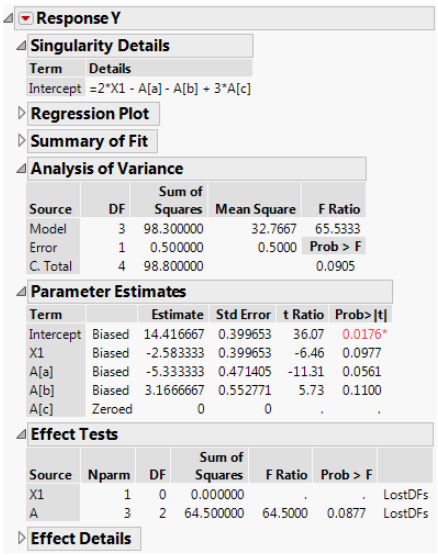
To gain more insight on LostDFs, follow the steps below or run the data table script Fit Model Report:

1. Select **Help > Sample Data Folder** and open Singularity.jmp.
2. Click **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Select X1 and A and click **Add**.
5. Set the **Emphasis** to **Minimal Report**.
6. Click **Run**.

Portions of the report are shown in (Figure 3.66). The Singularity Details report shows that there is a linear dependency involving X1 and the three terms associated with the effect A. For more information about how a nominal effect is coded, see "Statistical Details for the Custom Test Example". The Analysis of Variance report shows that there are three model degrees of freedom. The Parameter Estimates report shows Biased estimates for the three terms X1, A[a], and A[b] and a Zeroed estimate for the fourth, A[c].

The Effect Tests report shows that X1 cannot be tested, because A must be entered first and A accounts for the three model degrees of freedom. However, A can be tested, but with only two degrees of freedom. (X1 must be entered first and it accounts for one of the model degrees of freedom.) The test for A is partial, so it must be interpreted with care.

Figure 3.66 Fit Least Squares Report for Model with X1 and A



Response Y

Singularity Details

Term	Details
Intercept	=2*X1 - A[a] - A[b] + 3*A[c]

Regression Plot

Summary of Fit

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	98.300000	32.7667	65.5333
Error	1	0.500000	0.5000	Prob > F
C. Total	4	98.800000		0.0905

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob>|t|
Intercept	Biased	14.416667	0.399653	36.07	0.0176*
X1	Biased	-2.583333	0.399653	-6.46	0.0977
A[a]	Biased	-5.333333	0.471405	-11.31	0.0561
A[b]	Biased	3.1666667	0.552771	5.73	0.1100
A[c]	Zeroed	0	0	.	.

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F	
X1	1	0	0.000000	.	.	LostDFs
A	3	2	64.500000	64.5000	0.0877	LostDFs

Effect Details

Statistical Details for the Standard Least Squares Personality

This section provides statistical details for the Standard Least Squares personality of the Fit Model platform.

- "Statistical Details for Emphasis Rules"
- "Statistical Details for the Custom Test Example"
- "Statistical Details for Correlation of Estimates"
- "Statistical Details for Nominal Effects Coding"
- "Statistical Details for Leverage Plots"
- "Statistical Details for the Kackar-Harville Correction"
- "Statistical Details for Power Analysis"

Statistical Details for Emphasis Rules

In the Fit Model launch window, the Emphasis option determines the initial Fit Model report. The default Emphasis for the Standard Least Squares personality of the Fit Model launch window is based on the number of rows, n, the number of effects (k) entered in the Construct Model Effects list, and the attributes applied to effects.

- If $n > 1000$, the Emphasis is set to Minimal Report.
- If $n \leq 1000$ and $k \leq 4$, the Emphasis is set to Effect Leverage
- If $n \leq 1000$ and $k \geq 10$, the Emphasis is set to Effect Screening.
- If $n \leq 1000$ and $4 < k < 10$ and $n - k > 20$, the Emphasis is set to Effect Leverage.
- If any effect has a Random Effect attribute, the Emphasis is set to Minimal Report.
- If none of these conditions hold, the Emphasis is set to Effect Screening.

Statistical Details for the Custom Test Example

This section contains additional details related to an example of specifying a custom test in the Standard Least Squares personality of the Fit Model platform. In "Example of a Custom Test", you are interested in testing three contrasts using the Cholesterol.jmp sample data table. Specifically, you want to compare:

- the mean responses for treatments A and B,
- the mean response for treatments A and B combined to the mean response for the control group,

- the mean response for treatments A and B combined to the mean response for the combined control and placebo groups.

To derive the contrast coefficients that you enter into the Custom Test columns, do the following. Denote the theoretical effects for the four treatment groups as α_A, α_B, $\alpha_{Control}$, and $\alpha_{Placebo}$. These are the treatment effects, so they are constrained to sum to 0. Because the parameters associated with the indicator variables represent only the first three effects, you need to formulate your contrasts in terms of these first three effects. See "Statistical Details for the Custom Test Example" and "Interpretation of Parameters".

The hypotheses that you want to test can be written in terms of model effects as follows:

- Compare treatment A to treatment B:

$$\alpha_A - \alpha_B = 0$$

- Compare treatments A and B to the control group:

$$0.5(\alpha_A + \alpha_B) - \alpha_{Control} = 0$$

- Compare treatments A and B to the control and placebo groups:

$$0.5(\alpha_A + \alpha_B) - 0.5(\alpha_{Control} + \alpha_{Placebo}) = \alpha_A + \alpha_B = 0$$

To obtain contrast coefficients for this contrast, you need to write the placebo effect in terms of the model effects. Specifically, use the fact that $\alpha_A + \alpha_B + \alpha_{Control} + \alpha_{Placebo} = 0$. Then $\alpha_{Placebo} = -\alpha_A - \alpha_B - \alpha_{Control}$. It follows that:

$$\begin{aligned} 0.5(\alpha_A + \alpha_B) - 0.5(\alpha_{Control} + \alpha_{Placebo}) &= 0.5(\alpha_A + \alpha_B) - 0.5(\alpha_{Control} - \alpha_A - \alpha_B - \alpha_{Control}) \\ &= 0.5(\alpha_A + \alpha_B) - 0.5\alpha_{Control} + 0.5(\alpha_A + \alpha_B + \alpha_{Control}) \\ &= \alpha_A + \alpha_B \\ &= 0 \end{aligned}$$

Statistical Details for Correlation of Estimates

This section contains details regarding the correlation of estimates in a standard least squares model. Consider a data set with n observations and p–1 predictors. Define the matrix **X** to be the design matrix. That is, **X** is the n by p matrix whose first column consists of 1s and whose remaining p–1 columns consist of the p–1 predictor values. (Nominal columns are coded in terms of indicator predictors. Each of these is a column in the matrix X.)

The estimate of the vector of regression coefficients is

$$\hat{\beta} = (X'X)^{-1}X'Y$$

where Y represents the vector of response values.

Under the usual regression assumptions, the covariance matrix of $\hat{\beta}$ is

$$Cov(\hat{\beta}) = \sigma^2(X'X)^{-1}$$

where σ^2 represents the variance of the response.

The correlation matrix for the estimates is obtained by dividing each entry in the covariance matrix by the product of the square roots of the diagonal entries. Define V to be the diagonal matrix whose entries are the square roots of the diagonal entries of the covariance matrix:

$$V = Sqrt(Diag(Cov(\hat{\beta})))$$

Then the correlation matrix for the parameter estimates is given by the following:

$$Corr(\hat{\beta}) = \sigma^2 V^{-1}(X'X)^{-1}V^{-1}$$

Statistical Details for Nominal Effects Coding

When you enter a column with a nominal modeling type in the Fit Model launch window, JMP represents it internally as a set of continuous indicator variables. Each variable assumes only the values –1, 0, and 1. (Note that this coding is one of many ways to use indicator variables to code nominal variables.) If your nominal column has n levels, then n–1 of these indicator variables are needed to represent it. (The need for n–1 indicator variables relates directly to the fact that the main effect associated with the nominal column has n–1 degrees of freedom.) Full details are covered in "Nominal Factors".

Tip: You can view the coding by selecting Save Columns > Save Coding Table from the red triangle menu for the main report. See "Save Coding Table".

Suppose that you have a nominal column with four levels. Take, as an example, the treatment column in the Cholesterol.jmp sample data table. The treatment column has four levels: A, B, Control, and Placebo. Each of the first three levels is represented by an indicator variable. These indicator variables are named treatment[A], treatment[B], and treatment[Control].

The indicator variable for a given level assigns the values 1 to that level, –1 to the last level, and 0 to the remaining levels. Table 3.1 shows the definitions of the treatment[A], treatment[B], and treatment[Control] indicator variables for this example. For example, consider the indicator variable treatment[A]. As shown in Table 3.1, this variable assigns the following values:

- The value 1 is assigned to rows that have treatment = A
- The value 0 is assigned to rows that have treatment = B or Control
- The value –1 is assigned to rows that have treatment = Placebo

Table 3.1 Illustration of Indicator Variables for treatment in Cholesterol.jmp

Treatment Assigned to Row	treatment[A]	treatment[B]	treatment[Control]
A	1	0	0
B	0	1	0
Control	0	0	1
Placebo	–1	–1	–1

The order of the levels is determined either by the Value Order column property, if you have assigned one, or by the default ordering assigned by JMP. The default ordering is typically the numeric sorting order for numbers and the alphanumeric sorting order for character data. However, certain categorical values, such as the names of months, are sorted appropriately by default. For more information about value ordering, see *Using JMP*.

These variables are used to parametrize the model. They do not typically appear in the data table, but the estimated coefficients for these variables are given in the Parameter Estimates and other reports. Although many other codings are possible, this coding has proven to be practical and interpretable.

For information about the coding of ordinal effects, see "Ordinal Factors".

Statistical Details for Leverage Plots

The Standard Least Squares personality of the Fit Model platform produce effect leverage plots, which are also referred to as *partial-regression residual leverage plots* (Belsley et al. 1980) or *added variable plots* (Cook and Weisberg 1982). Sall (1990) generalized these plots to apply to any linear hypothesis.

JMP provides two types of leverage plots:

- Effect Leverage plots show observations relative to the hypothesis that the effect is not in the model, given that all other effects are in the model.
- The Whole Model leverage plot, given in the Actual by Predicted Plot report, shows the observations relative to the hypothesis of no factor effects.

In the Effect leverage plot, only one effect is hypothesized to be zero. However, in the Whole Model Actual by Predicted plot, all effects are hypothesized to be zero. Sall (1990) generalizes the idea of a leverage plot to arbitrary linear hypotheses, of which the Whole Model leverage plot is an example. The details from that paper, summarized in this section, specialize to the two types of plots found in JMP.

Construction

Suppose that the estimable hypothesis of interest is

$$L\beta = 0$$

The leverage plot characterizes this test by plotting points so that the distance of each point to the sloped regression line displays the unconstrained residual. The distance to the horizontal line at 0 displays the residual when the fit is constrained by the hypothesis. The difference between the sums of squares of these two sets of residuals is the sum of squares due to the hypothesis. This value becomes the main component of the *F* test.

The parameter estimates constrained by the hypothesis can be written

$$b_0 = b - (X'X)^{-1}L'\lambda$$

Here b is the least squares estimate

$$b = (X'X)^{-1}X'y$$

and lambda is the Lagrangian multiplier for the hypothesis constraint, calculated by

$$\lambda = (L(X'X)^{-1}L')^{-1}Lb)$$

The unconstrained and hypothesis-constrained residuals are, respectively,

$$r = y - Xb$$
$$r_0 = r + X(X'X)^{-1}L'\lambda$$

For each observation, consider the point with horizontal axis value v_x and vertical axis value v_y where:

- v_x is the constrained residual minus the unconstrained residual, r_0 - r, reflecting information left over once the constraint is applied
- v_y is the horizontal axis value plus the unconstrained residual

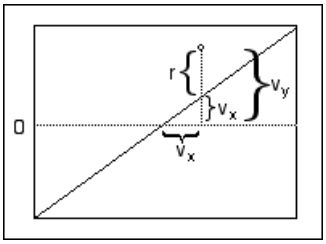
Thus, these points have x and y coordinates

$$v_x = X(X'X)^{-1}\mathbf{L}'\lambda \text{ and } v_y = r + v_x$$

These points form the basis for the leverage plot. This construction is illustrated in Figure 3.67, where the response mean is 0 and slope of the solid line is 1.

Leverage plots in JMP have a dotted horizontal line at the mean of the response, $\bar{y}$. The plotted points are given by $(v_x + \bar{y}, v_y)$.

Figure 3.67 Construction of Leverage Plot



Superimposing a Test on the Leverage Plot

In simple linear regression, you can plot the confidence limits for the expected value of the response as a smooth function of the predictor variable **x**

$$\text{Upper (x)} = \mathbf{xb} + t_{\alpha/2} s\sqrt{x(X'X)^{-1}x'}$$

$$\text{Lower (x)} = \mathbf{xb} - t_{\alpha/2} s\sqrt{x(X'X)^{-1}x'}$$

where **x** = [1 x] is the 2-vector of predictors.

These confidence curves give a visual assessment of the significance of the corresponding hypothesis test, illustrated in Figure 3.51:

- Significant: If the slope parameter is significantly different from zero, the confidence curves cross the horizontal line at the response mean.

- Borderline: If the *t* test for the slope parameter is sitting right on the margin of significance, the confidence curve is asymptotic to the horizontal line at the response mean.
- Not Significant: If the slope parameter is not significantly different from zero, the confidence curve does not cross the horizontal line at the response mean.

Leverage plots mirror this thinking by displaying confidence curves. These are adjusted so that the plots are suitably centered. Denote a point on the horizontal axis by *z*. Define the functions

$$\text{Upper}(z) = z + \sqrt{s^2 t^2_{\alpha/2} \bar{h} + (F_\alpha / F) z^2}$$

and

$$\text{Lower}(z) = z - \sqrt{s^2 t^2_{\alpha/2} \bar{h} + (F_\alpha / F) z^2}$$

where F is the F statistic for the hypothesis and F_α is the reference value for significance level α.

And $\bar{h} = \bar{x}(X'X)^{-1}\bar{x}'$, where $\bar{x}$ is a row vector consisting of suitable middle values for the predictors, such as their means.

These functions behave in the same fashion as do the confidence curves for simple linear regression:

- If the F statistic is greater than the reference value, the confidence functions cross the horizontal axis.
- If the F statistic is equal to the reference value, the confidence functions have the horizontal axis as an asymptote.
- If the F statistic is less than the reference value, the confidence functions do not cross.

Also, it is important that Upper(*z*) - Lower(*z*) is a valid confidence interval for the predicted value at z.

Statistical Details for the Kackar-Harville Correction

In the Standard Least Squares personality of the Fit Model platform, the variance matrix of the fixed effects is always modified to include a Kackar-Harville correction. The variance matrix of the BLUPs, and the covariances between the BLUPs and the fixed effects, are not Kackar-Harville corrected. The rationale for this approach is that corrections for BLUPs can be computationally and memory intensive when the random effects have many levels. In SAS, the Kackar-Harville correction is done for both fixed effects and BLUPs only when the `DDFM=KENWARDROGER` is set.

Because JMP implements the Kenward-Roger first-order adjustment, note the following:

- Standard errors for linear combinations that involve only fixed effects parameters match `PROC MIXED DDFM=KENWARDROGER(FIRSTORDER)`. This presumes that one has taken care to transform between the different parameterizations used by `PROC MIXED` and JMP.
- Standard errors for linear combinations that involve only BLUP parameters match `PROC MIXED DDFM=SATTERTHWAITE`.
- Standard errors for linear combinations that involve both fixed effects and BLUPS do not match `PROC MIXED` for any `DDFM` option if the data are unbalanced. However, these standard errors are between what you get with the `DDFM=SATTERTHWAITE` and `DDFM=KENWARDROGER(FIRSTORDER)` options. If the data are balanced, JMP matches SAS for balanced data, regardless of the `DDFM` option, because the Kackar-Harville correction is null.

Degrees of Freedom

The degrees of freedom for tests involving only linear combinations of fixed effect parameters are calculated using the Kenward and Roger correction. Therefore, the JMP results for these tests match `PROC MIXED` using the `DDFM=KENWARDROGER(FIRSTORDER)` option. If there are BLUPs in the linear combination, JMP uses a Satterthwaite approximation to get the degrees of freedom. The results then follow a pattern similar to what is described for standard errors in the preceding paragraph.

For more information about the Kackar-Harville correction and the Kenward-Roger DF approach, see Kenward and Roger (1997). The Satterthwaite method is described in detail in the MIXED Procedure chapter in SAS Institute Inc. (2022d).

Statistical Details for Power Analysis

In the Standard Least Squares personality of the Fit Model platform, options that relate to power calculations are available only for continuous-response models. These are the contexts in which power and related test details are available:

Parameter Estimate

To obtain retrospective test details for each parameter estimate, select **Estimates > Parameter Power** from the report's red triangle menu. This option displays the least significant value, the least significant number, and the adjusted power for the 0.05 significance level test for each parameter based on current study data.

Effect or Effect Details

To obtain either prospective or retrospective details for the *F* test of a specific effect, select **Power Analysis** from the effect's red triangle menu. Keep in mind that, for the Effect Screening and Minimal Report personalities, the report for each effect is found under Effect Details. For the Effect Leverage personality, the report for an effect is found to the right of the first (Whole Model) column in the report.

LS Means Contrast

To obtain either prospective or retrospective details for a test of one or more contrasts, select **LSMeans Contrast** from the effect's red triangle menu. Define the contrasts of interest and click Done. From the Contrast red triangle menu, select **Power Analysis**.

Custom Test

To obtain either prospective or retrospective details for a custom test, select **Estimates > Custom Test** from the response's red triangle menu. Define the contrasts of interest and click **Done**. From the Custom Test red triangle menu, select **Power Analysis**.

In all cases except the first, selecting Power Analysis opens the Power Details window. You then enter information in the Power Details window to modify the calculations according to your needs.

Effect Size

The effect size, denoted by δ, is a measure of the difference between the null hypothesis and the true values of the parameters involved. The null hypothesis might be formulated in terms of a single linear contrast that is set equal to zero, or of several such contrasts. The value of δ reflects the difference between the true values of the contrasts and their hypothesized values of 0.

In general terms, the effect size is given by:

$$\delta = \sqrt{SS_{Hyp(Pop)}/n}$$

where $SS_{Hyp(Pop)}$ is the sum of squares for the hypothesis being tested given in terms of population parameters and n is the total number of observations.

When observations are available, the estimated effect size is calculated by substituting the calculated sum of squares for the hypothesis into the formula for δ.

Balanced One-Way Layout

For example, in the special case of a balanced one-way layout with k levels where the i^{th} group has mean response α_i,

$$\delta^2 = \frac{\sum(\alpha_i - \bar{\alpha})^2}{k}$$

Recall that JMP codes parameters so that, for i =1, 2,..., k-1

$$\beta_i = (\alpha_i - \bar{\alpha})$$

and

$$\beta_k = -\sum_{m=1}^{k-1} \alpha_m$$

So, in terms of these parameters, δ for a two-level balanced layout is given by:

$$\delta^2 = \frac{\beta_1^2 + (-\beta_1)^2}{2} = \beta_1^2$$

or $\delta = |\beta_1|$

Unbalanced One-Way Layout

In the case of an unbalanced one-way layout with k levels, and where the i^{th} group has mean response α_i and n_i observations, and where $n = \sum n_i$:

$$\delta^2 = \sum \frac{n_i}{n}(\alpha_i - \bar{\alpha})^2$$

Effect Size and Power

The power is the probability that the F test of a hypothesis is significant at the α significance level, when the true effect size is a specified value. If the true effect size equals δ, then the test statistic has a noncentral F distribution with noncentrality parameter

$$\lambda = (n\delta^2)/\sigma^2$$

When the null hypothesis is true (that is, when the effect size is zero), the noncentrality parameter is zero and the test statistic has a central F distribution.

The power of the test increases with λ. In particular, the power increases with sample size n and effect size δ, and decreases with error variance σ^2.

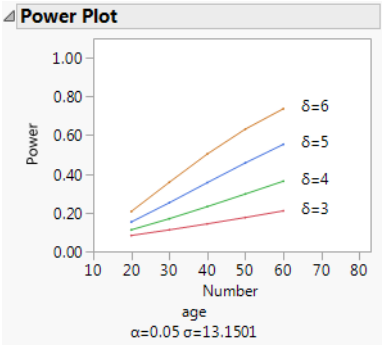
Some books, such as Cohen (1977), use a standardized effect size, $\Delta = \delta/\sigma$, rather than the raw effect size used by JMP. For the standardized effect size, the noncentrality parameter equals $\lambda = n\Delta^2$.

In the Power Details window, δ is initially set to $\sqrt{SS_{Hyp}/n}$. SS_{Hyp} is the sum of squares for the hypothesis, and n is the number of observations in the current study. SS_{Hyp} is an estimate of δ computed from the data, but such estimates are biased (Wright and O'Brien 1988). To calculate power using a sample estimate for δ, you might want to use the Adjusted Power and Confidence Interval calculation rather than the Solve for Power calculation. The adjusted power calculation uses an estimate of δ that is partially corrected for bias. See "Computations for the Adjusted Power".

Plot of Power by Sample Size

To see a plot of power by sample size, select the Power Plot option from the red triangle menu at the bottom of the Power report. JMP plots the Power and Number columns from the Power table. The plot shown in Figure 3.68 results from plotting the Power table obtained in "Example of Retrospective Power Analysis".

Figure 3.68 Plot of Power by Sample Size



The Least Significant Number (LSN)

The *least significant number* (LSN) is the smallest number of observations that leads to a significant test result, given the specified values of delta, sigma, and alpha. Recall that delta, sigma, and alpha represent, respectively, the effect size, the error standard deviation, and the significance level.

Note: LSN is *not* a recommendation of how large a sample to take because it does not take into account the probability of significance. It is computed based on specified values of delta and sigma.

The LSN has these characteristics:

- If the LSN is less than the actual sample size n, then the effect is significant.
- If the LSN is greater than n, the effect is not significant. If you believe that more data will show essentially the same structural results as does the current sample, the LSN suggests how much data you would need to achieve significance.
- If the LSN is equal to n, then the p-value is equal to the significance level alpha. The test is on the border of significance.
- The power of the test for the effect size, calculated when n = LSN, is always greater than or equal to 0.5. Note, however, that the power can be close to 0.5, which is considered low for planning purposes.

The Least Significant Value (LSV)

The LSV, or *least significant value*, is computed for single-degree-of-freedom hypothesis tests. These include tests for the significance of individual model parameters, as well as more general linear contrasts. The LSV is the smallest effect size, in absolute value, that would be significant at level alpha. The LSV gives a measure of the sensitivity of the test on the scale of the parameter, rather than on a probability scale.

The LSV has these characteristics:

- If the absolute value of the parameter estimate or contrast is greater than or equal to the LSV, then the p-value of the significance test is less than or equal to alpha.
- The absolute value of the parameter estimate or contrast is equal to the LSV if and only if its significance test has p-value equal to alpha.
- The LSV is the radius of a $1 - \alpha$ confidence interval for the parameter or linear combination of parameters. The $1 - \alpha$ confidence interval is centered at the estimate of the parameter or contrast.

Power

The power of a test is the probability that the test gives a significant result. The power is a function of the effect size δ, the significance level α, the error standard deviation σ, and the sample size n. The power is the probability that you will detect a specified effect size at a given significance level. In general, you would like to design studies that have high power of detecting differences that are of practical or scientific importance.

Power has these characteristics:

- If the true value of the parameter is in fact the hypothesized value, the power equals the significance level of the test. The significance level is usually a small value, such as 0.05. The small value is appropriate, because you want a low probability of seeing a significant result when the postulated hypothesis is true.

- If the true value of the parameter is *not* the hypothesized value, in general, you want the power to be as large as possible.
- Power increases as: sample size increases; error variance decreases; the difference between the true parameter value and the hypothesized value increases.

The Adjusted Power and Confidence Intervals

In retrospective power analysis, you typically substitute sample estimates for the population parameters involved in power calculations. This substitution causes the noncentrality parameter estimate to have a positive bias (Wright and O'Brien 1988). The adjusted power calculation is based on a form of the estimated noncentrality parameter that is partially corrected for this bias.

You can also construct a confidence interval for the adjusted power. Such confidence intervals tend to be wide. See Wright and O'Brien (1988).

Note that the adjusted power and confidence interval calculations are relevant only for the value of δ estimated from the data (the value provided by default). For other values of delta, the adjusted power and confidence interval are not provided.

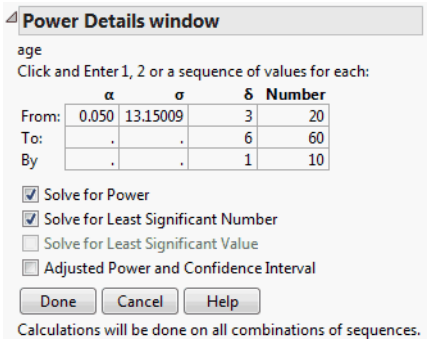
See "Computations for the Adjusted Power".

Example of Retrospective Power Analysis

This example illustrates a retrospective power analysis using the Big Class.jmp sample data table. The Power Details window (Figure 3.69) permits exploration of various quantities over ranges of values for α, σ, δ, and Number, or study size. Clicking Done replaces the window with the results of the calculations.

1. Select **Help > Sample Data Folder** and open Big Class.jmp.
2. Select **Analyze > Fit Model**.
3. Select weight and click **Y**.
4. Add age, sex, and height as the effects.
5. Click **Run**.
6. Click the age red triangle and select **Power Analysis**.

Figure 3.69 Power Details Window for Age



7. Replace the δ value in the **From** box with 3, and enter 6 and 1 in the **To** and **By** boxes as shown in Figure 3.69.
8. Replace the **Number** value in the **From** box with 20, and enter 60 and 10 in the **To** and **By** boxes as shown in Figure 3.69.
9. Select **Solve for Power** and **Solve for Least Significant Number**.
10. Click **Done**.
11. The Power Details report is replaced by the Power Details report.

Figure 3.70 Power Details Report for Age

Power Details

Test age

Power

α	σ	δ	Number	Power
0.0500	13.15009	3	20	0.0828
0.0500	13.15009	3	30	0.1117
0.0500	13.15009	3	40	0.1426
0.0500	13.15009	3	50	0.1755
0.0500	13.15009	3	60	0.2099
0.0500	13.15009	4	20	0.1117
0.0500	13.15009	4	30	0.1694
0.0500	13.15009	4	40	0.2317
0.0500	13.15009	4	50	0.2969
0.0500	13.15009	4	60	0.3630
0.0500	13.15009	5	20	0.1524
0.0500	13.15009	5	30	0.2515
0.0500	13.15009	5	40	0.3554
0.0500	13.15009	5	50	0.4575
0.0500	13.15009	5	60	0.5529
0.0500	13.15009	6	20	0.2063
0.0500	13.15009	6	30	0.3572
0.0500	13.15009	6	40	0.5035
0.0500	13.15009	6	50	0.6320
0.0500	13.15009	6	60	0.7368

Least Significant Number

α	σ	δ	Number(LSN)
0.0500	13.15009	3	216.8578
0.0500	13.15009	4	123.8892
0.0500	13.15009	5	80.93391
0.0500	13.15009	6	57.6814

This analysis is a retrospective power analysis because the calculations assume a study with a structure identical to that of the Big Class.jmp sample data table. For example, the calculation of power in this example depends on the effects entered into the model and the number of participants in each age and sex grouping. Also, the value of σ was derived from the current study, though you could have replaced it with a value that would be representative of a future study.

For more information about the power results shown in Figure 3.70, see "Power". For more information about the least significant number (LSN), see "The Least Significant Number (LSN)".

Prospective Power Analysis

Prospective analysis helps you answer the question, "If differences of a specified size exist, will I detect them given my proposed sample size, alpha level, and estimate of error variance?" In a prospective power analysis, you must provide estimates of the group means and sample sizes in a data table. You must also provide an estimate of the error standard deviation σ in the Power Details window.

Equal Group Sizes

Consider a situation where you are comparing the means of three independent groups. To obtain sample sizes to achieve a given power, select **DOE > Design Diagnostics > Sample Size and Power** and then select **k Sample Means**. Next to Std Dev, enter your estimate of the error standard deviation. In the Prospective Means list, enter means that reflect the smallest differences that you want to detect. If, for example, you want to detect a difference of 8 units between any two means, enter the extreme values of the means (for example, 40, 40, and 48). Because the power is based on deviations from the grand mean, you can enter only values that reflect the desired differences (for example, 0, 0, and 8).

If you click **Continue**, you obtain a graph of power versus sample size. If instead you specify either power or sample size in the Sample Size window, the other quantity is computed and displayed in the Sample Size window. In particular, if you specify power, the sample size that is provided is the total required sample size. The k Sample Means calculation assumes equal group sizes. For three groups, you would divide the sample size by 3 to obtain the individual group sizes. For more information about k Sample Means, see the *Design of Experiments Guide*.

Unequal Group Sizes

Suppose that you want to design a study that uses groups of different sizes. You need to plan an experiment to study two treatments that reportedly reduce bacterial counts. You want to compare the effect of these treatments with results from a control group that receives no treatment. You also want to detect a difference of at least 8 units between the means of either treatment group and the control group. But the control group must be twice as large as either treatment group. The two treatment groups also must be equal in size. Previous studies suggest that the error standard deviation is on the order of 5 or 6.

To obtain a prospective power analysis for this situation, create a data table containing some basic information, as shown in the Bacteria.jmp sample data table.

Figure 3.71 Bacteria.jmp Data Table

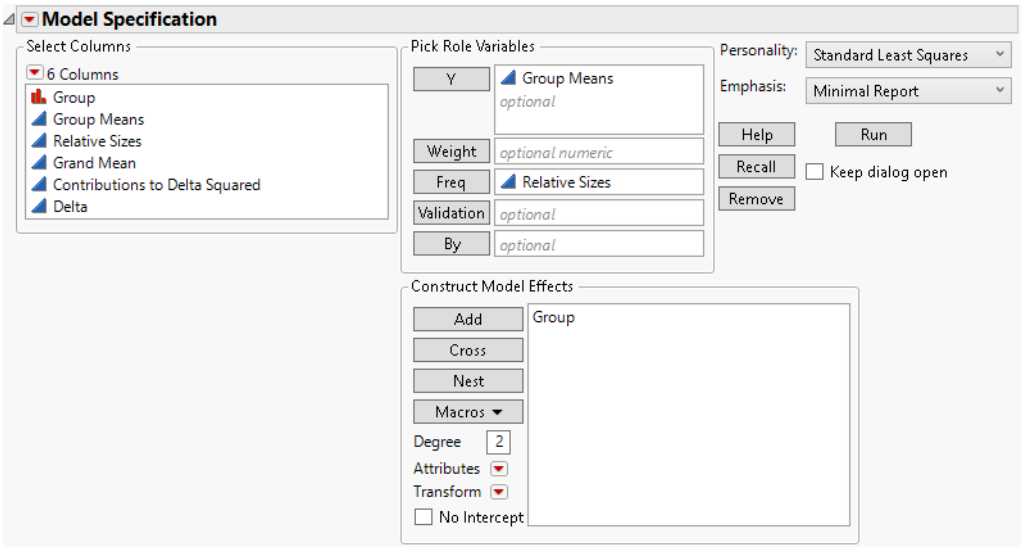
3/0 Cols	Group	Group Means	Relative Sizes
1	Control	40	2
2	Treatment A	40	1
3	Treatment B	48	1

- The Group column identifies the groups.
- The Means column reflects the smallest difference among the columns that it is important to detect. Here, it is assumed that the control group has a mean of about 40. You want the test to be significant if either treatment group has a mean that is at least 8 units higher than the mean of the control group. For this reason, you assign a mean of 48 to one of the two treatment groups. Set the mean of the other treatment group equal to that of the control group. (Alternatively, you could assign the control group and one of the treatment groups means of 0 and the remaining treatment group a mean of 8.) Note that the differences in the group means are population values.
- The Relative Sizes column shows the desired relative sizes of the treatment groups. This column indicates that the control group needs to be twice as large as each of the treatment groups. (Alternatively, you could start out with an initial guess for the treatment sizes that respects the relative size criterion.)

> **Note:** The Relative Sizes column must be assigned the role of a Freq (frequency). See the symbol to the right of the column name in the Columns panel.

Next, use Fit Model to fit a one-way analysis of variance model (Figure 3.72). Note that Relative Sizes is declared as **Freq** in the launch window. Also, the Minimal Report emphasis option is selected.

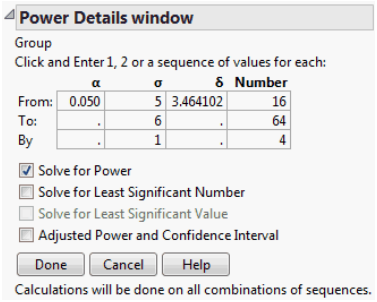
Figure 3.72 Fit Model Launch Window for Bacteria Study



Click **Run** to obtain the Fit Least Squares report. The report shows Root Mean Square Error and Sum of Squares for Error as 0.0, because you specified a data table with no error variation within the groups. You must enter a proposed range of values for the error variation to obtain the power analysis. Specifically, you have information that the error variation will be about 5 but might be as large as 6.

1. Click the disclosure icon next to Effect Details to open this report.
2. Click the Group red triangle and select **Power Analysis**.
3. To explore the range of error variation suspected by the scientist, under σ, enter 5 in the first box and 6 in the second box (Figure 3.73).
4. Note that δ is entered as 3.464102. This is the effect size that corresponds to the specified difference in the group means. The data table contains three hidden columns that illustrate the calculation of the effect size. See "Unbalanced One-Way Layout".
5. To explore power over a range of study sizes, under **Number**, enter 16 in the first box, 64 in the second box, and an increment of 4 in the third box (Figure 3.73).
6. Select **Solve for Power**.
7. Click **Done**.

Figure 3.73 Power Details Window for Bacteria Study



The Power Details report, shown in Figure 3.74, replaces the Power Details window. This report gives power calculations for $\alpha = 0.05$, for all combinations of $\sigma = 5$ and 6, and sample sizes of 16 to 64 in increments of size 4. When σ is 5, to obtain about 90% power, you need a total sample size of about 32. You need 16 participants in the control group and 8 in each of the treatment groups. On the other hand, if σ is 6, then a total of 44 participants is required.

Figure 3.74 Power Details Report for Bacteria Study

Power Details

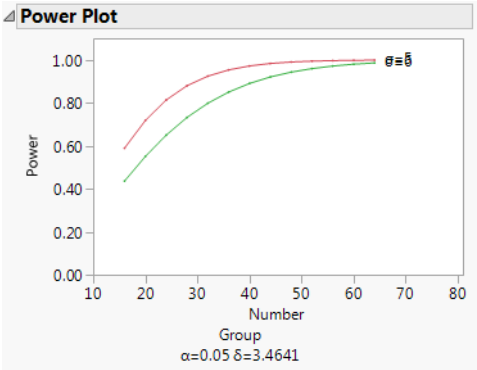
Test Group

Power

α	σ	δ	Number	Power
0.0500	5	3.464102	16	0.5894
0.0500	5	3.464102	20	0.7184
0.0500	5	3.464102	24	0.8134
0.0500	5	3.464102	28	0.8799
0.0500	5	3.464102	32	0.9246
0.0500	5	3.464102	36	0.9537
0.0500	5	3.464102	40	0.9721
0.0500	5	3.464102	44	0.9835
0.0500	5	3.464102	48	0.9903
0.0500	5	3.464102	52	0.9944
0.0500	5	3.464102	56	0.9968
0.0500	5	3.464102	60	0.9982
0.0500	5	3.464102	64	0.9990
0.0500	6	3.464102	16	0.4361
0.0500	6	3.464102	20	0.5510
0.0500	6	3.464102	24	0.6501
0.0500	6	3.464102	28	0.7323
0.0500	6	3.464102	32	0.7986
0.0500	6	3.464102	36	0.8506
0.0500	6	3.464102	40	0.8906
0.0500	6	3.464102	44	0.9208
0.0500	6	3.464102	48	0.9433
0.0500	6	3.464102	52	0.9598
0.0500	6	3.464102	56	0.9717
0.0500	6	3.464102	60	0.9803
0.0500	6	3.464102	64	0.9864

Click the arrow at the bottom of the table in the Power Details report to obtain a plot of power versus sample size for the two values of σ, shown in Figure 3.75. Here, the red markers correspond to $\sigma = 5$ and the green correspond to $\sigma = 6$.

Figure 3.75 Power Plot for Bacteria Study



Chapter 4

Standard Least Squares Examples

Analyze Common Classes of Models

This chapter provides examples with instructional material for several models fit using the Standard Least Squares personality of the Fit Model platform.

Contents

Example of Simple Linear Regression 213
Example of a Polynomial Effects Model 213
Example of One-Way Analysis of Variance 213
Example of Two-Way Analysis of Variance 216
Example of Two-Way Analysis of Variance with an Interaction 217
Example of a Three-Way Full Factorial Model 217
Example of Analysis of Covariance with Equal Slopes 218
Example of Analysis of Covariance with Unequal Slopes 221
Example of a Response Surface Model 223
Example of a Two-Factor Nested Random Effects Model 231
Example of a Split Plot Design Analysis 232
Example of a Simple Repeated Measures Model 235
Example of Estimating Random Effect Parameters 236
Example of a Knotted Spline Effect 238
Example of a Bayes Plot for Active Factors 240

Example of Simple Linear Regression

Use the Standard Least Squares personality of the Fit Model platform to fit a linear regression model. You are interested in the relationship between population and ozone level for a set of cities.

1. Select **Help > Sample Data Folder** and open Polycity.jmp.
2. Select **Analyze > Fit Model**.
3. In the Select Columns list, select Ozone and click Y.
4. In the Select Columns list, select POP.
5. Click **Add**.

When you click **Run**, the Fit Least Squares report appears, showing various results, including a Regression Plot. The Regression Plot shows the data and a simple linear regression model fit to the data.

Example of a Polynomial Effects Model

Use the Standard Least Squares personality of the Fit Model platform to fit a cubic polynomial model to bivariate data.

1. Select **Help > Sample Data Folder** and open Growth.jmp.
2. Select **Analyze > Fit Model**.
3. In the Select Columns list, select ratio and click **Y**.
4. Type 3 in the text box next to **Degree**.
5. In the Select Columns list, select age.
6. Select **Macros > Polynomial to Degree**.

Example of One-Way Analysis of Variance

Use the Standard Least Squares personality of the Fit Model platform to fit a one-way analysis of variance model, specifying a continuous response column and a nominal effect column. In a one-way analysis of variance, a different mean is fit to each of the different groups, as identified by a nominal variable.

1. Select **Help > Sample Data Folder** and open Drug.jmp.
2. Select **Analyze > Fit Model**.

3. Select y and click **Y**.
4. Select Drug and click **Add**.
5. Click **Run**.

In this example, Drug has three levels, a, d, and f. The standard least squares fitting method translates this specification into a linear model: The nominal variables define a sequence of indicator variables, which assume only the values 1, 0, and –1. The linear model is specified as follows:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$$

where:

- y_i is the observed response for the i^{th} observation
- x_{1i} is the value of the first indicator variable for the i^{th} observation
- x_{2i} is the value of the second indicator variable for the i^{th} observation
- β_0, β_1, and β_2 are parameters for the intercept, the first indicator variable, and the second indicator variable, respectively
- ε_i are the independent and normally distributed error terms

The first indicator variable, x_1, is defined as follows. Note that Drug = a contributes a value 1, Drug = d contributes a value 0, and Drug = f contributes a value –1 to the indicator variable:

$$x_{1i} = \begin{cases} 1, & \text{if Drug = a} \\ 0, & \text{if Drug = d} \\ -1, & \text{if Drug = f} \end{cases}$$

The second indicator variable, x_2, is given the following values:

$$x_{2i} = \begin{cases} 0, & \text{if Drug = a} \\ 1 & \text{if Drug = d} \\ -1, & \text{if Drug = f} \end{cases}$$

The estimates of the means for the three levels in terms of this parameterization are defined as follows:

$$\mu_a = \beta_0 + \beta_1$$

$$\mu_d = \beta_0 + \beta_2$$

$$\mu_f = \beta_0 - \beta_1 - \beta_2$$

Solving for β_i yields the following:

$$\beta_0 = \frac{(\mu_a + \mu_d + \mu_f)}{3} = \mu \text{ (the average over levels)}$$

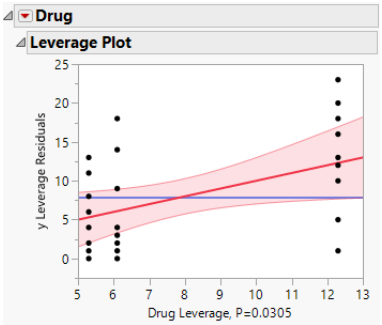
$$\beta_1 = \mu_a - \mu$$

$$\beta_2 = \mu_d - \mu$$

Therefore, if regressor variables are coded as indicators for each level minus the indicator for the last level, then the parameter for a level is interpreted as the difference between that level's response and the average response across all levels. See the appendix "Statistical Details" for additional information about the interpretation of the parameters for nominal factors.

Figure 4.1 shows the Leverage Plot and the LS Means Table for the Drug effect. Figure 4.2 shows the Parameter Estimates and the Effect Tests reports for the one-way analysis of the drug data.

Figure 4.1 Leverage Plot and LS Means Table for Drug



Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
a	5.300000	1.9197801	5.3000
d	6.100000	1.9197801	6.1000
f	12.300000	1.9197801	12.3000

Figure 4.2 Parameter Estimates and Effect Tests for Drug.jmp

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	7.9	1.108386	7.13	<.0001*
Drug[a]	-2.6	1.567494	-1.66	0.1088
Drug[d]	-1.8	1.567494	-1.15	0.2609

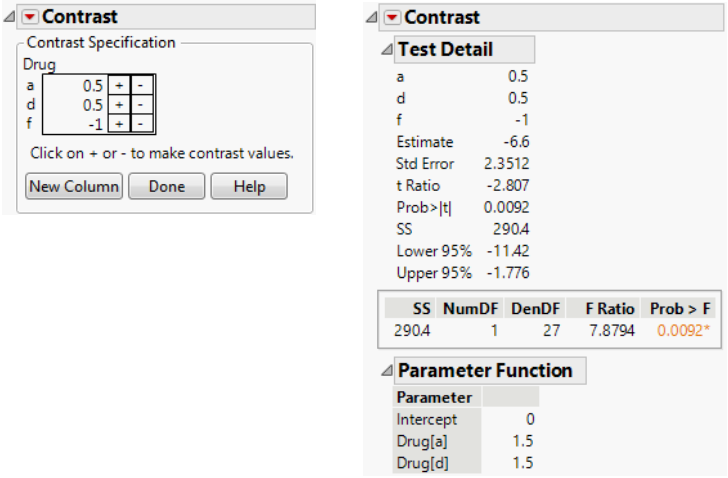
Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Drug	2	2	293.60000	3.9831	0.0305*

The Drug effect can be studied in more detail by using a contrast of the least squares means:

1. Click the Drug red triangle and select **LSMeans Contrast**.
2. Click the + boxes for drugs a and d, and the - box for drug f to define the contrast that compares the average of drugs a and d to f (shown in Figure 4.3).
3. Click **Done**.

Figure 4.3 Contrast Example for the Drug Experiment



The Contrast report shows that the LSMean for drug f is significantly different from the average of the LSMeans of the other two drugs.

Example of Two-Way Analysis of Variance

Use the Standard Least Squares personality of the Fit Model platform to fit a two-way analysis of variance model. You then use the model to explore predictions based on settings of the effect variables.

1. Select **Help > Sample Data Folder** and open Analgesics.jmp.

2. Select **Analyze > Fit Model**.
3. Select pain and click **Y**.
4. Select gender and drug and click **Add**.
5. Click **Run**.
6. From the red triangle menu next to Response pain, select **Factor Profiling > Profiler**.

Example of Two-Way Analysis of Variance with an Interaction

Use the Standard Least Squares personality of the Fit Model platform to fit a two-way analysis of variance model with an interaction term. You are interested in whether the type of popcorn and the size of the batch have an effect on the yield of popcorn.

1. Select **Help > Sample Data Folder** and open Popcorn.jmp.
2. Select **Analyze > Fit Model**.
3. In the Select Columns list, select yield and click **Y**.
4. In the Select Columns list, select popcorn and batch.
5. Select **Macros > Full Factorial**.
6. Click **Run**.
7. From the red triangle menu next to Response yield, select **Factor Profiling > Profiler**.

Example of a Three-Way Full Factorial Model

Use the Standard Least Squares personality of the Fit Model platform to fit a three-way full factorial model. You are interested in whether speed, angle, and material, or their interactions, have an effect on the amount of wear on a cutting tool.

1. Select **Help > Sample Data Folder** and open Tool Wear.jmp.
2. Select **Analyze > Fit Model**.
3. In the Select Columns list, select Wear and click **Y**.
4. In the Select Columns list, select Speed, Angle, and Material.
5. Select **Macros > Full Factorial**.
6. Click **Run**.
7. From the red triangle menu next to Response Wear, select **Factor Profiling > Surface Profiler**.

Tip: To add points to the plot, open the Appearance panel and click **Actual**. If you want to make the points appear larger, right-click in the plot, select **Settings** and adjust the **Marker Size**.

The Surface Profiler plots show the predicted response for Wear in terms of the two continuous effects Speed and Angle. The points for which Material is A are colored red, whereas those for which Material is B are colored blue. The difference in the form of the response surfaces across the levels of Material is a consequence of the three-way interaction.

To show plots for both Materials A and B, use the slider marked Material in the Independent Variables panel, setting it at 0 for Material A and 1 for Material B. Note that the table contains two data table scripts that produce Surface Profiler plots: **Prediction and Surface Profilers** and **Surface Profilers for Two Materials**.

Example of Analysis of Covariance with Equal Slopes

Use the Standard Least Squares personality of the Fit Model platform to fit an analysis of variance model with a continuous regressor term. This model is called an *analysis of covariance*. The covariate adds an additional term, x_{3i}, to the model equation. The model analysis of covariance model is written this way:

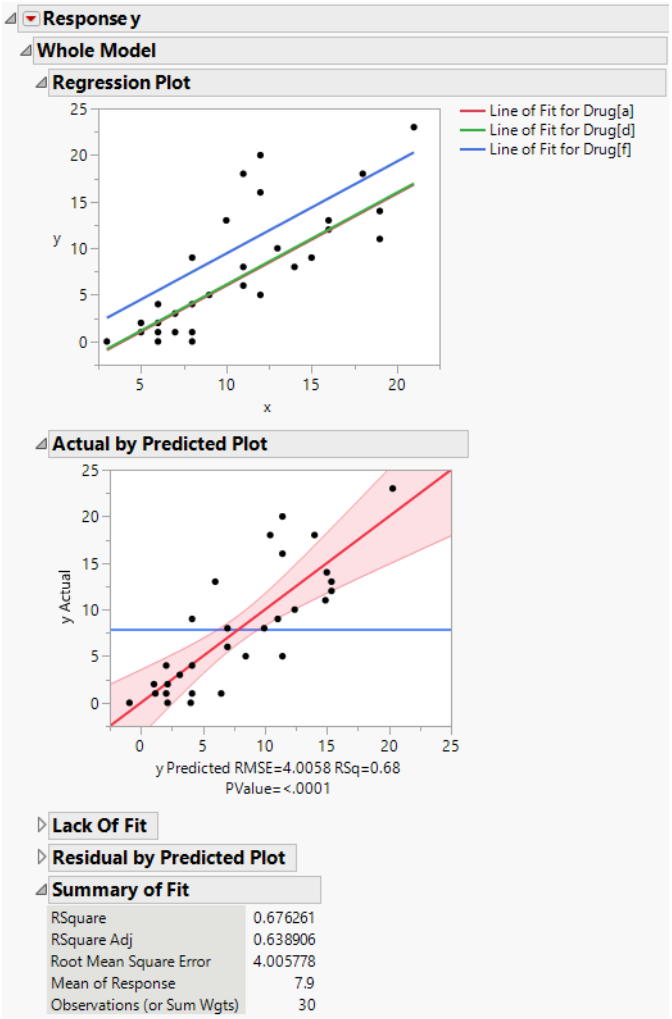
$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i$$

There are two model effects: one is a nominal main effect involving two parameters, and the other is continuous covariate associated with one parameter.

1. Select **Help > Sample Data Folder** and open Drug.jmp.
2. Select **Analyze > Fit Model**.
3. Select y and click **Y**.
4. Select both Drug and x and click **Add**.
5. Click **Run**.

The Regression Plot in the report shows that you have fit a model with equal slopes (Figure 4.4). Compared to the main effects model (Drug effect only), RSquare increases from 22.8% to 67.6%. The Root Mean Square Error decreases from 6.07 to 4.0. As shown in Figure 4.4, the *F* test significance probability for the whole model decreases from 0.03 to less than 0.0001.

Figure 4.4 Analysis of Covariance for Drug Data



The drug data table contains replicated observations. For example, rows 1 and 9 both have Drug = a and x = 11. When fitting models, replicated observations can be used to construct a *pure error* estimate of variation. Another estimate of error can be constructed for unspecified functional forms of covariates, or interactions of nominal effects. These estimates form the basis for a lack of fit test. If the lack of fit error is significant, this indicates that there is some effect in your data not explained by your model.

The Lack of Fit report shows the results of this test for the drug data. The lack of fit error is not significant, as seen by the Prob > F value of 0.7507.

The covariate, x, accounts for much of the variation in the response previously accounted for by the Drug variable. Thus, even though the model is fit with much less error, the Drug effect is no longer significant. The significance of Drug observed in the main effects model appears to be explained by the covariate.

The least squares means in the covariance model differ from the ordinary means. This is because they are adjusted for the effect of x, the covariate, on the response, y. The least squares means are values predicted for each of the three levels of Drug, when the covariate, x, is held at some neutral value. The neutral value is chosen to be the mean of the covariate, which is 10.7333.

The least squares means are calculated using the parameter estimates given in the Parameter Estimates report:

Prediction Expression: `-2.696 - 1.185*Drug[a] - 1.0761*Drug[d] + 0.98718*x`

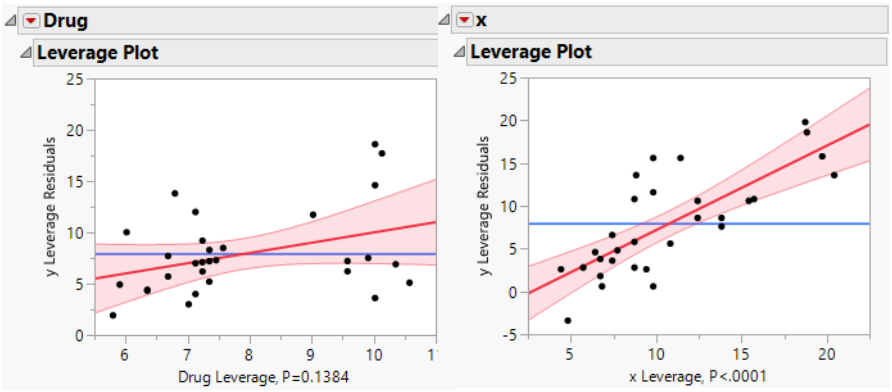
For a: `-2.696 - 1.185*(1) -1.0761*(0) + 0.98718*(10.7333) = 6.71`

For d: `-2.696 - 1.185*(0) -1.0761*(1) + 0.98718*(10.7333) = 6.82`

For f: `-2.696 - 1.185*(-1) -1.0761*(-1) + 0.98718*(10.7333) = 10.16`

Figure 4.5 shows a leverage plot for each effect. Because the covariate is significant, the leverage values for Drug are dispersed somewhat from their least squares means.

Figure 4.5 Comparison of Leverage Plots for Drug Test Data



Example of Analysis of Covariance with Unequal Slopes

Use the Standard Least Squares personality of the Fit Model platform to fit an analysis of covariance model with unequal slopes. This example fits a model where the slope for a continuous regressor depends on the level of a nominal variable. After fitting the model, the example compares the least square means for the levels of the nominal variable at a specific value of the regressor.

1. Select **Help > Sample Data Folder** and open Drug.jmp.
2. Select **Analyze > Fit Model**.
3. Select y and click **Y**.
4. Select both Drug and x and click **Macros > Factorial to Degree**.

   This adds terms up to the degree specified in the **Degree** box to the model. The default value for **Degree** is 2. Thus, the main effects of Drug and x, and their interaction, Drug*x, are added to the model effects list.
5. Click **Run**.

This specification adds two columns to the linear model (call them x_{4i} and x_{5i}) that allow the slopes for the covariate to differ by Drug level. The new variables are formed by multiplying the indicator variables for Drug by the covariate values, giving the following formula:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \beta_5 x_{5i} + \varepsilon_i$$

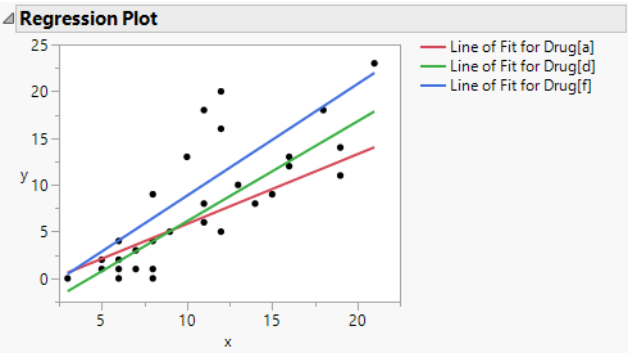
Table 4.1, shows the coding for this model. The mean of X is 10.7333. It is used in centering continuous terms.

Table 4.1 Coding of Analysis of Covariance with Separate Slopes

Regressor	Effect	Values
X_1	Drug[a]	+1 if a, 0 if d, –1 if f
X_2	Drug[d]	0 if a, +1 if d, –1 if f
X_3	X	the values of X
X_4	Drug[a]*(X - 10.733)	X – 10.7333 if a, 0 if d, –(X – 10.7333) if f
X_5	Drug[d]*(X - 10.733)	0 if a, X – 10.7333 if d, –(X – 10.7333) if f

A portion of the report is shown in Figure 4.6. The Regression Plot shows fitted lines with different slopes. The Effect Tests report gives a *p*-value for the interaction of 0.56. This is not significant, indicating the model does not need to include different slopes.

Figure 4.6 Plot with Interaction



Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Drug	2	2	52.05637	1.5713	0.2284
x	1	1	564.56753	34.0821	<.0001*
Drug*x	2	2	19.64465	0.5930	0.5606

Perform a Spotlight Analysis

You now want to compare the least square means for the levels of Drug at a specific value of the covariate x. This type of comparison in an analysis of covariance model is sometimes referred to as *spotlight analysis*. For more information about spotlight analysis, see Spiller et al. (2013).

1. Click the Response y red triangle and select **Multiple Comparisons**.
2. In the Multiple Comparisons window, select **User-Defined Estimates**.
3. Select all three values below **Choose Drug Levels**.
4. In the first box below x, enter 12.5.
5. Click **Add Estimates**.

   This adds comparisons of the three levels of Drug at x = 12.5.
6. Click **OK**.

   The User-Defined Estimates report shows least square means estimates for each level of Drug with the covariate x set to 12.5. The red triangle next to Multiple Comparisons for User-Defined Estimates contains options that enable you to test for differences among the estimates.

Figure 4.7 User-Defined Estimates Report

Multiple Comparisons for User-Defined Estimates

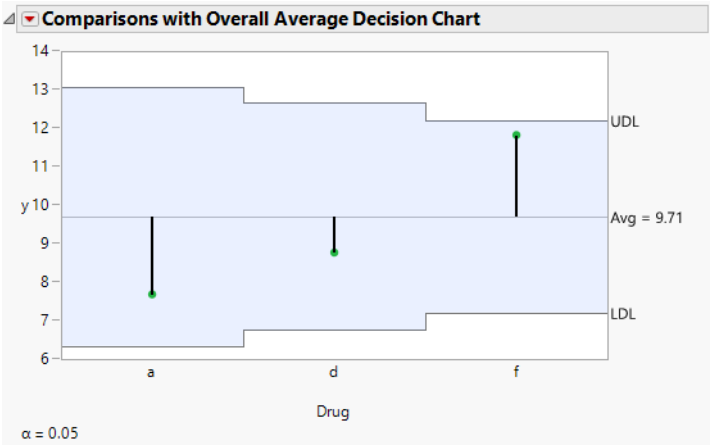
User-Defined Estimates

x = 12.5

Drug	Estimate	Std Error	DF	Lower 95%	Upper 95%
a	7.684713	1.5772057	24	4.4295208	10.939906
d	8.771371	1.4401229	24	5.7991033	11.743639
f	11.822214	1.2943345	24	9.1508392	14.493590

7. Click the red triangle next to Multiple Comparisons for User-Defined Estimates and select **Comparisons with Overall Average**.

Figure 4.8 Comparisons with Overall Average Decision Chart



The Comparisons with Overall Average option creates an analysis of means (ANOM) chart for differences between the average and the three least squares means. From the ANOM chart, you conclude that there is not a significant effect of Drug on the response at x = 12.5.

Example of a Response Surface Model

Use the Standard Least Squares personality of the Fit Model platform to fit a response surface model. Your objective is to minimize the response.

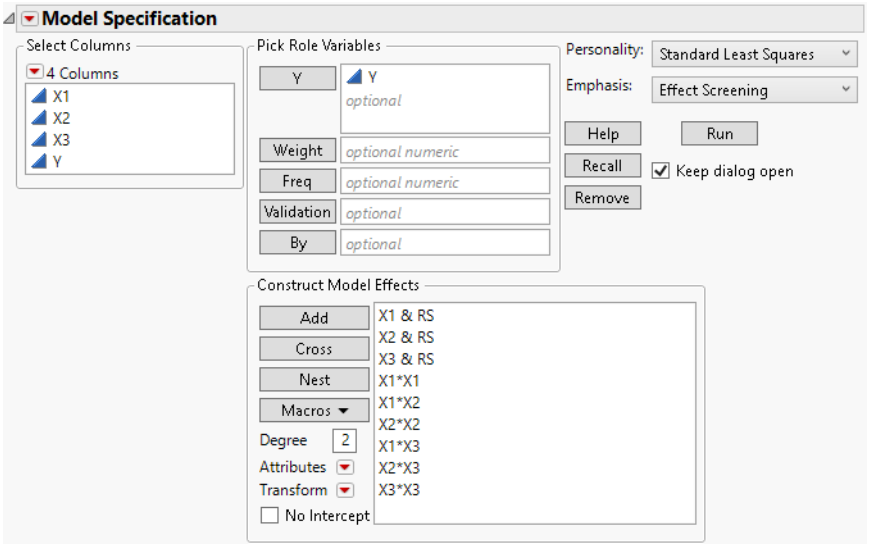
Fit the Full Response Surface Model

1. Select **Help > Sample Data Folder** and open Design Experiment/Custom RSM.jmp.
2. Select **Analyze > Fit Model**.

   Because the data table contains a **Model** script, the Model Specification window is filled out as specified in the **Model** script. Note the following:

- Main effects appear in the **Construct Model Effects** list with a **&RS** suffix, indicating that the Response Surface macro has been applied.
- The effects are those for a full response surface in the three predictors X1, X2, and X3.
- Because the model contains terms with the **&RS** suffix, the analysis results will include a Response Surface report.

Figure 4.9 Fit Model Launch Window for the Response Surface Analysis

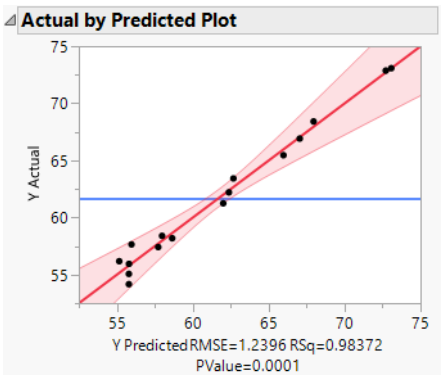


3. Click **Run**.

Reduce the Model

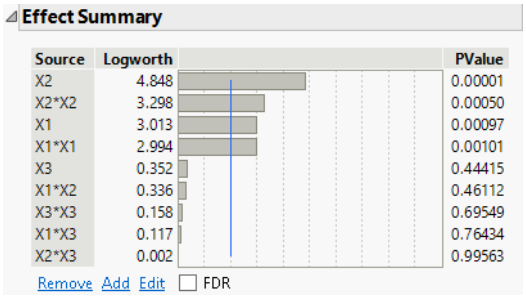
The Actual by Predicted Plot shows that the model is significant. There is no evidence of lack of fit.

Figure 4.10 Actual by Predicted Plot for Full Model



The Effect Summary report suggests that a number of effects are not significant. In particular, X2*X3 is the least significant effect with a PValue of 0.99563. Next, you will systematically reduce the model using the Effect Summary report interactively.

Figure 4.11 Effect Summary Report



1. In the Effect Summary report, click X2*X3 and click **Remove**.

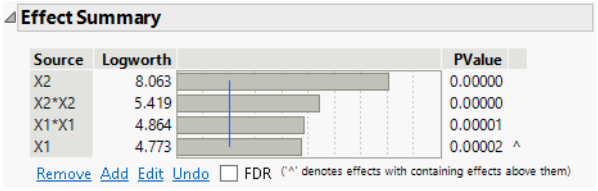
   The model updates.

   The PValue column in the Effect Summary report indicates that X1*X3 is not significant.
2. Click X1*X3 and click **Remove**.

   The PValue for X3*X3 indicates that it is not significant.
3. Click X3*X3 and click **Remove**.
4. Click X1*X2 and click **Remove**.

   Notice that X3 is not significant. It is not contained in any higher-order effects, so you can remove it without violating the Effect Heredity principle. See "Effect Heredity".
5. Click X3 and click **Remove**.

Figure 4.12 Effect Summary Report After Reducing Model



All remaining effects are significant.

Examine the Response Surface Report

1. Click the Response Y red triangle and select **Estimates > Show Prediction Expression**.

Figure 4.13 Prediction Expression

Prediction Expression

55.721136364
+ 2.349 • X1
+ 5.003 • X2
+ X1 • (X1 • 4.4365909091)
+ X2 • (X2 • 5.0765909091)

Tip: The Prediction Expression is the prediction formula. You can also obtain this formula by selecting **Save Columns > Prediction Formula**.

In the following steps, you can refer to the prediction expression to see the model coefficients.

2. Open the Response Surface report and then the Canonical Curvature report.

Figure 4.14 Response Surface Report

Response Surface

Coef

	X1	X2	Y
X1	4.4365909	0	2.349
X2	.	5.0765909	5.003

Solution

Variable	Critical Value
X1	-0.26473
X2	-0.492752

Solution is a Minimum
Predicted Value at Solution 54.177592

Canonical Curvature

Eigenvalues and Eigenvectors

Eigenvalue	5.0766	4.4366
X1	0.00000	1.00000
X2	1.00000	0.00000

The first table gives the second-order model coefficients in matrix form. The coefficient of X1*X1 is 4.4365909, the coefficient of X2*X2 is 5.0765909, and the coefficient of X1*X2 is 0. The coefficients of the linear effects, 2.349 for X1 and 5.003 for X2, are given in the column labeled Y.

The Solution report shows the critical values. These are the values where a maximum, a minimum, or a saddle point occur. In this example, the Solution report indicates that the response surface achieves a minimum of 54.18 at the critical value, where X1 = -0.265 and X2 = -0.493.

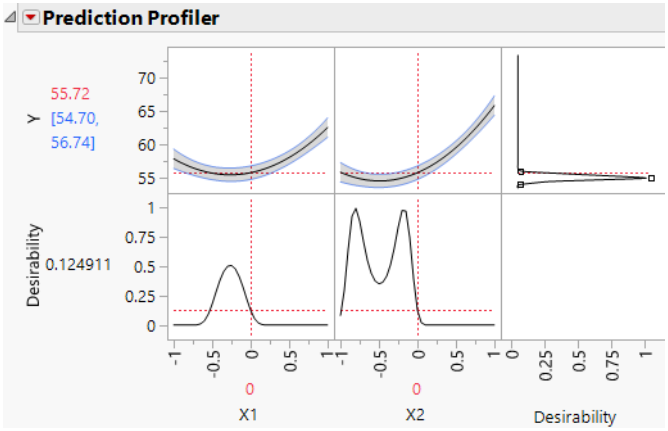
The Canonical Curvature report shows the eigenstructure of the matrix of second-order parameter estimates. The eigenstructure is useful for identifying the shape and orientation of the curvature. See "Canonical Curvature Report".

In this example, both eigenvalues are positive, which indicates that the surface achieves a minimum. The direction of greatest curvature corresponds to the largest eigenvalue (5.0766). That direction is defined by the corresponding eigenvector components. For the first direction, X2, with an eigenvector value of 1.00, determines the direction. The second direction is determined by X1, also with an eigenvector value of 1.00.

Find the Critical Point Using the Prediction Profiler

The Prediction Profiler report shows the quadratic behavior of the response surface along traces for X1 and X2. Because the Response Limits column property is set for Y, the profiler also shows desirability functions.

Figure 4.15 Prediction Profiler with Match Target as Goal

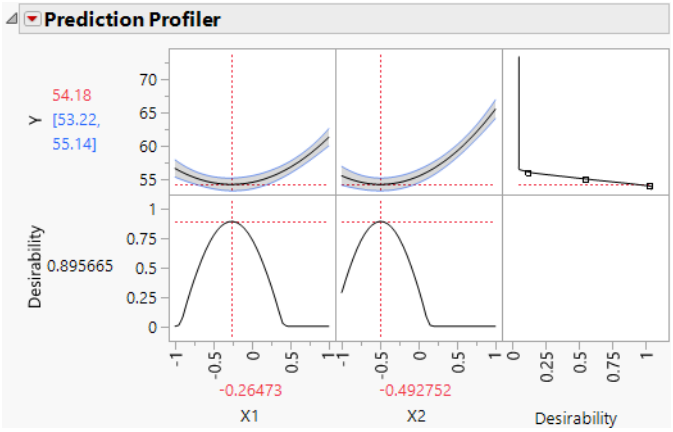


The goal for the Response Limits column property is set to Match Target. For this example, you are interested in minimizing Y, not matching a target. Therefore, you must change the setting:

1. Press Ctrl and click in the top right cell of the Prediction Profiler.
2. In the Response Goal dialog, select **Minimize** from the list of options.
3. Click **OK**.

   The desirability function now reflects your goal of minimizing Y.
4. Click the Prediction Profiler red triangle and select **Optimization and Desirability > Maximize Desirability**.

Figure 4.16 Prediction Profiler with Minimize as Goal and Desirability Maximized



Settings within the design region that minimize Y appear under the profiler. Note that these are precisely the Critical Values given in the Solution report.

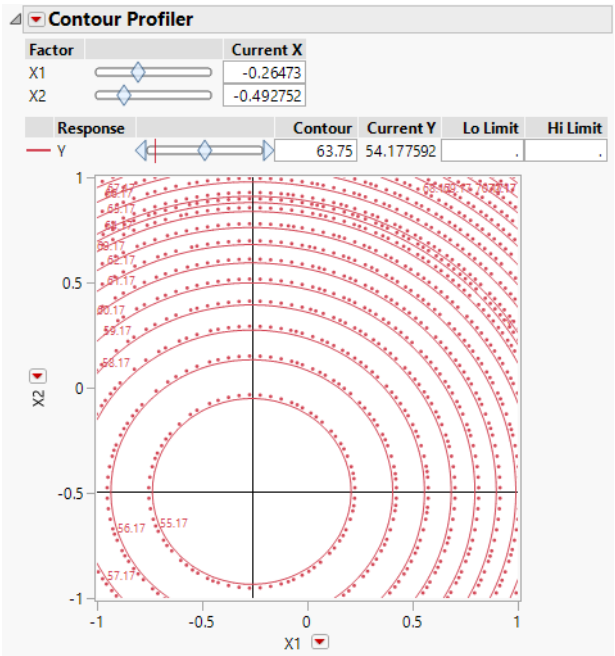
View the Surface Using the Contour Profiler

The Contour Profiler shows contours of the response surface. It gives an alternate profiler visualization of the predicted response in the area of the critical point.

1. Click the Response Y red triangle and select **Factor Profiling > Contour Profiler**.
2. Click the Contour Profiler red triangle and select **Contour Grid**.
3. For **Increment**, type 1.
4. Click **OK**.

   The contours are plotted at one unit intervals. See Figure 4.17.
5. Click the Prediction Profiler red triangle and select **Factor Settings > Link Profilers**.

Figure 4.17 Contour Profiler with Crosshairs at Critical Point

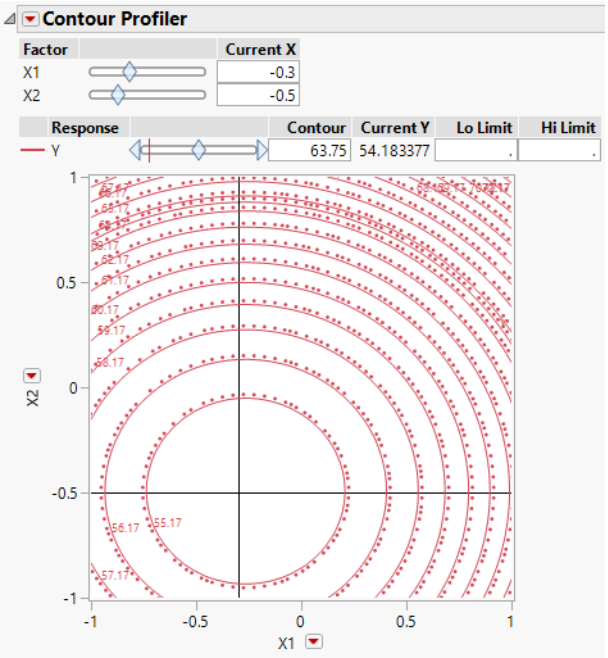


Linking the Contour Profiler to the Prediction Profiler links the **Current X** values in the Contour Profiler to the X values shown in the Prediction Profiler. The X values in the Prediction Profiler give the critical point where Y is minimized. The crosshairs in the Contour Profiler show the critical point. Notice that the **Current Y** value is 54.177592, the predicted minimum value according to the Prediction Profiler.

Often, it is not possible to set your process factors to exactly the values that optimize the response. The Contour Profiler can help you identify alternate settings of the process factors. In the next steps, suppose that you can set your process to X1 and X2 values with only one decimal place precision, and that your process settings can vary by one decimal place in either direction of those settings.

6. In the Contour Profiler report, under **Current X**, type -0.3 next to X1 and -0.5 next to X2.

Figure 4.18 Contour Profiler Showing X1 = -0.3 and X2 = -0.5



The crosshairs are well within the innermost contour and the Current Y (the predict Y value at the Current X settings) is 54.183377, only slightly different from the predicted minimum of 54.177592.

7. In the Contour Profiler, click and drag the crosshairs to explore Current X values within a 0.1 unit radius of X1 = -0.3 and X2 = -0.5.

   The predicted Y values are all below 54.4. In fact, if the settings wander to any point within the innermost contour, the predicted Y is less than the contour value of 55.17.

Example of a Two-Factor Nested Random Effects Model

Use the Standard Least Squares personality of the Fit Model platform to fit a two-factor nested random effects model. As part of a measurement systems analysis study, 24 randomly chosen parts are measured. These parts are evenly divided among the six operators who typically measure these parts. Each operator makes three independent measurements of each of the four assigned parts.

Since the parts measured by one operator are measured only by that specific operator, Part is nested within Operator. Since the parts are a random sample of production, Part is considered a random effect. Since these specific six operators are of interest, Operator is treated as a fixed effect. Specify the appropriate model.

1. Select **Help > Sample Data Folder** and open Variability Data/2 Factors Nested.jmp.
2. Select **Analyze > Fit Model**.
3. In the Select Columns list, select Y and click **Y**.
4. In the Select Columns list, select Operator and Part.
5. Click **Add**.
6. To nest Part within Operator: In the Construct Model Effects list, select Part. In the Select Columns list, select Operator. The two effects should be highlighted.
7. Click **Nest**.
8. With Part[Operator] highlighted in the Construct Model Effects list, select **Attributes > Random Effect**.
9. Click **Run**.

The figure shows two plots. The first is a Variability Chart showing the three measurements by each Operator on each of the four parts. Horizontal line segments show the mean measurement for each Operator.

To construct the Variability Chart, in the 2 Factors Nested.jmp sample data table, run the data table script **Variability Chart - Nested**. Click the Variability Gauge red triangle, deselect **Show Range Bars**, and select **Show Group Means**.

The second plot is the Fit Least Squares report Prediction Profiler plot for Operator. This plot shows the predicted response for each operator. The vertical dashed red line set at Jane indicates that Jane's predicted response is 0.997. You can see the correspondence between the model predictions given in the Prediction Profiler plot and the raw data in the Variability Chart.

To obtain the Prediction Profiler plot, click the Response Y red triangle and select **Factor Profiling > Profiler**.

These plots show how the predicted measurements for each Operator are modeled. However, keep in mind that you are not only interested in whether the operators differ in how they measure parts. You are also interested in the variability of the part measurements themselves, which requires estimation of the variance component associated with Part.

Example of a Split Plot Design Analysis

Use the Standard Least Squares personality of the Fit Model platform to analyze a split plot design. Levels of random effects are randomly selected from a larger population of levels. For the purpose of inference, the distribution of a random effect is assumed to be normal, with mean zero and some variance (called a *variance component*).

In a sense, every model has at least one random effect, which is the effect that makes up the residual error. The individual observations are assumed to be randomly selected from a much larger population, and the error term is assumed to have a mean of zero and variance σ^2.

The most common random effects model is the repeated measures or split plot model. Table 4.2 lists the types of effects in a split plot model. In these models, the experiment has two layers. Some effects are applied on the whole plots or subjects of the experiment. Then these plots are divided or the subjects are measured at different times and other effects are applied within those subunits. The effects describing the whole plots or subjects are whole plot effects, and the subplots or repeated measures are subplot effects. Usually, the subunit effect is omitted from the model and absorbed as residual error.

Table 4.2 Types of Effects in a Split Plot Model

Split Plot Model	Type of Effect	Repeated Measures Model
whole plot treatment	fixed effect	across subjects treatment
whole plot ID	random effect	subject ID
subplot treatment	fixed effect	within subject treatment
subplot ID	random effect	repeated measures ID

Each of these cases can be treated as a layered model, and there are several traditional ways to fit them in a fair way. The situation is treated as two different experiments:

1. The whole plot experiment has whole plot or subjects as the experimental unit to form its error term.
2. Subplot treatment has individual measurements for the experimental units to form its error term (left as residual error).

The older, traditional way to test whole plots is to do any one of the following:

- Take means across the measurements and fit these means to the whole plot effects.
- Form an *F*-ratio by dividing the whole plot mean squares by the whole plot ID mean squares.
- Organize the data so that the split or repeated measures form different columns. Fit a MANOVA model, and use the univariate statistics.

These approaches work if the structure is simple and the data are complete and balanced. However, there is a more general model that works for any structure of random effects. This more generalized model is called the *mixed model*, because it has both fixed and random effects.

The most common type of layered design is a balanced split plot, often in the form of repeated measures across time. One experimental unit for some of the effects is subdivided (sometimes by time period) and other effects are applied to these subunits.

Consider the data in the Animals.jmp sample data table (the data are fictional). The study collected information about differences in the seasonal hunting habits of foxes and coyotes. Each season for one year, three foxes and three coyotes were marked and observed periodically. The average number of miles that they wandered from their dens during different seasons of the year was recorded (rounded to the nearest mile). The model is defined by the following aspects:

- The continuous response variable called miles
- The species effect with values fox or coyote
- The season effect with values fall, winter, spring, and summer
- An animal identification code called subject, with nominal values 1, 2, and 3 for both foxes and coyotes

There are two layers to the model:

1. The top layer is the between-subject layer, in which the effect of being a fox or coyote (species effect) is tested with respect to the variation from subject to subject.
2. The bottom layer is the within-subject layer, in which the repeated-measures factor for the four seasons (season effect) is tested with respect to the variation from season to season within a subject. The within-subject variability is reflected in the residual error.

The season effect can use the residual error for the denominator of its *F* statistics. However, the between-subject variability is not measured by residual error and must be captured with the subject within species (subject[species]) effect in the model. The *F* statistic for the between-subject effect species uses this nested effect instead of residual error for its *F* ratio denominator.

Note: JMP Pro users can construct this model using the Mixed Model personality.

To specify the split plot model for this data, follow these steps:

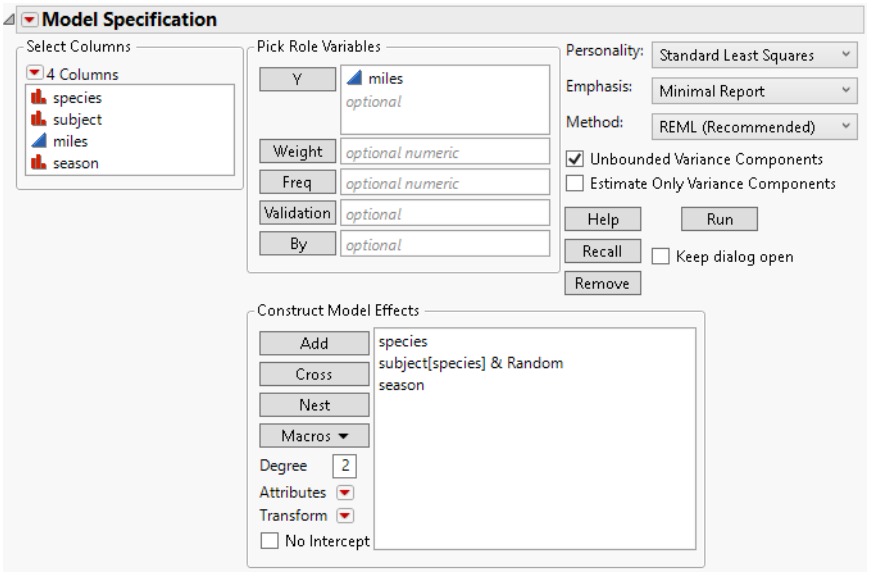
1. Select **Help > Sample Data Folder** and open Animals.jmp.
2. Select **Analyze > Fit Model**.
3. Select miles and click **Y**.
4. Select species and subject and click **Add**.
5. In the Select Columns list, select species.
6. In the Construct Model Effects list, select subject.
7. Click **Nest.**

   This adds the subject within species (subject[species]) effect to the model.
8. Select the nested effect subject[species].
9. Select **Attributes > Random Effect**.

   This nested effect is now identified as an error term for the species effect and appears as subject[species]&Random.
10. In the Select Columns list, select season and click **Add**.

    When you define an effect as random using the **Attributes** menu, the Method options (**REML** and **EMS**) appear at the top right of the dialog. The **REML** option is selected as the default. The completed launch window is shown in Figure 4.19.

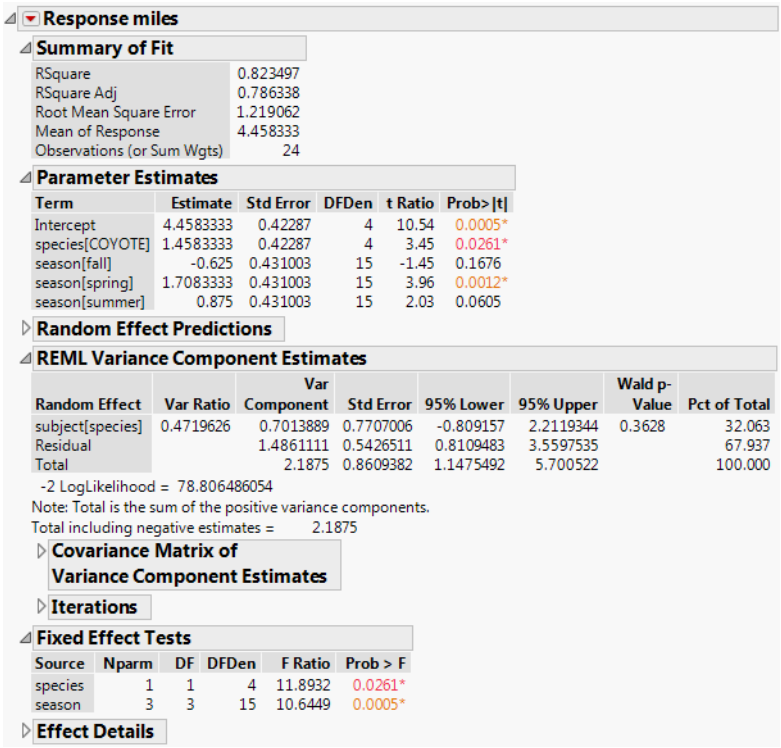
Figure 4.19 Fit Model Dialog



11. Click **Run**.

The report is shown in Figure 4.20. Both fixed effects, species and season, are significant. The REML Variance Component Estimates report gives estimates of the subject within species and residual variances.

Figure 4.20 Partial Report of REML Analysis



Response miles

Summary of Fit

RSquare	0.823497
RSquare Adj	0.786338
Root Mean Square Error	1.219062
Mean of Response	4.458333
Observations (or Sum Wgts)	24

Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|
Intercept	4.4583333	0.42287	4	10.54	0.0005*
species[COYOTE]	1.4583333	0.42287	4	3.45	0.0261*
season[fall]	-0.625	0.431003	15	-1.45	0.1676
season[spring]	1.7083333	0.431003	15	3.96	0.0012*
season[summer]	0.875	0.431003	15	2.03	0.0605

Random Effect Predictions

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Wald p-Value	Pct of Total
subject[species]	0.4719626	0.7013889	0.7707006	-0.809157	2.2119344	0.3628	32.063
Residual		1.4861111	0.5426511	0.8109483	3.5597535		67.937
Total		2.1875	0.8609382	1.1475492	5.700522		100.000

-2 LogLikelihood = 78.806486054
Note: Total is the sum of the positive variance components.
Total including negative estimates = 2.1875

Covariance Matrix of Variance Component Estimates

Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
species	1	1	4	11.8932	0.0261*
season	3	3	15	10.6449	0.0005*

Effect Details

Example of a Simple Repeated Measures Model

Use the Standard Least Squares personality of the Fit Model platform to fit a model to repeated measures data. You have data on the distance traveled by each of six animals in each of the four seasons. There are two species and the animal identifier is nested within species. Since these six animals are representatives of larger species populations, you decide to treat subject as a random effect. You want to model the response, miles, as a function of species and season, accounting for the fact that there are repeated measures for each animal.

1. Select **Help > Sample Data Folder** and open Animals.jmp.
2. Select **Analyze > Fit Model**.
3. In the Select Columns list, select miles and click **Y**.
4. In the Select Columns list, select species and subject.

5. Click **Add**.
6. To nest subject within species: In the Construct Model Effects list, select subject. In the Select Columns list, select species. The two effects should be highlighted.
7. Click **Nest**.
8. In the Construct Model Effects list, select subject[species].
9. Select **Attributes > Random Effect**.
10. In the Select Columns list, select season.
11. Click **Add**.
12. In the Construct Model Effects list, select season. In the Select Columns list, click species. Both effects should be highlighted.
13. Click **Cross**.
14. Click **Run**.

Example of Estimating Random Effect Parameters

Use the Standard Least Squares personality of the Fit Model platform to estimate parameters in a random effects model. Random effects have a dual character. In one characterization, they represent residual error, such as the error associated with a whole-plot experimental unit. In another characterization, they are like fixed effects, associating a parameter with each level of the random effect. As parameters, you have extra information about them—they are derived from a normal distribution with mean zero and the variance estimated by the variance component. The effect of this extra information is that the estimates of the parameters are shrunken toward zero.

The parameter estimates associated with random effects are called *BLUPs* (Best Linear Unbiased Predictors). Some researchers consider these BLUPs as parameters of interest, and others consider them uninteresting by-products of the methodology.

BLUP parameter estimates are used to estimate random-effect least squares means, which are therefore also shrunken toward the grand mean. The degree of shrinkage depends on the variance of the effect and the number of observations per level in the effect. With large variance estimates, there is little shrinkage. If the variance component is small, then more shrinkage takes place. If the variance component is zero, the effect levels are shrunk to exactly zero. It is even possible to obtain highly negative variance components where the shrinkage is reversed. You can consider fixed effects as a special case of random effects where the variance component is very large.

The REML method balances the information about each individual level with the information about the variances across levels. If the number of observations per level is large, the estimates shrink less. If there are very few observations per level, the estimates shrink more. If there are infinitely many observations, there is no shrinkage and the estimates are identical to fixed effects.

Suppose that you have batting averages for different baseball players. The variance component for the batting performance across players describes how much variation is typical between players in their batting averages. Suppose that the player plays only a few times and that the batting average is unusually small or large. Then you tend not to trust that estimate, because it is based on only a few at-bats. But if you mix that estimate with the grand mean, that is, shrink the estimate toward the grand mean, you would trust the estimate more. For players who have a long batting record, you would shrink much less than those with a short record.

You can explore this behavior yourself.

1. Select **Help > Sample Data Folder** and open Baseball.jmp.
2. Select **Analyze > Fit Model**.
3. Select Batting and click **Y, Response**.
4. Select Player and click **Add**.
5. Select Player in the Construct Model Effects box, and select **Random Effect** from the Attributes list.
6. Click **Run**.

Table 4.3 shows the Least Squares Means from the Player report for a REML (Recommended) fit. Also shown are the Method of Moment estimates, obtained using the EMS Method. The Method of Moment estimates are the ordinary Player means. Note that the REML estimate for Suarez, who has only three at-bats, is shrunken more toward the grand mean than estimates for other players with more at-bats.

Table 4.3 Comparison of Estimates between Method of Moments and REML

	Method of Moments	REML	N
Variance Component	0.01765	0.019648	
Anderson	0.29500000	0.29640407	6
Jones	0.20227273	0.20389793	11
Mitchell	0.32333333	0.32426295	6
Rodriguez	0.55000000	0.54713393	6
Smith	0.35681818	0.35702094	11
Suarez	0.55000000	0.54436227	3
Least Squares Means	same as ordinary means	shrunken from means	

Example of a Knotted Spline Effect

Use the Standard Least Squares personality of the Fit Model platform to fit a model with a knotted spline effect. The Knotted Spline Effect attribute enables you to fit a segmentation of smooth polynomials to a specified effect. When you select this attribute, a window appears, enabling you to specify the number of knot points. (Knotted splines are implemented only for main-effect continuous terms.)

JMP follows the advice in the literature in positioning the points. The knotted spline is also referred to as a Stone spline or a Stone-Koo spline. See Stone and Koo (1985). If there are 100 or fewer points, the first and last knots are the fifth point inside the minimum and maximum, respectively. Otherwise, the first and last knots are placed at the 0.05 and 0.95 quantiles if there are 5 or fewer knots, or the 0.025 and 0.975 quantiles for more than 5 knots. The default number of knots is 5 unless there are 30 or fewer points. In that case, the default is 3 knots.

Knotted splines have the following properties in contrast to smoothing splines:

- Knotted splines work inside general models with many terms, whereas smoothing splines are for bivariate regressions.
- The regression basis is not a function of the response.
- Knotted splines are parsimonious, adding only $k - 2$ terms for curvature for k knot points.
- Knotted splines are conservative compared to pure polynomials in the sense that the extrapolation outside the range of the data is a straight line, rather than a polynomial.

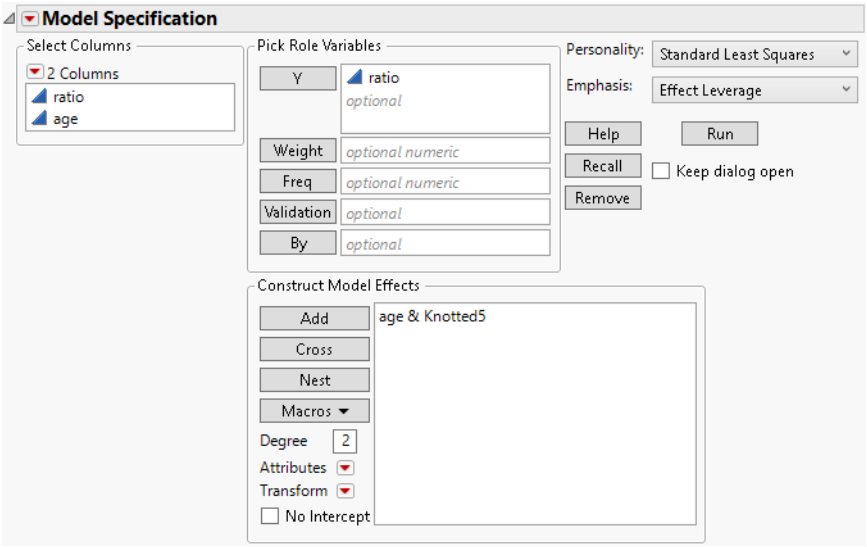
- There is an easy test for curvature.

Example Using the Knotted Spline Effect to Test for Curvature

To test for curvature, follow these steps:

1. Select **Help > Sample Data Folder** and open Growth.jmp.
2. Select **Analyze > Fit Model**.
3. Select the ratio column and click **Y**.
4. Select the age column and click **Add**.
5. Select age in the **Effects** pane and select **Attributes > Knotted Spline Effect**.
6. For the number of knots, type 5 and click **OK**.

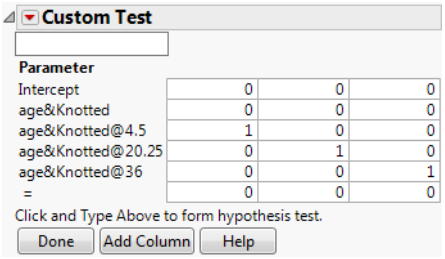
Figure 4.21 Fit Model Launch Window



7. Click **Run**.
8. Click the Response ratio red triangle and select **Estimates > Custom Test**.

   In the Custom Test report, notice that there is only one column. You want three columns.
9. Click the **Add Column** button twice to produce a total of three columns.
10. In the first column, type 1 for **age&Knotted@4.5**.
11. In the second column, type 1 for **age&Knotted@20.25**.
12. In the third column, type 1 for **age&Knotted@36**.

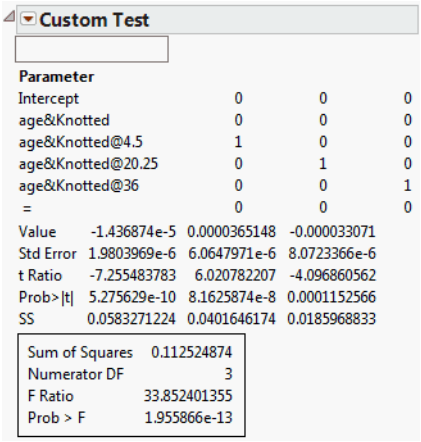
Figure 4.22 Values for the Custom Test for Curvature



13. Click **Done**.

    The small Prob > F value in the Custom Test report indicates that there is curvature.

Figure 4.23 Curvature Report

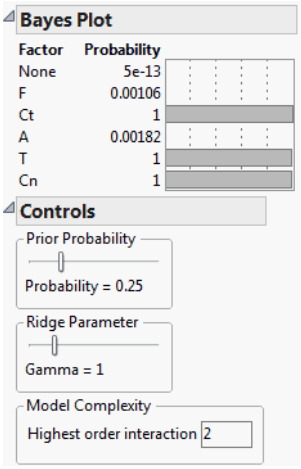


Example of a Bayes Plot for Active Factors

Use the Standard Least Squares personality of the Fit Model platform to determine which factors in an experimental design are active. This example illustrates a script that can be used to identify active factors using a Bayesian approach.

1. Select **Help > Sample Data Folder** and open Reactor.jmp.
2. In the Samples > Scripts folder, open the BayesPlotforFactors.jsl sample script.
3. Select **Edit > Run Script**.
4. Select Y and click **Y, Response**.
5. Select F, Ct, A, T, and Cn and click **X, Factor**.
6. Click **OK**.

Figure 4.24 Bayes Plot for Factor Activity



The Model Complexity indicates that the highest order interaction to consider is two. Therefore, all possible models that include up to second-order interactions are constructed. Based on the value assigned to Prior Probability, a posterior probability is computed for each of the possible models. The probability for a factor is the sum of the probabilities for each of the models where it was involved.

This approach identifies Ct, T, and Cn as active factors, and A and F as inactive.

If the ridge parameter were zero (not allowed), all the models would be fit by least squares. As the ridge parameter increases, the parameter estimates for any model shrink toward zero. For more information on the ridge parameter, and why it cannot be zero, see Box and Meyer (1993).

Chapter 5

Stepwise Regression Models

Find a Model Using Variable Selection

The Stepwise personality of the Fit Model platform enables you to fit stepwise regression models, explore all possible models for a set of regressors, and conduct model averaging.

Stepwise regression is an approach to selecting a subset of effects for a regression model. It can be useful in the following situations:

- There is little theory to guide the selection of terms for a model.
- You want to interactively explore which predictors seem to provide a good fit.
- You want to improve a model's prediction performance by reducing the variance caused by estimating unnecessary terms.

For categorical predictors, you can do the following:

- Choose from among various rules to determine how associated terms enter the model.
- Enforce effect heredity.

Contents

Overview of Stepwise Regression 245
Example Using Stepwise Regression 245
The Stepwise Report 247
Stepwise Platform Options 247
Stepwise Regression Control Panel 248
Current Estimates Report 255
Step History Report 256
Models with Crossed, Interaction, or Polynomial Terms 257
Models with Nominal and Ordinal Effects 257
Construction of Hierarchical Terms 258
Perform Binary and Ordinal Logistic Stepwise Regression 259
The All Possible Models Option 260
The Model Averaging Option 260
Validation Options in Stepwise Regression 261
Validation Set with Two or Three Values in Stepwise Regression 261
K-Fold Cross Validation in Stepwise Regression 264
Additional Examples of the Stepwise Personality 265
Example of the Combine Rule 266
Example of a Model with a Nominal Term 267
Example of the Restrict Rule for Hierarchical Terms 272
Example of Logistic Stepwise Regression 274
Example of the All Possible Models Option 275
Example of the Model Averaging Option 277

Overview of Stepwise Regression

You can perform stepwise regression in the Stepwise personality of the Fit Model platform. The Stepwise personality computes estimates that are the same as those of other least squares platforms, but it facilitates searching and selecting among many models.

The approach has side effects of which you need to be aware. The significance levels on the statistics for selected models violate the standard statistical assumptions because the model has been selected rather than tested within a fixed model. On the positive side, the approach has been helpful for 30 years in reducing the number of terms. The book *Subset Selection in Regression* (Miller 1990) brings statistical sense to model selection statistics.

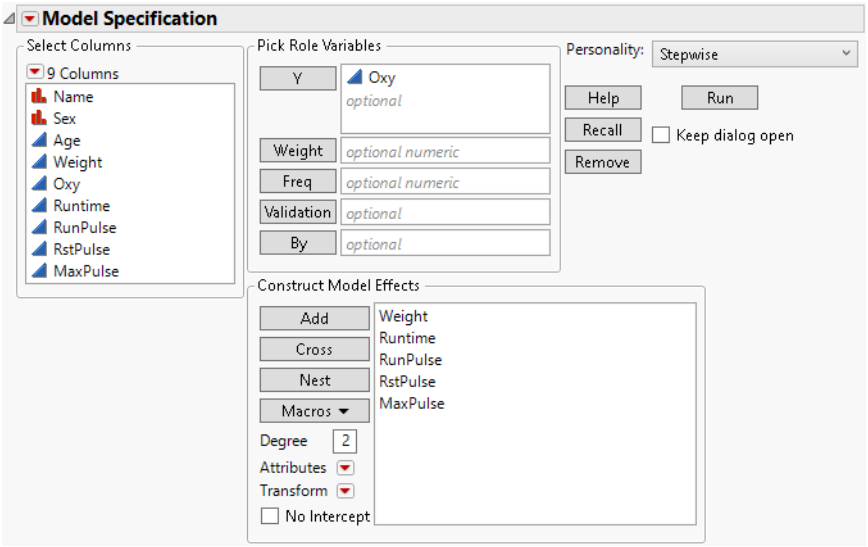
This chapter uses the term *significance probability* in a mechanical way to represent that the calculation would be valid in a fixed model, recognizing that the true significance probability could be nowhere near the reported one.

Example Using Stepwise Regression

In this example, you use the Stepwise personality of the Fit Model platform to select significant variables in a linear regression model modeling oxygen uptake. Aerobic fitness can be evaluated using a special test that measures the oxygen uptake of a person running on a treadmill for a prescribed distance. However, it would be more economical to find a formula that uses simpler measurements that evaluate fitness and predict oxygen uptake. To identify such an equation, measurements of age, weight, run time, and pulse were taken for 31 participants who ran 1.5 miles.

1. Select **Help > Sample Data Folder** and open Fitness.jmp.
2. Select **Analyze > Fit Model**.
3. Select Oxy and click **Y**.
4. Select Weight, Runtime, RunPulse, RstPulse, MaxPulse, and click **Add**.
5. For **Personality**, select **Stepwise**.

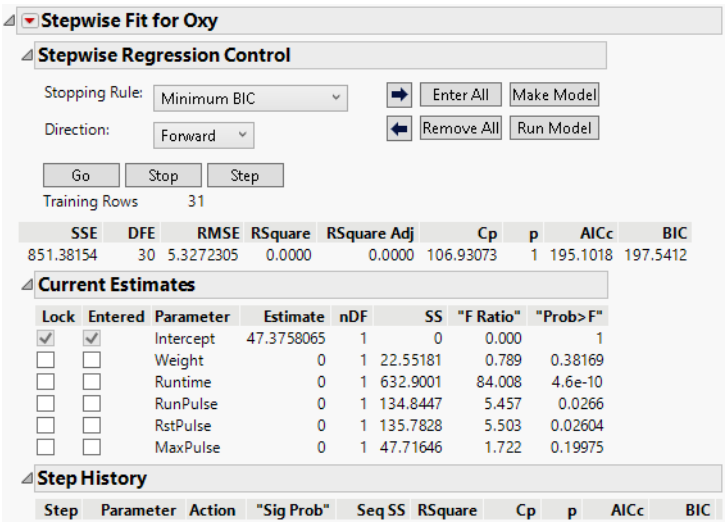
Figure 5.1 Completed Fit Model Launch Window



Validation is available only in JMP Pro.

6. Click **Run**.

Figure 5.2 Stepwise Report Window



To find a good oxygen uptake prediction equation, you need to compare many different regression models. Use the options in the Stepwise report window to search through models with combinations of effects and choose the model that you want.

The Stepwise Report

The Stepwise Fit report contains platform options, a control panel, and reports that contain the current estimates and the step history.

Platform options The Stepwise Fit red triangle menu contains options that affect all of the variables. See "Stepwise Platform Options".

Stepwise Regression Control Controls that limit regressor effect probabilities, determine the method of selecting effects, start or stop the selection process, and create a model. See "Stepwise Regression Control Panel".

Current Estimates A report that enables you to add, remove, and lock in model effects. See "Current Estimates Report".

Step History A report that records the effect of adding a term to the model. See "Step History Report".

Stepwise Platform Options

The Stepwise Fit red triangle menu contains the following options:

K-Fold Crossvalidation (Available only for continuous responses.) Performs K-Fold cross validation in the selection process. When selected, this option enables the Max K-Fold RSquare stopping rule ("Stepwise Regression Control Panel"). For more information about validation, see "Validation Options in Stepwise Regression".

All Possible Models (Available only for continuous responses.) Fits all possible models up to specified limits and shows the best models for each number of terms. Enter values for the maximum number of terms to fit in any one model. Also enter values for the maximum number of best model results to show for each number of terms in the model. Categorical variables are represented using indicator variables. See "Models with Nominal and Ordinal Effects". You can restrict the models that appear to those that satisfy strong effect heredity. See "The All Possible Models Option".

Model Averaging (Available only for continuous responses.) Enables you to average the fits for a number of models, instead of selecting a single best model. See "The Model Averaging Option".

Plot Criterion History Creates a plot of AICc and BIC versus the number of parameters. The Criterion History plot contains two shaded zones. Define the minimum AICc value as V^{best}. The green zone is defined by the range [V^{best}, V^{best}+4]. The yellow zone is defined by the range (V^{best}+4, V^{best}+10].

Plot RSquare History (Available only for continuous responses.) Creates a plot of training and validation R-square versus the number of parameters.

Clear History Clears and resets the step history.

JMP PRO **Export Model with Validation** (Available only when you have entered a Validation column in the Stepwise launch window.) Adds the Validation column to the Model Specification window when you select Make Model. Runs the model with the Validation column when you select Run Model.

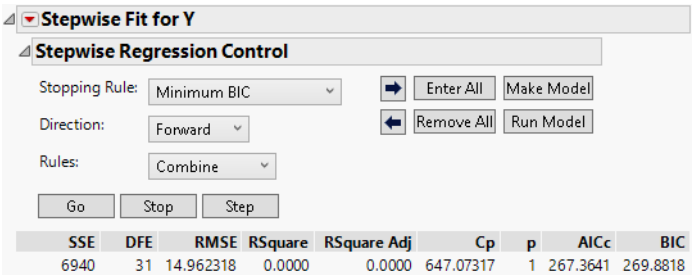
This option is selected by default.

Model Dialog Shows the completed Fit Model launch window for the current model.

Stepwise Regression Control Panel

Use the Stepwise Regression Control panel to limit regressor effect probabilities, determine the method of selecting effects, begin or stop the selection process, and run a model. A note appears beneath the Go button to indicate whether you have excluded or missing rows.

Figure 5.3 Stepwise Regression Control Panel



Stopping Rule

The Stopping Rule determines which model is selected. For all stopping rules other than P-value Threshold, only the Forward and Backward directions are allowed. The only stopping rules that use validation are Max Validation RSquare and Max K-Fold RSquare. See "Validation Options in Stepwise Regression".

P-value Threshold Uses *p*-values (significance levels) to enter and remove effects from the model. Two other options appear when you choose P-value Threshold:

Prob to Enter Specifies the maximum *p*-value that an effect must have to be entered into the model during a forward step.

Prob to Leave Specifies the minimum *p*-value that an effect must have to be removed from the model during a backward step.

Minimum AICc Uses the minimum corrected Akaike Information Criterion to choose the best model. For more details, see "Likelihood, AICc, and BIC".

Minimum BIC Uses the minimum Bayesian Information Criterion to choose the best model. For more details, see "Likelihood, AICc, and BIC".

JMP PRO **Max Validation RSquare** Uses the maximum R-square from the validation set to choose the best model. This is available only when you use a validation column with two or three distinct values. For more information about validation, see "Validation Set with Two or Three Values in Stepwise Regression".

Max K-Fold RSquare Uses the maximum RSquare from K-fold cross validation to choose the best model. You can access the Max K-Fold RSquare stopping rule by selecting this option from the Stepwise red triangle menu. JMP Pro users can access the option by using a validation set with four or more values. When you select this option, you are asked to specify the number of folds. For more information about validation, see "K-Fold Cross Validation in Stepwise Regression".

Direction

The Direction you choose controls how effects enter and leave the model. Select one of the following options:

Forward Enters the term with the smallest *p*-value. If the P-value Threshold stopping rule is selected, that term must be significant at the level specified by **Prob to Enter**. See "Forward Selection Example".

Backward Removes the term with the largest *p*-value. If the P-value Threshold stopping rule is selected, that term must not be significant at the level specified in **Prob to Leave**. See "Backward Selection Example".

Note: When Backward is selected as the Direction, you must click Enter All before clicking Go or Step.

Mixed Available only when the P-value Stopping Rule is selected. It alternates the forward and backward steps. It includes the most significant term that satisfies **Prob to Enter** and removes the least significant term satisfying **Prob to Leave**.

Go, Stop, Step Buttons

The Go, Stop, and Step buttons enable you to control how terms are entered or removed from the model.

Note: All Stopping Rules consider only models defined by *p*-value entry (Forward direction) or removal (Backward direction). Stopping rules do not consider all possible models.

Go Automates the process of entering (Forward direction) or removing (Backward direction) terms. Among the fitted models, the model that is considered best based on the selected Stopping Rule is listed last. Except for the P-value Threshold stopping rule, the model selected as Best is one that overlooks local dips in the behavior of the stopping rule statistic. The button to the right the Best model selects it for the Make Model and Run Model options, but you are free to change this selection.

- For P-value Threshold, the best model is based on the Prob to Enter and Prob to Leave criteria. See "P-value Threshold".
- For Min AICc and Min BIC, the automatic fits continue until a Best model is found. The Best model is one with a minimum AICc or BIC that can be followed by as many as ten models with larger values of AICc or BIC, respectively. This model is designated by the terms Best in the Parameter column and Specific in the Action column.
- For Max Validation RSquare (JMP Pro only) and Max K-Fold RSquare, the automatic fits continue until a Best model is found. The Best model is one with an RSquare Validation or RSquare K-Fold value that can be followed by as many as ten models with smaller values of RSquare Validation or RSquare K-Fold, respectively. This model is designated by the terms Best in the Parameter column and Specific in the Action column.

Stop Stops the automatic selection process started by the Go button.

Step Enters terms one-by-one in the Forward direction or removes them one-by one in the Backward direction. At any point, you can select a model by clicking its button on the right in the Step History report. The selection of model terms is updated in the Current Estimates report. This is the model that is used once you click Make Model or Run Model.

Rules

Note: Appears only if your model contains related terms. When you have a nominal or ordinal variable, related terms are constructed and appear in the Current Estimates table.

Use Rules to change the rules that are applied when there is a hierarchy of terms in the model. A hierarchy can occur in the following ways:

- A hierarchy results when a variable is a component of another variable. For example, if your model contains variables A, B, and A*B, then A and B are *precedent* terms to A*B in the hierarchy.
- A hierarchy also results when you include nominal or ordinal variables. A term that is above another term in the tree structure is a *precedent* term. See "Construction of Hierarchical Terms".

Select one of the following options:

Combine Calculates *p*-values for two separate tests when considering entry for a term that has precedents. The first *p*-value, p_1, is calculated by grouping the term with its precedent terms and testing the group's significance probability for entry as a joint *F* test. The second *p*-value, p_2, is the result of testing the term's significance probability for entry after the precedent terms have already entered into the model. The final significance probability for entry for the term that has precedents is max(p_1, p_2).

Tip: The Combine rule avoids including non-significant interaction terms, whose precedent terms can have particularly strong effects. In this scenario, the strong main effects might make the group's significance probability for entry, p_1, very small. However, the second test finds that the interaction by itself is not significant. As a result, p_2 is large and is used as the final significance probability for entry.

Caution: The degrees of freedom value for a term that has precedents depends on which of the two significance probabilities for entry is larger. The test used for the final significance probability for entry determines the degrees of freedom, nDF, in the Current Estimates table. Therefore, if p_1 is used, nDF equals the number of terms in the group for the joint test, and if p_2 is used, nDF equals 1.

The Combine option is the default rule. See "Models with Crossed, Interaction, or Polynomial Terms".

Restrict Restricts the terms that have precedents so that they cannot be entered until their precedents are entered. See "Models with Nominal and Ordinal Effects" and "Example of the Restrict Rule for Hierarchical Terms".

No Rules Gives the selection routine complete freedom to choose terms, regardless of whether the routine breaks a hierarchy or not.

Whole Effects Enters only whole effects, when terms involving that effect are significant. This rule applies only when categorical variables with more than two levels are entered as possible model effects. See "Rules".

Buttons

The Stepwise Control Panel contains the following buttons:

Go Automates the selection process to completion.

Stop Stops the selection process.

Step Increments the selection process one step at a time.

Arrow buttons  Step forward and backward one step in the selection process.

Enter All Enters all unlocked terms into the model.

Remove All Removes all unlocked terms from the model.

Make Model Creates a model for the Fit Model window from the model currently showing in the Current Estimates table. In cases where there are nominal or ordinal terms, **Make Model** creates temporary transform columns that contain terms that are needed for the model.

Run Model Runs the model currently showing in the Current Estimates table. In cases where there are nominal or ordinal terms, **Run Model** creates temporary transform columns that contain terms that are needed for the model.

Statistics

The following statistics appear below the Stepwise Regression Control panel.

SSE Sum of squared errors for the current model.

DFE Error degrees of freedom for the current model.

RMSE Root mean square error (residual) for the current model.

RSquare Proportion of the variation in the response that can be attributed to terms in the model rather than to random error.

RSquare Adj Adjusts R^2 to make it more comparable over models with different numbers of parameters by using the degrees of freedom in its computation. The adjusted R^2 is useful in stepwise procedure because you are looking at many different models and want to adjust for the number of terms in the model.

Cp Mallow's C_p criterion for selecting a model. It is an alternative measure of total squared error and can be defined as follows:

$$C_p = \left(\frac{\text{SSE}_p}{s^2}\right) - (N - 2p)$$

where s^2 is the MSE for the full model and SSE_p is the sum-of-squares error for a model with p variables, including the intercept. Note that p is the number of x-variables+1. If C_p is graphed with p, Mallows (1973) recommends choosing the model where C_p first approaches p.

p Number of parameters in the model, including the intercept.

AICc Corrected Akaike's Information Criterion. For more details, see "Likelihood, AICc, and BIC".

BIC Bayesian Information Criterion. For more details, see "Likelihood, AICc, and BIC".

Forward Selection Example

In forward selection, terms are entered into the model and most significant terms are added until all of the terms are significant.

1. Complete the steps in "Example Using Stepwise Regression".

   Notice that the default selection for **Direction** is Forward.

2. Click **Step**.

   In Figure 5.4, you can see that after one step, the most significant term, Runtime, is entered into the model.

3. Click **Go**.

   In Figure 5.5 you can see that all of the terms have been added, except RstPulse and Weight.

Figure 5.4 Current Estimates Table for Forward Selection after One Step

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	82.4217727	1	0	0.000	1
☐	☐	Weight	0	1	1.323628	0.171	0.68267
☐	☑	Runtime	-3.3105554	1	632.9001	84.008	4.6e-10
☐	☐	RunPulse	0	1	15.36208	2.118	0.15673
☐	☐	RstPulse	0	1	0.130138	0.017	0.89814
☐	☐	MaxPulse	0	1	1.567361	0.202	0.65632

Figure 5.5 Current Estimates Table for Forward Selection after Three Steps

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	80.9007896	1	0	0.000	1
☐	☐	Weight	0	1	4.989591	0.827	0.37137
☐	☑	Runtime	-2.9701867	1	443.2028	73.971	3.25e-9
☐	☑	RunPulse	-0.3751142	1	55.14175	9.203	0.00529
☐	☐	RstPulse	0	1	0.350744	0.056	0.81399
☐	☑	MaxPulse	0.35421891	1	41.34703	6.901	0.01403

Backward Selection Example

In backward selection, all terms are entered into the model and then the least significant terms are removed until all of the remaining terms are significant.

1. Complete the steps in "Example Using Stepwise Regression".
2. Click **Enter All**.

Figure 5.6 All Effects Entered into the Model

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	82.3936054	1	0	0.000	1
☐	☑	Weight	-0.0509071	1	4.83788	0.772	0.38784
☐	☑	Runtime	-2.9518165	1	366.3375	58.489	5.29e-8
☐	☑	RunPulse	-0.3970425	1	59.51519	9.502	0.00495
☐	☑	RstPulse	0.01239004	1	0.199033	0.032	0.85995
☐	☑	MaxPulse	0.38479281	1	45.83023	7.317	0.01212

3. For **Direction**, select Backward.
4. Click **Step** two times.

   The first backward step removes RstPulse and the second backward step removes Weight.

Figure 5.7 Current Estimates with Terms Removed and Step History Table

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	80.9007896	1	0	0.000	1
☐	☐	Weight	0	1	4.989591	0.827	0.37137
☐	☑	Runtime	-2.9701867	1	443.2028	73.971	3.25e-9
☐	☑	RunPulse	-0.3751142	1	55.14175	9.203	0.00529
☐	☐	RstPulse	0	1	0.350744	0.056	0.81399
☐	☑	MaxPulse	0.35421891	1	41.34703	6.901	0.01403

Step History

Step	Parameter	Action	"Sig Prob"	Seq SS	RSquare	Cp	p	AICc	BIC	
1	All	Entered	.	.	0.8161	6	6	157.051	162.22	○
2	RstPulse	Removed	0.8600	0.199033	0.8158	4.0318	5	153.721	158.825	○
3	Weight	Removed	0.3714	4.989591	0.8100	2.8284	4	151.592	156.362	◉

The Current Estimates and Step History tables shown in Figure 5.7 summarize the backward stepwise selection process. Note the BIC value of 156.362 for the third step in the Step History table. If you click Step again to remove another parameter from the model, the BIC value increases to 159.984. For this reason, you choose the step 3 model. This is also the model that the Go button produces.

Current Estimates Report

In the Stepwise Fit report, use the Current Estimates section to enter, remove, and lock in model effects. (The intercept is permanently locked into the model.) This report contains the following columns:

Figure 5.8 Current Estimates Table

SSE	DFE	RMSE	RSquare	RSquare Adj	Cp	p	AICc	BIC
851.38154	30	5.3272305	0.0000	0.0000	106.93073	1	195.1018	197.5412

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	47.3758065	1	0	0.000	1
☐	☐	Weight	0	1	22.55181	0.789	0.38169
☐	☐	Runtime	0	1	632.9001	84.008	4.6e-10
☐	☐	RunPulse	0	1	134.8447	5.457	0.0266
☐	☐	RstPulse	0	1	135.7828	5.503	0.02604
☐	☐	MaxPulse	0	1	47.71646	1.722	0.19975

Lock Locks a term in or out of the model. A checked term cannot be entered or removed from the model.

Entered Indicates whether a term is currently in the model. You can click a term's check box to manually bring an effect in or out of the model.

Parameter The effect names.

Estimate The current parameter estimate, which is zero if the effect is not currently in the model.

nDF The number of degrees of freedom for a term. A term has more than one degree of freedom if its entry into a model also forces other terms into the model.

Wald/Score ChiSq (Shown only when response is categorical.) The test statistic for each term in the model. For terms not already included in the model, the test statistic is based on a score test of including the term in the model. For terms that are already included in the model, the test statistic is based on a Wald test of removing the term from the model.

"Sig Prob" (Shown only when response is categorical.) The *p*-value associated with the Wald/Score ChiSq test statistic based on nDF degrees of freedom. The "Sig Prob" is used to determine the next term to be included in the model.

SS (Shown only when response is continuous.) The reduction in the error (residual) sum of squares (SS) if the term is entered into the model or the increase in the error SS if the term is removed from the model. If a term is restricted in some fashion, it could have a reported SS of zero.

"F Ratio" (Shown only when response is continuous.) The traditional test statistic to test that the term effect is zero. It is the square of a *t*-ratio. It is in quotation marks because it does not have an *F*-distribution for testing the term because the model was selected as it was fit.

"Prob>F" (Shown only when response is continuous.) The *p*-value associated with the *F* statistic. Like the "F Ratio," it is in quotation marks because it is not to be trusted as a real significance probability.

R The multiple correlation with the other effects in the model. This column appears only if you right-click in the report and select Columns > R.

Step History Report

As each step is taken, the Step History section of the Stepwise Fit report records the effect of adding a term to the model. The Step History report in Figure 5.9 shows the order in which the terms entered the model and shows the statistics for each model in "Example Using Stepwise Regression".

Use the radio buttons on the right to choose a model.

Figure 5.9 Step History Report

Step History

Step	Parameter	Action	"Sig Prob"	Seq SS	RSquare	Cp	p	AICc	BIC	
1	Runtime	Entered	0.0000	632.9001	0.7434	7.8825	2	155.397	158.81	○
2	RunPulse	Entered	0.1567	15.36208	0.7614	7.4298	3	155.787	159.984	○
3	MaxPulse	Entered	0.0140	41.34703	0.8100	2.8284	4	151.592	156.362	◉

Step History for Categorical Responses

The Step History report for models with a categorical response contains four additional columns: L-R ChiSquare, "Sig Prob", Entry ChiSquare, and Entry "Sig Prob".

The L-R ChiSquare and "Sig Prob" columns contain the full-versus-reduced likelihood ratio test statistic and *p*-value. Here, the full model is the one that contains the specified term and the reduced model does not contain the specified term.

The Entry ChiSquare and Entry "Sig Prob" columns contain the Wald/Score ChiSquare and "Sig Prob" values that were used to choose the most recent term to include in the model.

Figure 5.10 Step History for a Categorical Response before First Step

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	Wald/Score ChiSq	"Sig Prob"
☑	☑	Intercept[M]	-0.0645385	1	0	1
☐	☐	Oxy	0	1	7.275453	0.00699
☐	☐	Weight	0	1	10.13425	0.00146
☐	☐	Runtime	0	1	5.764797	0.01635
☐	☐	RunPulse	0	1	1.865024	0.17205
☐	☐	RstPulse	0	1	2.693096	0.10078
☐	☐	MaxPulse	0	1	1.467541	0.22573

Step History

Step	Parameter	Action	L-R ChiSquare	"Sig Prob"	Entry ChiSquare	Entry "Sig Prob"	RSquare	p	AICc	BIC

Figure 5.11 Step History for a Categorical Response after First Step

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	Wald/Score ChiSq	"Sig Prob"
☑	☑	Intercept[M]	-16.228294	1	0	1
☐	☐	Oxy	0	1	8.750599	0.0031
☐	☑	Weight	0.20792272	1	6.994297	0.00818
☐	☐	Runtime	0	1	6.371395	0.0116
☐	☐	RunPulse	0	1	1.428737	0.23197
☐	☐	RstPulse	0	1	3.510878	0.06097
☐	☐	MaxPulse	0	1	0.365726	0.54534

Step History

Step	Parameter	Action	L-R ChiSquare	"Sig Prob"	Entry ChiSquare	Entry "Sig Prob"	RSquare	p	AICc	BIC
1	Weight	Entered	12.16669	0.0005	10.1342	0.00146	0.2833	2	35.2047	37.6441

Note that the Entry ChiSquare and Entry "Sig Prob" values for Weight in the Step History table in Figure 5.11 match the Wald/Score ChiSq and "Sig Prob" values for Weight in Figure 5.10.

Models with Crossed, Interaction, or Polynomial Terms

Some stepwise regression models, especially those associated with experimental designs, involve interaction terms. For continuous factors, these are products of the columns representing the effects. For nominal and ordinal factors, interactions are defined by model terms that involve products of terms representing the categorical levels.

When there are interaction terms, you often want to impose a restriction on the model selection process so that lower-order components of higher-order effects are included in the model. This is suggested by the principle of Effect Heredity. See the *Design of Experiments Guide*. For example, if a two-way interaction is included in a model, its component main effects (*precedents*) should be included as well.

See "Example of the Combine Rule".

Models with Nominal and Ordinal Effects

Traditionally, stepwise regression has not addressed the situation where there are categorical effects in the model. Note the following:

- When a regression model contains nominal or ordinal effects, those effects are represented by sets of indicator columns.
- When a categorical effect has only two levels, that effect is represented by a single column.

- When a categorical effect has *k* levels, where *k* > 2, then it must be represented by k-1 columns.

The convention in JMP for standard platforms is to represent nominal variables by terms whose parameter estimates average to zero across all the levels.

In the Stepwise platform, categorical variables (nominal and ordinal) are coded in a *hierarchical* fashion. This differs from coding in other least squares fitting platforms. In hierarchical coding, the levels of the categorical variable are successively split into groups of levels that most separate the means of the response. The splitting process achieves the goal of representing a *k*-level categorical variable by *k* - 1 terms.

Note: In hierarchical coding, the initial terms that are constructed represent the groups responsible for the greatest separation. The advantage of this coding scheme is that these informative terms have the potential to enter the model early.

Construction of Hierarchical Terms

In the Stepwise personality of the Fit Model platform, hierarchical terms are constructed using a tree structure that is analogous to a Partition analysis. However, the criterion that is maximized is the sum of squares between groups (SSB).

For a nominal variable with *k* levels, the *k* levels are split into two groups of levels that have maximum SSB. Call these two groups of levels A1 and A2, where A1 has the smaller mean and A2 has the larger mean. The two groups of levels in A1 and A2 are used to define an indicator variable with values of 1 for the levels in A1 and -1 for the levels in A2. This variable is the first hierarchical term for the nominal variable.

For the levels within each of the initial two groups A1 and A2, the split into two groups of levels with the maximum SSB is identified. Suppose that the groups of levels with maximum SSB are among the levels in A1. Call the two groups B1 and B2, where B1 has the smaller mean and B2 has the larger mean. The two groups of levels in B1 and B2 are used to define a hierarchical variable with values of 1 for the levels in B1, -1 for the levels in B2, and 0 for the levels in A2. To construct the next variable, splits of the levels in B1, B2, and A2 are considered. The split that maximizes SSB defines the next hierarchical variable. The process continues until *k-1* hierarchical terms are constructed.

For an ordinal variable, the groups of levels considered in splitting contain only levels that are contiguous in the ordering. This ensures that the constructed terms respect the level ordering.

Rules and Hierarchical Terms

When you use the **Combine** rule or the **Restrict** rule, a term cannot enter the model unless all the terms above it in the hierarchy have been entered. When you use the **Whole Effects** rule and enter a term for a categorical variable, all of its associated terms are entered. For an example, see "Construction of Hierarchical Terms in Example".

Perform Binary and Ordinal Logistic Stepwise Regression

The Stepwise personality of Fit Model performs ordinal logistic stepwise regression when the response is ordinal or nominal. Nominal responses are treated as ordinal responses in the logistic stepwise regression fitting procedure. When a response has only two levels, ordinal logistic regression models are equivalent to nominal logistic regression models. To run a logistic stepwise regression, specify an ordinal or nominal response, add terms to the model as usual, and choose Stepwise from the Personality menu.

The Stepwise reports for a logistic model are similar to those provided when the response is continuous. The following elements are specific to logistic regression results:

- When the response is categorical, the overall fit of the model is given by its negative log-likelihood (-LogLikelihood). This value is calculated based on the full iterative maximum likelihood fit. See "Likelihood, AICc, and BIC".
- The Current Estimates report shows chi-square statistics (Wald/Score ChiSq) and their *p*-values (Sig Prob). The test statistic column shows score statistics for parameters that are not in the current model and shows Wald statistics for parameters that are in the current model. The regression estimates (Estimate) are based on the full iterative maximum likelihood fit.
- The Step History report shows the L-R ChiSquare. This value is the test statistic for the likelihood ratio test of the hypothesis that the corresponding regression parameter is zero, given the other terms in the model. The Sig Prob is the *p*-value for this test.

Note: If the response is nominal, you can fit the current model using the Nominal Logistic personality of Fit Model by clicking the Make Model button. In the Fit Model launch window that appears, click Run.

See "Example of Logistic Stepwise Regression".

The All Possible Models Option

In the Stepwise personality of the Fit Model platform, use the All Possible Models option to investigate all models that can be constructed using your predictors. This option is accessed from the Stepwise red triangle menu.

Note the following:

- This option is not practical for large problems, when the number of models is greater than 5 million.
- Categorical predictors are represented by indicator variables. See "Models with Nominal and Ordinal Effects".

The following options restrict the number of models that appear:

Maximum number of terms in a model Enter a value for the maximum number of terms in a model.

Number of best models to see Enter the maximum number of models of each size to display. The best models according to RSquare value appear.

Restrict to models where interactions imply lower order effects (Heredity Restriction)

Shows only models that contain all lower-order effects when a higher-order effect is included. These models satisfy strong effect heredity. This option is useful when your predictors include interaction or polynomial terms.

See "Example of the All Possible Models Option".

The Model Averaging Option

In the Stepwise personality of the Fit Model platform, the model averaging technique enables you to average the fits for several models instead of selecting a single model. Often, the resulting average model has better prediction capability than each of the single models that were averaged. The Model Averaging feature is useful for avoiding a model that over fits your data. When many terms are selected into a model, the fit tends to inflate the parameter estimates. Model averaging tends to shrink the estimates on the weaker terms, which yields better predictions. The models are averaged with respect to the AICc Weight of each model, which is calculated as follows:

$$\text{AICcWeight} = \exp[-0.5(\text{AICc} - \text{AICcBest})]$$

AICcBest is the smallest AICc value among the fitted models. The AICc Weight values are calculated for each model, sorted in decreasing order, and scaled to sum to 1. The scaled AICc Weight values that sum to less than one minus the specified Cumulative AICc Weight Cutoff value are set to zero. This eliminates the use of weak models in the averaged model. The parameters for the averaged model are the weighted averages of the parameter estimates across the models that have nonzero AICcWeights.

See "Example of the Model Averaging Option".

Validation Options in Stepwise Regression

To perform cross validation for stepwise regression models in JMP, click the Stepwise Fit red triangle and select **K-Fold Crossvalidation**. See "K-Fold Cross Validation in Stepwise Regression".

JMP PRO In JMP Pro, you can specify a Validation column in the Fit Model window. A validation column must have a numeric data type and should contain at least two distinct values.

- If the column contains two values, the smaller value defines the training set and the larger value defines the validation set.
- If the column contains three values, the values define the training, validation, and test sets in order of increasing size.
- If the column contains four or more distinct values and the response is continuous, these values define folds for k-fold validation.

For more information about using a Validation column, see "Validation Set with Two or Three Values in Stepwise Regression".

Validation Set with Two or Three Values in Stepwise Regression

JMP PRO If you specify a Validation column with two or three values, the Stepwise personality fits models based on the training set. Model fit statistics are reported for the validation and test sets. See "Validation and Test Set Statistic Definitions" for details about how these statistics are defined.

If the response is continuous, the following statistics appear in the Stepwise Regression Control panel:

- RSquare Validation (also shown in the Step History report)
- RASE Validation
- RSquare Test (if there is a test set)
- RASE Test (if there is a test set)

If the response is binary nominal or ordinal, the following statistics appear in the Stepwise Regression Control panel:

- RSquare Validation (also shown in the Step History report)
- Avg Log Error Validation
- RSquare Test (if there is a test set)
- Avg Log Error Test (if there is a test set)

Max Validation RSquare

If you specify a validation column with two or three values in the Fit Model window, the Stopping Rule defaults to Max Validation RSquare. This rule attempts to find a model that maximizes the RSquare statistic for the validation set. The rule can be applied with the Direction set to Forward or Backward.

Note: Max Validation RSquare considers only the models defined by *p*-value entry (Forward direction) or removal (Backward direction). It does not consider all possible models.

You can use the Step button to enter terms one-by-one in the Forward direction or to remove them one-by one in the Backward direction. At any point, you can select a model by clicking the button to the right of RSquare Validation in the Step History report. The selection of model terms is updated in the Current Estimates report. This is the model that is used once you click Make Model or Run Model.

Forward Direction

In the Forward direction, Stepwise constructs successive models by adding terms based on the next smallest *p*-value.

If you click Go rather than Step, the process of entering terms proceeds automatically. Among the fitted models, the model that is considered best is listed last. This model is obtained by overlooking local dips in RSquare Validation. Specifically, it is the model with the largest RSquare Validation that can be followed by as many as ten models with lower RSquare Validation values. This model is designated by the terms Best in the Parameter column and Specific in the Action column. The button to the right of RSquare Validation selects this Best model, though you are free to change this selection.

Backward Direction

In the Backward direction, Stepwise constructs successive models by removing terms based on the next largest *p*-value.

To use the Backward direction, you must first click Enter All to enter all of the terms into the model. The Backward direction behaves in a similar fashion to the Forward direction. If you click Go rather than Step, the process of removing terms proceeds automatically. The model designated as Best is the one with the largest RSquare Validation that can be followed by as many as ten models with lower RSquare Validation values.

Validation and Test Set Statistic Definitions

RSquare Validation and RASE Validation are defined in this section. RSquare Test and RASE Test are computed for the test set in a completely analogous fashion.

Continuous Response

RSquare Validation An RSquare measure for the validation set computed as follows:

- For each observation in the validation set, compute the prediction error. This is the difference between the actual response and the response predicted by the training set model.
- Square and sum the prediction errors to obtain $SSE_{Validation}$.
- Square and sum the differences between the actual responses in the validation set and their mean. This is the $SST_{Validation}$.
- RSquare Validation is:

$$\text{RSquare Validation} = 1 - \frac{SSE_{Validation}}{SST_{Validation}}$$

Note: It is possible for RSquare Validation to be negative.

RASE Validation The square root of the mean squared prediction error for the validation set. This is computed as follows:

- For each observation in the validation set, compute the prediction error. This is the difference between the actual response and the response predicted by the training set model.
- Square and sum the prediction errors to obtain the $SSE_{Validation}$.
- Denote the number of observations in the validation set by $n_{Validation}$.
- RASE Validation is:

$$\text{RASE Validation} = \sqrt{\frac{SSE_{Validation}}{n_{Validation}}}$$

Binary Nominal or Ordinal Response

RSquare Validation An Entropy RSquare measure (also known as McFadden's R^2) for the validation set computed as follows:

- A model is fit using the training set.
- Predicted probabilities are obtained for all observations.
- Using the predicted probabilities based on the training set model, the likelihood for the model is computed for observations in the validation set. Call this quantity $Likelihood_Full_{Validation}$.
- Using the data in the validation set, the likelihood of the reduced model (no predictors) is computed. Call this quantity $Likelihood_Reduced_{Validation}$.
- RSquare Validation is:

$$\text{RSquare Validation} = 1 - \frac{\log(Likelihood_Full_{Validation})}{\log(Likelihood_Reduced_{Validation})}$$

Note: It is possible for RSquare Validation to be negative.

Avg Log Error Validation The average log error for the validation set is computed as follows:

- For each observation in the validation set, compute the log of its predicted probability as determined by the model based on the training set.
- Sum these logs, divide by the number of observations in the validation set, and take the negative of the resulting value.

Tip: Smaller values of Avg Log Error Validation are desirable.

K-Fold Cross Validation in Stepwise Regression

K-fold cross validation randomly divides the data into k subsets. In turn, each of the k sets is used as a validation set while the remaining data are used as a training set to fit the model. In total, k models are fit and k validation statistics are obtained. The model giving the best validation statistic is chosen as the final model. This method is useful for small data sets, because it makes efficient use of limited amounts of data.

Note: K-fold cross validation is available only for continuous responses.

In JMP, click the Stepwise Fit red triangle and select **K-Fold Crossvalidation**.

In JMP Pro, you can access *k*-fold cross validation in two ways:

- Click the Stepwise Fit red triangle and select **K-Fold Crossvalidation**.
- Specify a validation column with four or more distinct values.

RSquare K-Fold Statistic

If you conduct k-fold cross validation, the RSquare K-Fold statistic appears to the right of the other statistics in the Stepwise Regression Control panel. RSquare K-Fold is calculated as follows:

1 - Sum(SSE)/Sum(SST)

where:

SSE represents a vector of the error sum of squares in each of the *k* folds

SST represents a vector of the total sum of squares in each of the *k* folds

Max K-Fold RSquare

When you use *k*-fold cross validation, the Stopping Rule defaults to Max K-Fold RSquare. This rule attempts to maximize the RSquare K-Fold statistic.

Note: Max K-Fold RSquare considers only the models defined by *p*-value entry (Forward direction) or removal (Backward direction). It does not consider all possible models.

The Max K-Fold RSquare stopping rule behaves in a fashion similar to the Max Validation RSquare stopping rule. See "Max Validation RSquare". Replace references to RSquare Validation with RSquare K-Fold.

Additional Examples of the Stepwise Personality

This section contains examples using the Stepwise personality of the Fit Model platform.

- "Example of the Combine Rule"
- "Example of a Model with a Nominal Term"
- "Example of the Restrict Rule for Hierarchical Terms"
- "Example of Logistic Stepwise Regression"
- "Example of the All Possible Models Option"
- "Example of the Model Averaging Option"

Example of the Combine Rule

This example illustrates the Combine rule in the Stepwise personality of the Fit Model platform.

1. Select **Help > Sample Data Folder** and open Reactor.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. In the Degree box, type 2.
5. Select F, Ct, A, T, and Cn and click **Macros > Factorial to Degree**.
6. For **Personality**, select **Stepwise**.
7. Click **Run**.

Figure 5.12 Initial Current Estimates Report Using Combine Rule

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	65.5	1	0	0.000	1
☐	☐	F	0	1	15.125	0.066	0.79972
☐	☐	Ct	0	1	3042	23.412	3.67e-5
☐	☐	A	0	1	3.125	0.014	0.90823
☐	☐	T	0	1	924.5	4.611	0.03998
☐	☐	Cn	0	1	312.5	1.415	0.24363
☐	☐	F*Ct	0	1	15.125	0.066	0.79972
☐	☐	F*A	0	3	22.75	0.031	0.9926
☐	☐	F*T	0	1	6.125	0.027	0.87178
☐	☐	F*Cn	0	1	0.125	0.001	0.98161
☐	☐	Ct*A	0	1	6.125	0.027	0.87178
☐	☐	Ct*T	0	1	1404.5	7.612	0.00979
☐	☐	Ct*Cn	0	1	32	0.139	0.71193
☐	☐	A*T	0	1	36.125	0.157	0.69476
☐	☐	A*Cn	0	1	6.125	0.027	0.87178
☐	☐	T*Cn	0	1	968	4.863	0.03525

The model in Figure 5.12 contains all terms for up to two-factor interactions for the five continuous factors. The Combine, Restrict, and Whole Effects rules described in "Rules" enable you to control entry of interaction terms.

The Combine rule determines the entry of interaction terms based on two tests. See "Combine". You can determine which of the two tests was used for the *p*-value based on the degrees of freedom, nDF. For example, the interaction term F*A has an nDF value of 3. This means that F*A is grouped with its precedent terms F and A, and is considered for entry based on the 3 degree of freedom joint *F* test. In contrast, the interaction term Ct*T has an nDF value of 1. This means that Ct*T is considered for entry based on the 1 degree of freedom *F* test that tests the significance of Ct*T after its precedent terms, Ct and T, are already included in the model. Click **Step** once to see that Ct is entered by itself.

8. Click **Step** again to see that Ct*T is entered, along with T (Ct is already in the model).

Figure 5.13 Current Estimates Report Using Combine Rule, One Step

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	65.5	1	0	0.000	1
☐	☐	F	0	1	15.125	0.263	0.61236
☐	☑	Ct	9.75	2	4446.5	39.676	6.74e-9
☐	☐	A	0	1	3.125	0.054	0.81819
☐	☑	T	5.375	2	2329	20.781	2.93e-6
☐	☐	Cn	0	1	312.5	6.715	0.01524
☐	☐	F*Ct	0	2	30.25	0.256	0.7764
☐	☐	F*A	0	3	22.75	0.123	0.9459
☐	☐	F*T	0	2	21.25	0.178	0.83755
☐	☐	F*Cn	0	1	0.125	0.002	0.96335
☐	☐	Ct*A	0	2	9.25	0.077	0.92601
☐	☑	Ct*T	6.625	1	1404.5	25.064	2.72e-5
☐	☐	Ct*Cn	0	1	32	0.562	0.45989
☐	☐	A*T	0	2	39.25	0.334	0.7194
☐	☐	A*Cn	0	1	6.125	0.106	0.74747
☐	☐	T*Cn	0	1	968	43.488	4.49e-7

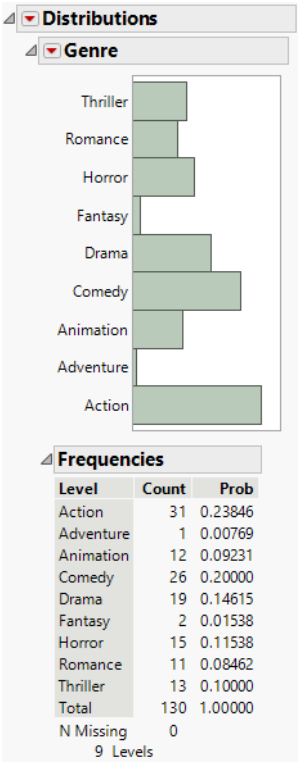
When there are significant interaction terms, several terms can enter at the same step. If the **Step** button is clicked twice, Ct*T is entered along with its two contained effects Ct and T. However, a step back is not symmetric because a crossed term can be removed without removing its two component terms. Notice that Ct and T now each have 2 degrees of freedom. This is because if Stepwise removes Ct or T, it must also remove Ct*T. If you change the Direction to **Backward** and click **Step**, Ct*T is removed and the degrees of freedom for Ct and T change to 1.

Example of a Model with a Nominal Term

This example illustrates fitting a model with a nominal term in the Stepwise personality of the Fit Model platform. You are interested in the global gross receipts on movies that were released in 2011. Your potential predictors are two ratings variables that are continuous and one variable for movie genre that is nominal. Before you attempt to reduce your model using stepwise regression, you want to explore the variables of interest.

1. Select **Help > Sample Data Folder** and open Hollywood Movies.jmp.
2. Select **Analyze > Distribution**.
3. Select Genre and click **Y, Columns**.
4. Click **OK**.

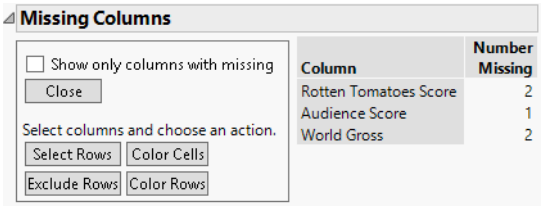
Figure 5.14 Distribution of Genre



Note that Genre has nine levels, so it is represented by eight model terms. Further data exploration will reveal that, because of missing data, only eight levels are considered by Stepwise.

5. In the data table's Columns panel, select the columns of interest: Rotten Tomatoes Score, Audience Score, and World Gross.
6. Select **Analyze > Screening > Explore Missing Values**.
7. Click **Y, Columns** and click **OK**.

Figure 5.15 Missing Columns Report



Note that Rotten Tomatoes Score is missing in 2 rows, Audience Score is missing in 1 row, and World Gross is missing in 2 rows.

8. In the Missing Columns report, select the three columns listed under **Column**.
9. Click **Select Rows**.

   In the data table's Rows panel, you can see that three rows are selected. Because these three rows contain missing data on the predictors or response, they are automatically excluded from the Stepwise analysis. Note that row 128 is the only entry in the Adventure category, which means that category is entirely removed from the analysis. For the purposes of the Stepwise analysis, it follows that Genre has only eight categories. Now that you have seen the effect of the missing data, you will conduct the Stepwise analysis.
10. Select **Analyze > Fit Model**.
11. Select Rotten Tomatoes Score, Audience Score, and Genre and click **Add**.

    If you fit a standard least squares model to World Gross using Rotten Tomatoes Score, Audience Score, and Genre as predictors, the residuals are highly heteroscedastic. (This is typical of financial data.) Use a log transformation to better satisfy the regression assumption of equal variance.
12. Right-click World Gross in the Select Columns list and select **Transform > Log**.

    The transformed variable *Log[World Gross]* appears at the bottom of the Select Columns list.
13. Select *Log[World Gross]* and click **Y**.
14. Select **Stepwise** from the Personality list.
15. Click **Run**.

Figure 5.16 Current Estimates Table Showing List of Model Terms

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	4.09998703	1	0	0.000	1
☐	☐	Rotten Tomatoes Score	0	1	10.78976	3.303	0.07154
☐	☐	Audience Score	0	1	8.715114	2.655	0.10577
☐	☐	Genre{Drama&Horror&Thriller&Fantasy&Romance&Comedy-Action&Animation}	0	1	49.78148	16.850	7.25e-5
☐	☐	Genre{Drama&Horror&Thriller-Fantasy&Romance&Comedy}	0	1	9.560063	2.918	0.09008
☐	☐	Genre{Drama-Horror&Thriller}	0	1	2.027141	0.608	0.43718
☐	☐	Genre{Horror-Thriller}	0	1	0.013011	0.004	0.95043
☐	☐	Genre{Fantasy&Romance-Comedy}	0	1	1.428612	0.428	0.51439
☐	☐	Genre{Fantasy-Romance}	0	1	0.000376	0.000	0.99157
☐	☐	Genre{Action-Animation}	0	1	1.362765	0.408	0.52426

In the Current Estimates table, note that Genre is represented by 7 terms. You will construct a model using two of these to see how these terms are defined.

16. Check the boxes under **Entered** next to the first two terms for Genre:
    - Genre{Drama&Horror&Thriller&Fantasy&Romance&Comedy-Action&Animation}
    - Genre{Drama&Horror&Thriller-Fantasy&Romance&Comedy}

17. Click **Make Model**.

    Notice that the two terms are added as temporary transform columns to the Model Effects list in the Model Specification window. These columns are discussed in the next section.

Construction of Hierarchical Terms in Example

Recall that because of missing values, Genre is a nominal variable with eight levels. In the Current Estimates table, Genre is represented by seven terms. This is appropriate, because Genre has eight levels. The first two terms that represent Genre are described below. Subsequent terms are defined in a similar fashion.

First Term

The first term that appears is Genre{Drama&Horror&Thriller&Fantasy&Romance&Comedy-Action&Animation}. This variable has the form Genre{A1 - A2}, where A1 and A2 are separated by a minus sign. The notation indicates that the maximum separation in terms of sum of squares between groups occurs between the following two sets of levels:

- Drama, Horror, Thriller, Fantasy, Romance, and Comedy (represented by A1)
- Action and Animation (represented by A2)

If you include the term Genre{Drama&Horror&Thriller&Fantasy&Romance&Comedy-Action&Animation} in a model, a temporary transform column representing that term is used in the model. The column contains the following values:

- 1 for Drama, Horror, Thriller, Fantasy, Romance, and Comedy
- -1 for Action and Animation

Second Term

The second term that appears is Genre{Drama&Horror&Thriller-Fantasy&Romance&Comedy}. This set of levels is entirely contained in the first split for the first term (A1). The notation contrasts the levels:

- Drama, Horror, and Thriller
- Fantasy, Romance, and Comedy

Among all the splits of the levels of Drama, Horror, Thriller, Fantasy, Romance, and Comedy (A1) and of the levels of Action and Animation (A2), the algorithm determines that this split has the largest sum of squares between groups.

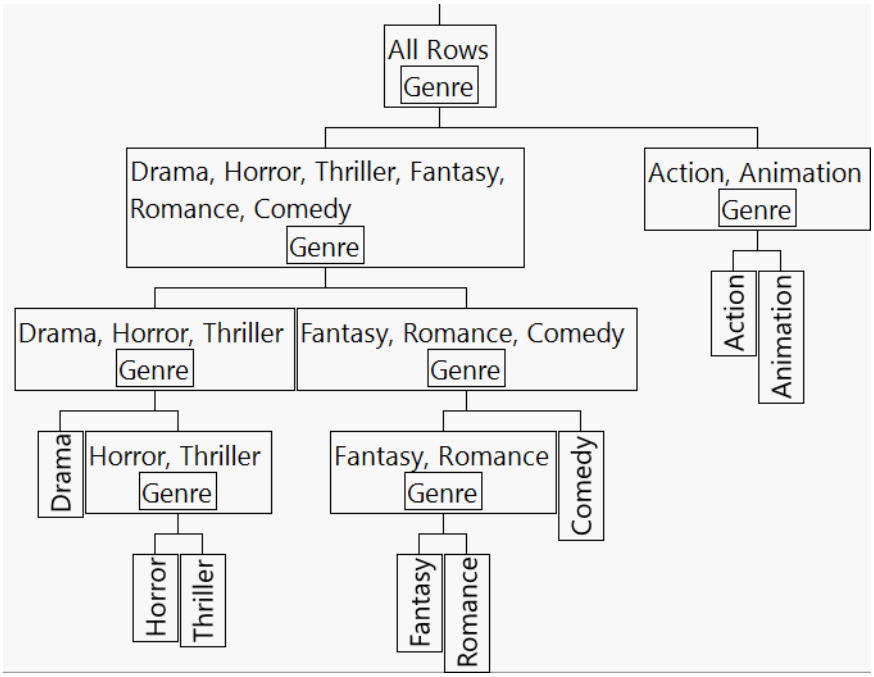
If you include this term in a model, a temporary transform column representing that term is used in the model. The column contains the following values:

- 1 for Drama, Horror, and Thriller
- -1 for Fantasy, Romance, and Comedy
- 0 for Action and Animation

Hierarchy of Terms

The splitting of terms continues, based on the sum of squares between groups criterion. The hierarchy that leads to the definition of the terms is illustrated in Figure 5.17.

Figure 5.17 Tree Showing Splits Used in Hierarchical Coding



Rules

When you use the **Combine** rule or the **Restrict** rule, a term cannot enter the model unless all the terms above it in the hierarchy have been entered. For example, if you enter Genre{Action-Animation}, then JMP enters Genre{Drama&Horror&Thriller&Fantasy&Romance&Comedy-Action&Animation} as well.

When you use the **Whole Effects** rule and enter any one of the Genre terms, all of the Genre terms are entered.

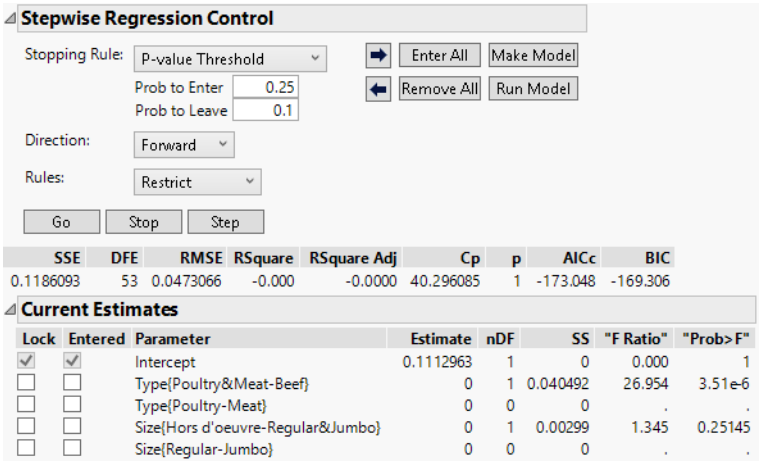
Example of the Restrict Rule for Hierarchical Terms

This example illustrates the Restrict rule for hierarchical terms in the Stepwise personality of the Fit Model platform. If you have a model with nominal or ordinal terms, when you make or run the model, temporary transform columns containing the hierarchical terms involved in the model are used in the model fit. The model itself appears in a new Fit Model window. This example further illustrates how Stepwise constructs a model with hierarchical effects.

A simple model examines the cost per ounce ($/oz) of hot dogs as a function of the Type of hot dog (Meat, Beef, Poultry) and the Size of the hot dog (Jumbo, Regular, Hors d'oeuvre).

1. Select **Help > Sample Data Folder** and open Hot Dogs2.jmp.
2. Select **Analyze > Fit Model**.
3. Select $/oz and click **Y**.
4. Select Type and Size and click **Add**.
5. For **Personality**, select **Stepwise**.
6. Click **Run**.
7. For **Stopping Rule**, select **P-value Threshold**.
8. For **Rules**, select **Restrict**.

Figure 5.18 Stepwise Control Panel with P-value Threshold and Restrict Rule



Notice that when you change from the default Rule of Combine to Restrict, the F Ratio and Prob > F values for two terms are shown as missing. These are the terms Type{Poultry-Meat} and Size{Regular-Jumbo}. This is because these two terms cannot enter the model until their precedent terms enter.

9. Click **Step**.

The term Type{Poultry&Meat-Beef} enters the model. This term has the smallest Prob>F value, and that value falls below the Prob to Enter threshold of 0.25.

Figure 5.19 Stepwise Control Panel with One Term Entered

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	0.11864706	1	0	0.000	1
☐	☑	Type{Poultry&Meat-Beef}	-0.0283529	1	0.040492	26.954	3.51e-6
☐	☐	Type{Poultry-Meat}	0	1	0.012426	9.648	0.00309
☐	☐	Size{Hors d'oeuvre-Regular&Jumbo}	0	1	0.001038	0.687	0.4112
☐	☐	Size{Regular-Jumbo}	0	0	0	.	.

The F Ratio and Prob > F values for the term Type{Poultry-Meat} appear. Since its precedent term has entered the model, Type{Poultry-Meat} is now allowed to enter.

10. Click **Step**.

    Since Type{Poultry-Meat} has the smallest Prob>F value among the remaining terms, and that value is below the Prob to Enter threshold, it is the next term to enter the model.

11. Click **Step**.

    The term Size{Hors d'oeuvre-Regular&Jumbo} enters the model, since its Prob>F value is 0.1577. Because its precedent term is now in the model, the term Size{Regular-Jumbo} is allowed to enter the model and its Prob>F value appears.

    However, the Prob>F value for the term Size{Regular-Jumbo} is 0.7566, which exceeds the Prob to Enter value of 0.25. For this reason, if you click Step again, it is not entered into the model.

Figure 5.20 Current Estimates Report for the Final Model

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	0.10849381	1	0	0.000	1
☐	☑	Type{Poultry&Meat-Beef}	-0.0275278	0	0	.	.
☐	☑	Type{Poultry-Meat}	-0.0205527	1	0.013985	11.083	0.00164
☐	☑	Size{Hors d'oeuvre-Regular&Jumbo}	-0.0121982	1	0.002596	2.057	0.1577
☐	☐	Size{Regular-Jumbo}	0	1	0.000125	0.097	0.7566

Tip: Use the Go button to run the entire stepwise process automatically. To see this in action, click **Remove All**. Then click **Go**.

12. Click **Make Model**.

    After you click Make Model, a Fit Model launch window appears, containing only the three model effects that were selected in the stepwise process. In the launch window, temporary transform columns that define the three hierarchical effects entered into the model are included in the model.

Example of Logistic Stepwise Regression

This example illustrates fitting a logistic regression model in the Stepwise personality of the Fit Model platform.

1. Select **Help > Sample Data Folder** and open Fitness.jmp.
2. Select **Analyze > Fit Model**.
3. Select Sex and click **Y**.
4. Select Weight, Runtime, RunPulse, RstPulse, and MaxPulse and click **Add**.
5. For **Personality**, select **Stepwise**.
6. Click **Run**.
7. Click **Go**.

Figure 5.21 Logistic Stepwise Report

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	Wald/Score ChiSq	"Sig Prob"
☑	☑	Intercept[M]	-35.807846	1	0	1
☐	☑	Weight	0.28229655	1	5.471429	0.01933
☐	☑	Runtime	1.32680366	1	2.545732	0.11059
☐	☐	RunPulse	0	1	0.470408	0.4928
☐	☐	RstPulse	0	1	0.007216	0.9323
☐	☐	MaxPulse	0	1	0.076497	0.7821

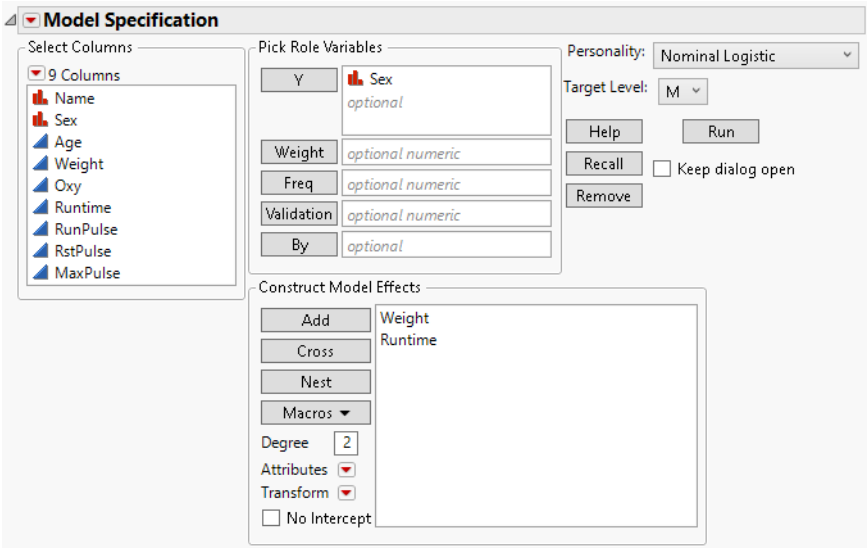
Step History

Step	Parameter	Action	L-R ChiSquare	"Sig Prob"	Entry ChiSquare	Entry "Sig Prob"	RSquare	p	AICc	BIC
1	Weight	Entered	12.16669	0.0005	10.1342	0.00146	0.2833	2	35.2047	37.6441
2	Runtime	Entered	8.017826	0.0046	6.37139	0.0116	0.4700	3	29.6472	33.0603
3	RunPulse	Entered	1.440088	0.2301	1.44684	0.22903	0.5036	4	30.8567	35.0542
4	MaxPulse	Entered	0.071809	0.7887	0.06994	0.79142	0.5052	5	33.6464	38.4164
5	RstPulse	Entered	0.007156	0.9326	0.00722	0.93228	0.5054	6	36.7393	41.8432
6	Best	Specific	.	.	0.00722	0.93228	0.4700	3	29.6472	33.0603

The two variables Weight and Runtime are entered into the model based on the Stopping Rule.

8. Click **Make Model**.

Figure 5.22 Model Specification Window for Reduced Model



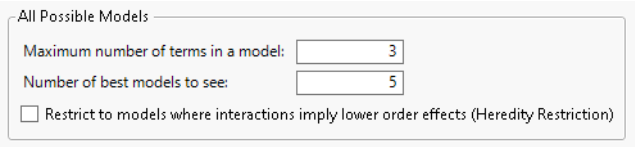
A model specification window appears containing the two variables as model effects. Note that the Personality is Nominal Logistic. If the response had been ordinal, the Personality would be Ordinal Logistic.

Example of the All Possible Models Option

This example illustrates the All Possible Models option in the Stepwise personality of the Fit Model platform.

1. Select **Help > Sample Data Folder** and open Fitness.jmp.
2. Select **Analyze > Fit Model**.
3. Select Oxy and click **Y**.
4. Select Runtime, RunPulse, RstPulse, and MaxPulse and click **Add**.
5. For **Personality**, select **Stepwise**.
6. Click **Run**.
7. Click the Stepwise red triangle menu and select **All Possible Models**.
8. Enter 3 for the maximum number of terms, and enter 5 for the number of best models.

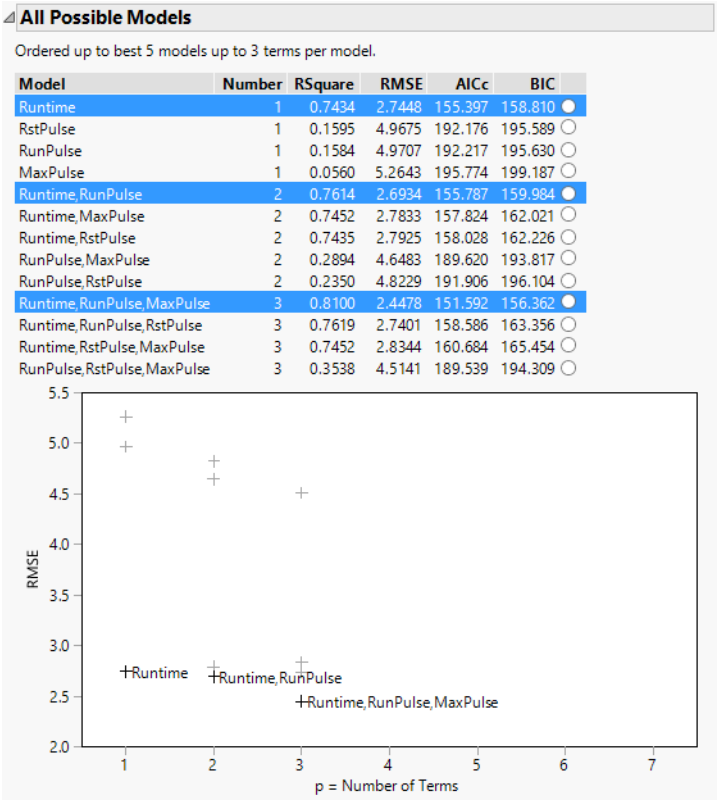
Figure 5.23 All Possible Models Pop-up Dialog



9. Click **OK**.

All possible models (up to three terms in a model) are fit.

Figure 5.24 All Possible Models Report



The models are listed in increasing order of the number of parameters that they contain. The model with the highest R^2 for each number of parameters is highlighted. The radio button column at the right of the table enables you to select one model at a time and check the results.

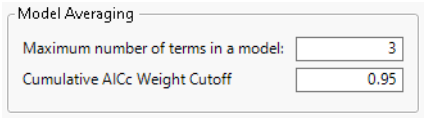
Note: The recommended criterion for selecting a model is to choose the one corresponding to the smallest BIC or AICc value. Some analysts also want to see the C_p statistic. Mallow's C_p statistic is computed, but initially hidden in the table. To make it visible, right-click in the table and select **Columns > Cp**.

Example of the Model Averaging Option

This example illustrates the Model Averaging option in the Stepwise personality of the Fit Model platform.

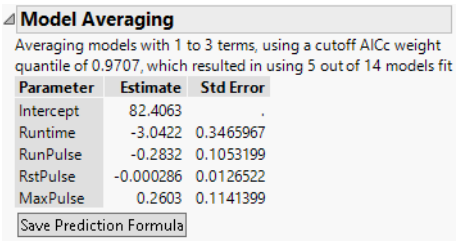
1. Select **Help > Sample Data Folder** and open Fitness.jmp.
2. Select **Analyze > Fit Model**.
3. Select Oxy and click **Y**.
4. Select Runtime, RunPulse, RstPulse, and MaxPulse and click **Add**.
5. For **Personality**, select **Stepwise**.
6. Click **Run**.
7. Click the Stepwise red triangle menu and select **Model Averaging**.
8. Enter 3 for the Maximum number of terms in a model.
9. Keep 0.95 for the Cumulative AICc Weight Cutoff.

Figure 5.25 Model Averaging Window



10. Click **OK**.

Figure 5.26 Model Averaging Report



In the Model Averaging report, average estimates and standard errors appear for each parameter. The standard errors shown reflect the bias of the estimates toward zero.

11. Click **Save Prediction Formula** to save the prediction formula in the original data table.

Chapter 6

JMP PRO Generalized Regression Models

Build Models Using Variable Selection Techniques

The Generalized Regression personality of the Fit Model platform is available only in JMP Pro.

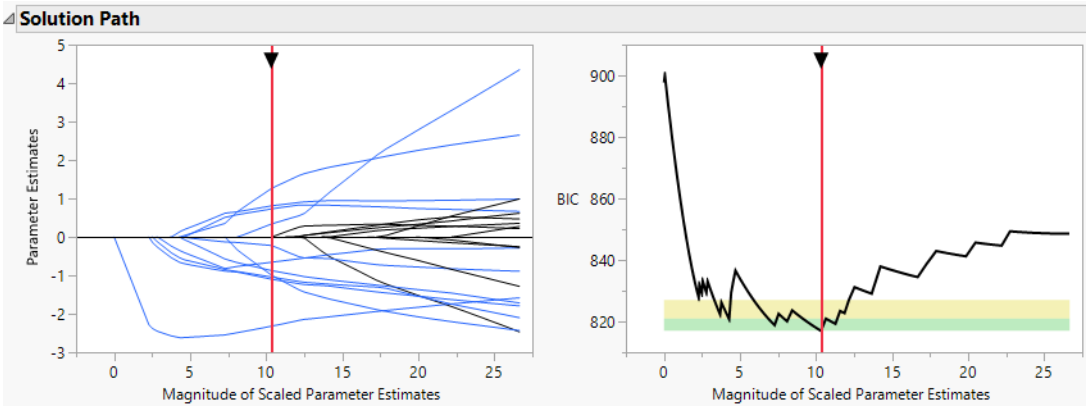
The Generalized Regression personality provides variable selection techniques, including shrinkage techniques, that specifically address modeling correlated and high-dimensional data. Two of these techniques, the Lasso and the Elastic Net, perform variable selection as part of the modeling procedure.

Large data sets that contain many variables typically exhibit multicollinearity issues. Modern data sets can include more variables than observations, requiring variable selection if traditional modeling techniques are to be used. The presence of multicollinearity and a profusion of predictors exposes the shortcomings of classical techniques.

Even for small data sets with little or no correlation, including designed experiments, the Lasso and Elastic Net are useful. They can be used to build predictive models or to select variables for model reduction or for future study.

The Generalized Regression personality is useful for many modeling situations. This personality enables you to specify a variety of distributions for your response variable. Use it when your response is continuous, binomial, a count, or zero-inflated. Use it when you are interested in variable selection or when you suspect collinearity in your predictors. More generally, use it to fit models that you compare to models obtained using other techniques.

Figure 6.1 The Solution Path for an Elastic Net Fit



Contents

Overview of the Generalized Regression Personality 281
Example of Generalized Regression 283
Launch the Generalized Regression Personality 286
Specify a Distribution 288
Generalized Regression Report Window 296
Generalized Regression Report Options 296
Model Launch Control Panel 297
Response Distribution 298
Estimation Method Options 298
Advanced Controls 303
Validation Method Options 306
Early Stopping 308
Go 308
Model Fit Reports 309
Regression Plot 309
Model Summary 310
Estimation Details 313
Solution Path 313
Parameter Estimates for Centered and Scaled Predictors 317
Parameter Estimates for Original Predictors 319
Active Parameter Estimates 320
Effect Tests 320
Model Fit Options 321
Self-Validated Ensemble Models 332
Overview of Self-Validated Ensemble Models 332
Reports for Self-Validated Ensemble Models 333
Model Fit Options for Self-Validated Ensemble Models 334
Statistical Details for the Generalized Regression Personality 336
Statistical Details for Estimation Methods 336
Statistical Details for Advanced Controls 338
Statistical Details for Distributions 339

JMP PRO Overview of the Generalized Regression Personality

The Generalized Regression personality of the Fit Model platform features regularized, or penalized, regression techniques. Such techniques attempt to fit better models by shrinking the model coefficients toward zero. The resulting estimates are biased. This increase in bias can result in decreased prediction variance, thus lowering overall prediction error compared to non-penalized models. Two of these techniques, the Elastic Net and the Lasso, include variable selection as part of the modeling procedure.

Modeling techniques such as the Elastic Net and the Lasso are particularly useful for large data sets, where collinearity is typically a problem. In addition, modern data sets often include more variables than observations. This situation is sometimes referred to as the $p > n$ problem, where n is the number of observations and p is the number of predictors. Such data sets require variable selection if traditional modeling techniques are to be used.

The Elastic Net and Lasso can also be used for small data sets with little correlation, including designed experiments. They can be used to build predictive models or to select variables for model reduction or for future study.

The personality provides the following classes of modeling techniques:

- Maximum Likelihood
- Step-Based Estimation
- Penalized Regression

The Elastic Net and Lasso are relatively recent techniques (Tibshirani 1996; Zou and Hastie 2005). Both techniques penalize the size of the model coefficients, resulting in a continuous shrinkage. The amount of shrinkage is determined by a *tuning parameter*. An optimal level of shrinkage is determined by one of several validation methods. Both techniques have the ability to shrink coefficients to zero. In this way, variable selection is built into the modeling procedure. The Elastic Net model subsumes both the Lasso and ridge regression as special cases. See "Statistical Details for Estimation Methods".

Details about Generalized Regression Modeling Techniques

- The Maximum Likelihood method is a classical approach. It provides a baseline to which you can compare the other techniques, and it is the most appropriate place for traditional inference techniques such as hypothesis testing.
- Forward Selection is a method of stepwise regression. In forward selection, terms are entered into the model. The most significant terms are added until all of the terms are in the model or there are no degrees of freedom left.

- The Lasso has two shortcomings. When several variables are highly correlated, it tends to select only one variable from that group. When the number of variables, p, exceeds the number of observations, n, the Lasso selects at most n predictors.
- The Elastic Net, on the other hand, tends to select all variables from a correlated group, fitting appropriate coefficients. It can also select more than n predictors when $p > n$.
- Ridge regression was among the first of the penalized regression methods proposed (Hoerl 1962; Hoerl and Kennard 1970). Ridge regression does not shrink coefficients to zero, so it does not perform variable selection.
- The Double Lasso attempts to separate the selection and shrinkage steps by performing variable selection with an initial Lasso model. The variables selected in the initial model are then used as the input variables for a second Lasso model.
- Two-Stage Forward Selection performs two stages of forward stepwise regression. It performs variable selection on the main effects in the first stage. Then, higher-order effects are allowed to enter the model in the second stage.

The Generalized Regression personality also fits an *adaptive* version of the Lasso and the Elastic Net. These adaptive versions attempt to penalize variables in the true active set less than variables not contained in the true active set. The *true active set* refers to the set of terms in a model that have an actual effect on the response. The adaptive versions of the Lasso and Elastic Net were developed to ensure that the oracle property holds. The oracle property guarantees the following: Asymptotically, your estimates are what they would have been had you fit the model to the true active set of predictors. More specifically, your model correctly identifies the predictors that should have zero coefficients. Your estimates converge to those that would have been obtained had you started with only the true active set. See "Adaptive Methods".

The Generalized Regression personality enables you to specify a variety of distributions for your response variable. The distributions fit include normal, Cauchy, Student's t, exponential, gamma, Weibull, lognormal, negative lognormal, beta, binomial, beta binomial, Poisson, negative binomial, zero-inflated binomial, zero-inflated beta binomial, zero-inflated Poisson, zero-inflated negative binomial, and zero-inflated gamma. This flexibility enables you to fit categorical and count responses, as well as continuous responses, and specifically, right-skewed continuous responses. You can also fit quantile regression and Cox proportional hazards models. For some of the distributions, you can fit models to censored data. The personality provides a variety of validation criteria for model selection and supports training, validation, and test columns. See "Specify a Distribution".

JMP PRO Example of Generalized Regression

This example shows how to develop a predictive model using generalized regression techniques. The data consist of measurements on 442 diabetics. The response of interest is disease progression measured one year after a baseline measure was taken. Ten variables thought to be related to disease progression are also measured at baseline.

1. Select **Help > Sample Data Folder** and open Diabetes.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y from the Select Columns list and click **Y**.
4. Select Age through Glucose and click **Macros > Factorial to Degree**.

   This adds all terms up to degree 2 (the default in the **Degree** box) to the model.
5. Select Validation from the Select Columns list and click **Validation**.
6. From the Personality list, select **Generalized Regression**.
7. Click **Run**.

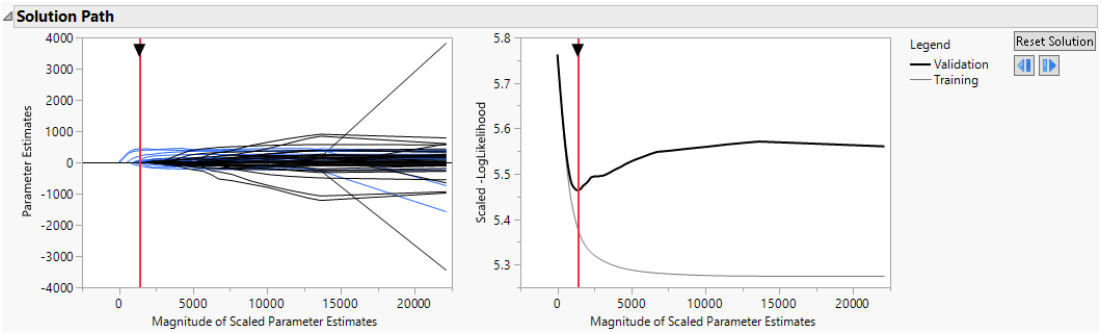
   The Generalized Regression report that appears contains a Model Comparison report, a Model Launch control panel, and a Normal Standard Least Squares with Validation Column report.

   In the Model Launch control panel, note the following:

   – The Response Distribution is set to Normal because you specified Normal as the Distribution in the Fit Model launch window.
   – The default Estimation Method is the Lasso.
   – The Validation Method is set to Validation Column because you specified a validation column in the Fit Model window.
8. Click **Go**.

   A Normal Lasso with Validation Column report appears. The Solution Path report (Figure 6.2) shows plots of the parameter estimates and scaled negative log-likelihood. The shrinkage increases as the Magnitude of Scaled Parameter Estimates decreases. The estimates at the far right of the plot are the maximum likelihood estimates. A vertical red line indicates those parameter values selected by the validation criterion, in this case, the holdback sample defined by the column Validation.

Figure 6.2 Solution Path Plot



9. Click the red triangle next to Normal Lasso with Validation Column and select **Select Nonzero Terms**.

   This option highlights the nonzero terms in the Parameter Estimates for Original Predictors report (Figure 6.3) and their paths in the Solution Path Plot. The corresponding columns in the data table are also selected. Note that only 11 of the 55 parameter estimates are nonzero. The scale parameter for the normal distribution (sigma) is also estimated and shown in a separate table at the bottom of the Parameter Estimates for Original Data report. Note that not all of the 55 parameter estimates appear in Figure 6.3.

Figure 6.3 Portion of Parameter Estimates for Original Predictors Report

Parameter Estimates for Original Predictors

Term	Estimate	Std Error	Wald ChiSquare	Prob > ChiSquare	Lower 95%	Upper 95%
Intercept	-247.8422	42.926975	33.334183	<.0001*	-331.9775	-163.7068
Age	0	0	0	1.0000	0	0
Gender[1-2]	8.7286488	6.9678967	1.5692446	0.2103	-4.928178	22.385475
BMI	5.702121	0.9303784	37.562422	<.0001*	3.8786128	7.5256293
BP	0.8156913	0.2526706	10.421787	0.0012*	0.320466	1.3109166
Total Cholesterol	0	0	0	1.0000	0	0
LDL	0	0	0	1.0000	0	0
HDL	-0.47425	0.3986141	1.4155011	0.2341	-1.25552	0.3070188
TCH	0	0	0	1.0000	0	0
LTG	41.286654	7.0844177	33.963392	<.0001*	27.401451	55.171858
Glucose	0	0	0	1.0000	0	0
(Age-48.5181)*Gender[1-2]	-0.082735	0.3621512	0.0521915	0.8193	-0.792538	0.6270682
(Age-48.5181)*(BMI-26.3758)	0	0	0	1.0000	0	0
(Age-48.5181)*(BP-94.647)	0.0043544	0.0171735	0.0642885	0.7998	-0.029305	0.0380137
(Age-48.5181)*(Total Cholesterol-189.14)	0	0	0	1.0000	0	0
(Age-48.5181)*(LDL-115.439)	0	0	0	1.0000	0	0
(Age-48.5181)*(HDL-49.7885)	0	0	0	1.0000	0	0
(Age-48.5181)*(TCH-4.07025)	0	0	0	1.0000	0	0
(Age-48.5181)*(LTG-4.64141)	0	0	0	1.0000	0	0
(Age-48.5181)*(Glucose-91.2602)	0.0231484	0.0233144	0.9858084	0.3208	-0.022547	0.0688439
Gender[1-2]*(BMI-26.3758)	0	0	0	1.0000	0	0
Gender[1-2]*(BP-94.647)	0	0	0	1.0000	0	0
Gender[1-2]*(Total Cholesterol-189.14)	0	0	0	1.0000	0	0
Gender[1-2]*(LDL-115.439)	0	0	0	1.0000	0	0
Gender[1-2]*(HDL-49.7885)	-0.221009	0.4728172	0.2184913	0.6402	-1.147714	0.7056955
Gender[1-2]*(TCH-4.07025)	0	0	0	1.0000	0	0
Gender[1-2]*(LTG-4.64141)	0	0	0	1.0000	0	0
Gender[1-2]*(Glucose-91.2602)	0	0	0	1.0000	0	0
(BMI-26.3758)*(BP-94.647)	0.0254032	0.0525392	0.2337819	0.6287	-0.077572	0.1283782
(BMI-26.3758)*(Total Cholesterol-189.14)	0	0	0	1.0000	0	0
(BMI-26.3758)*(LDL-115.439)	0	0	0	1.0000	0	0
(BMI-26.3758)*(HDL-49.7885)	0	0	0	1.0000	0	0
(BMI-26.3758)*(TCH-4.07025)	0	0	0	1.0000	0	0
(BMI-26.3758)*(LTG-4.64141)	0	0	0	1.0000	0	0
(BMI-26.3758)*(Glucose-91.2602)	0.0210214	0.0597655	0.1237153	0.7250	-0.096117	0.1381596

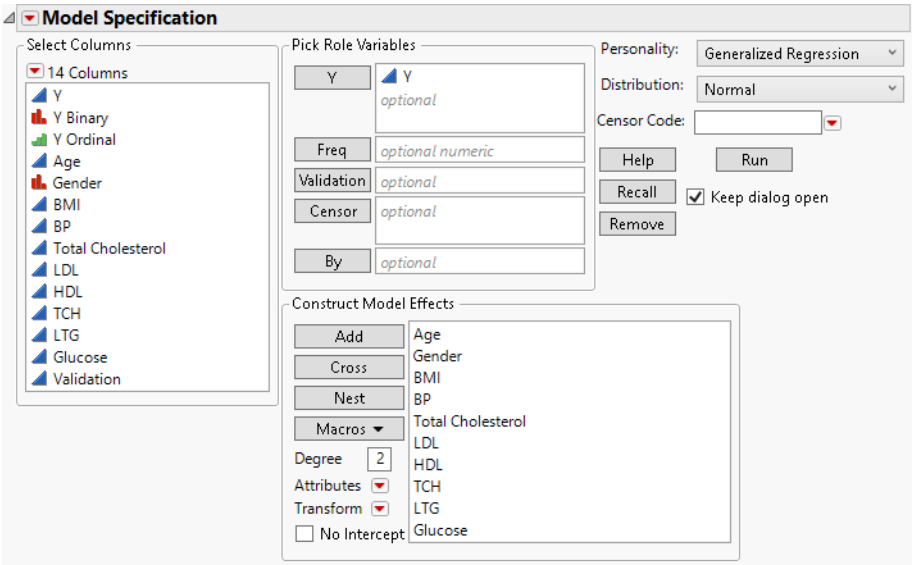
Normal Distribution Parameters	Estimate	Std Error	Wald ChiSquare	Prob > ChiSquare	Lower 95%	Upper 95%
Scale	52.512586	2.3230199	511.00007	<.0001*	47.959551	57.065621

To save the prediction formula, click the red triangle next to Normal Lasso with Validation Column report and select **Save Columns > Save Prediction Formula**.

JMP PRO Launch the Generalized Regression Personality

Launch the Generalized Regression personality by selecting **Analyze > Fit Model**, entering one or more columns for **Y**, and selecting **Generalized Regression** from the **Personality** menu.

Figure 6.4 Fit Model Launch Window with Generalized Regression Selected



For more information about aspects of the Fit Model window that are common to all personalities, see "Model Specification". For more information about the options in the Select Columns red triangle menu, see *Using JMP*. Information specific to the Generalized Regression personality is presented here.

The parameterization of nominal variables used in the Generalized Regression personality differs from their parameterization using other Fit Model personalities. The Generalized Regression personality uses indicator function parameterization. In this parameterization, the estimate that corresponds to the indicator for a level of a nominal variable is an estimate of the difference between the mean response at that level and the mean response at the last level. The last level is the level with the highest value order coding; it is the level whose indicator function is not included in the model.

If your model effects have missing values, you can treat these missing values as informative categories. Select the Informative Missing option from the Model Specification red triangle menu.

To specify a model without an intercept term, select the No Intercept option in the Construct Model Effects panel of the Fit Model window. If you select this option, note the following:

- The predictors are not centered and scaled.
- Odds ratios, hazard ratios, and incidence rate ratios are not available in the report window.
- The No Intercept option is not available for the Ordinal Logistic Distribution.

Caution: Using the No Intercept option with the Lasso or Elastic Net is not recommended because the results are sensitive to the scale of the model effects. The adaptive versions of these estimation methods are recommended instead.

Censoring

You can specify censoring for your response variable in one of the following ways:

- For interval-censored, right-censored, and left-censored responses, specify a column that contains a Detection Limits column property. The limits specified in the column property define the range of response values that are considered uncensored:
  - For interval-censored responses, specify a nonmissing value for both the Lower Detection Limit and the Upper Detection Limit in the Detection Limits column property.
  - For right-censored responses, specify a missing value for the Lower Detection Limit and a nonmissing value for the Upper Detection Limit in the Detection Limits column property.
  - For left-censored responses, specify a nonmissing value for the Lower Detection Limit and a missing value for the Upper Detection Limit in the Detection Limits column property.
- For right-censored responses, specify a column that contains indicators for right-censored observations as a Censor column in the launch window. Select the value in that column that designates right-censored observations from the Censor Code list.
- For interval-censored and left-censored responses, specify two columns that define the censoring interval in the Y column role:
  - For interval-censored responses, the first Y variable gives the lower limit.
  - For left-censored responses, the first Y variable contains a missing value.
  - For both interval-censored and left-censored responses, the second Y variable gives the upper limit for each response.

  If you specify two columns for Y and a Distribution that supports censoring, an Alert appears that asks whether the columns represent censoring. If you choose No, the columns are treated as separate responses.

Note: You can specify the default behavior for two responses using the Treatment of Two Response Columns preference in Generalized Regression platform preferences.

Censoring is available when the specified Distribution is Normal, Exponential, Gamma, Weibull, Lognormal, Beta, or Cox Proportional Hazards.

JMP PRO Specify a Distribution

In the Fit Model launch window, when you select Generalized Regression as the Personality, the Distribution option appears. Here you can specify a distribution for Y. The abbreviation *ZI* means *zero-inflated*. The distributions are separated into three categories based on their response: continuous, discrete, and zero-inflated. The options are described below.

Note: If you specify multiple Y variables in the Model Specification window, the same response distribution must be used for all of the specified Y variables. If you want to fit separate distributions to different response variables in the same Generalized Regression report, you must use a script.

JMP PRO Continuous

Normal Y has a normal distribution with mean μ and standard deviation σ. The normal distribution is symmetric and with a large enough sample size, can approximate a large variety of other distributions using the Central Limit Theorem. The link function for μ is the identity. That is, the mean of Y is expressed as a linear model.

Note: When the specified Distribution is Normal, Standard Least Squares replaces the Maximum Likelihood Estimation method.

The scale parameter for the normal distribution is σ. When there is no penalty in the estimation method, the estimate of the scale parameter σ is the root mean square error (RMSE). The RMSE is the square root of the usual unbiased estimator of σ^2. The results shown are equivalent to a standard least squares fit unless censored observations are involved.

Note: The parameterization of nominal variables used in the Generalized Regression personality differs from their parameterization using the Standard Least Squares personality. Because of this difference, parameter estimates differ for models that contain nominal or ordinal effects.

See "Statistical Details for Distributions".

Cauchy Y has a Cauchy distribution with location parameter μ and scale parameter σ. The Cauchy distribution has an undefined mean and standard deviation. The median and mode are both μ. Most data do not inherently follow a Cauchy distribution. However, it is useful for conducting a robust regression on data that contain a large proportion of outliers (up to 50%). The link function for μ is the identity. See "Statistical Details for Distributions".

t(5) Y has a Student's *t* distribution with 5 degrees of freedom, location parameter μ and scale parameter σ. The Student's *t* distribution is symmetric and is a robust option that spans the space between a normal distribution and a Cauchy distribution. As the degrees of freedom in the Student's *t* distribution approach infinity, the distribution is equivalent to the normal. When the degrees of freedom in the Student's *t* distribution equals 1, the distribution is equivalent to the Cauchy. The link function for μ is the identity. See "Statistical Details for Distributions".

Exponential Y has an exponential distribution with mean parameter μ. The exponential distribution is right-skewed and is often used to model lifetimes or the time between successive events. The link function for μ is the logarithm. See "Statistical Details for Distributions".

Gamma Y has a gamma distribution with mean parameter μ and dispersion parameter σ. The gamma is a flexible distribution and contains a family of other widely used distributions. For example, the exponential distribution is a special case of the gamma distribution where σ = μ. The chi-squared distribution can also be derived from the gamma distribution. The link function for μ is the logarithm. See "Statistical Details for Distributions".

Weibull Y has a Weibull distribution with mean parameter μ and scale parameter σ. The Weibull distribution is a flexible distribution and is often used to model lifetimes or the time until an event. The link function for μ is the identity. See "Statistical Details for Distributions".

LogNormal Y has a lognormal distribution with location parameter μ and scale parameter σ. The lognormal distribution is right-skewed and is often used to model lifetimes or the time until an event. The link function for μ is the identity. See "Statistical Details for Distributions".

Negative LogNormal Y has a negative lognormal distribution with location parameter μ and scale parameter σ. The negative lognormal distribution is left-skewed and available only for strictly negative observations. The link function for μ is the identity. The negative lognormal distribution is a negative transform of the lognormal distribution. See "Statistical Details for Distributions".

Beta Y has a beta distribution with mean parameter μ and dispersion parameter σ. The response for the beta is between 0 and 1 (not inclusive) and is often used to model

proportions or rates. The link function for μ is the logit. See "Statistical Details for Distributions".

Quantile Regression Quantile regression models a specified conditional quantile of the response. No assumption is made about the form of the underlying distribution. When you select Quantile Regression, a Quantile box appears beneath the Distribution menu. Specify the desired quantile.

If you specify 0.5 (the default) for the Quantile on the Model Specification window, quantile regression models the conditional median of the response. Quantile regression is particularly useful when the rate of change in the conditional quantile, expressed by the regression coefficients, depends on the quantile. An advantage of quantile regression over least squares regression is its flexibility for modeling data with heterogeneous conditional distributions.

Quantile Regression is fit by minimizing an objective function using an iterative approach. For more information about quantile regression, see Koenker and Hallock (2001) and Portnoy and Koenker (1997).

When you choose Quantile Regression, Maximum Likelihood is the only available Estimation Method, and None is the only available Validation Method.

Note: If a quantile regression fit is time intensive, a progress bar appears. The progress bar shows the relative change in the objective function. When you click Accept Current Estimates, the calculation stops and the reported parameter estimates correspond to the best model fit at that point.

Cox Proportional Hazards The Cox proportional hazards model is a regression model for time-to-event data with predictors. It is based on a multiplicative relationship between the predictors and the hazard function. It can be used to examine the effect of predictors on survival times. The model involves an arbitrary baseline hazard function that is scaled by the predictors to give a general hazard function. The proportional hazards model produces parameter estimates and standard errors for each predictor. The Cox proportional hazards model was first proposed by D. R. Cox (1972). For more information about proportional hazards models, see Kalbfleisch and Prentice (2002).

When you choose Cox Proportional Hazards, the only available Validation Methods are BIC and AICc. Also, the Ridge Estimation Method is not available.

Note: When there are ties in the response, the Efron likelihood is used. See Efron (1977). This is a different method for handling ties than is used in the Proportional Hazard personality of the Fit Model platform or in the Fit Proportional Hazards platform.

JMP PRO Discrete

Binomial Y has a binomial distribution with parameters p and n. The response, Y, indicates the total number of successes in n independent trials with a fixed probability, p, for all trials. This distribution allows for the use of a sample size column. If no column is listed, it is assumed that the sample size is one. The link function for p is the logit. When you select a binary response variable that has a Nominal modeling type, Binomial is the only available response distribution. See "Statistical Details for Distributions".

When you select Binomial as the Distribution, the response variable must be specified in one of the following ways.

- Unsummarized: If your data are not summarized as frequencies of events, specify a single binary column as the response. If this column has a modeling type of Nominal, you can designate one of the levels to be the Target Level. The default Target Level value is the higher of the two levels based on the order of the levels.
- Summarized with Freq column: If your data are summarized as frequencies of successes and failures, specify a single binary column as the response. If this column has a modeling type of Nominal, you can designate one of the levels to be the Target Level. The default Target Level value is the higher of the two levels based on the order of the levels. Assign the frequency column to the **Freq** role.
- Summarized with sample size column entered as second Y: If your data are summarized as frequencies of events (successes) and trials, specify two continuous columns as Y in this order: the count of the number of successes, and the count of the number of trials.

Note: When the specified Distribution is Binomial, Logistic Regression replaces the Maximum Likelihood Estimation method.

Beta Binomial Y has a beta binomial distribution with the probability of success, p, the number of trails, n, and overdispersion parameter, δ. This distribution is an overdispersed version of the binomial distribution.

Run demoBetaBinomial.jsl in the JMP Samples/Scripts folder to compare a beta binomial distribution with dispersion parameter δ to a binomial distribution with parameters p and $n = 20$.

The beta binomial distribution requires a sample size greater than one for each observation. Thus, the user must specify a sample size column. To insert a sample size column, specify two continuous columns as Y in this order: the count of the number of successes, and the count of the number of trials. The link function for p is the logit. See "Statistical Details for Distributions".

Multinomial Y has a multinomial distribution with three or more discrete levels. The response variable must have a nominal or ordinal modeling type. The model fits separate

intercepts and effects parameters for each level of the response variable. If the response variable has *k* levels, the model contains *k* - 1 intercepts and effects parameters. The link function for the Multinomial distribution is the multinomial logit. See "Nominal Responses".

Ordinal Logistic Y has a multinomial distribution with ordinal levels. The response variable must have an ordinal modeling type. The model fits an intercept for each level of the response variable. The effects parameters are common across all levels of the response variable. The link function for the Ordinal Logistic distribution is the ordered logit. See "Ordinal Responses".

Note: The intercept parameterization for Ordinal Logistic in Generalized Regression differs from that in the Ordinal Logistic personality of Fit Model. The first Intercept term in Generalized Regression corresponds to the first Intercept term in the Ordinal Logistic personality. The subsequent Intercept terms in Generalized Regression are the successive differences between the intercept terms for the ordered levels of the response variable.

Poisson Y has a Poisson distribution with mean λ. The Poisson distribution typically models the number of events in a given interval and is often expressed as count data. The link function for λ is the logarithm. Poisson regression is permitted even if Y assumes non-integer values. See "Statistical Details for Distributions".

Negative Binomial Y has a negative binomial distribution with mean μ and dispersion parameter σ. The negative binomial distribution typically models the number of successes before a specified number of failures. The negative binomial distribution is also equivalent to the Gamma Poisson distribution under certain conditions. For more information about the connection between negative binomial and Gamma Poisson, see *Basic Analysis*.

Run demoGammaPoisson.jsl in the JMP Samples/Scripts folder to compare a Gamma Poisson distribution with mean λ and dispersion parameter σ to a Poisson distribution with mean λ.

The link function for μ is the logarithm. Negative binomial regression is permitted even if Y assumes non-integer values. See "Statistical Details for Distributions".

JMP PRO Zero-Inflated

ZI Binomial Y has a zero-inflated binomial distribution with parameters *p*, *n*, and zero-inflation parameter π. The response, Y, indicates the total number of successes in *n* independent trials with a fixed probability, *p*, for all trials. This distribution allows for the use of a sample size column. If no column is listed, it is assumed that the sample size is one. The link function for *p* is the logit. See "Statistical Details for Distributions".

ZI Beta Binomial Y has a beta binomial distribution with the probability of success, *p*, the number of trails, *n*, overdispersion parameter, δ, and zero-inflation parameter π. This

distribution is an overdispersed version of the ZI binomial distribution. The ZI beta binomial distribution requires a sample size greater than one for each observation. Thus, the user must specify a sample size column. To insert a sample size column, specify two continuous columns as Y in this order: the count of the number of successes, and the count of the number of trials. The link function for p is the logit. See "Statistical Details for Distributions".

ZI Poisson Y has a zero-inflated Poisson distribution with mean parameter λ and zero-inflation parameter π. The parameter λ is the conditional mean based on the observations coming from the Poisson distribution and not the inflating zeros. The link function for λ is the logarithm. ZI Poisson regression is permitted even if Y assumes no observed zeros or non-integer values. See "Statistical Details for Distributions".

ZI Negative Binomial Y has a zero-inflated negative binomial with location parameter μ, dispersion parameter σ, and zero-inflation parameter π. The parameter μ is the conditional mean based on the observations coming from the negative binomial distribution and not the inflating zeros. The link function for μ is the logarithm. ZI negative binomial regression is permitted even if Y assumes no observed zeros or non-integer values. See "Statistical Details for Distributions".

ZI Gamma Y has a zero-inflated gamma distribution with mean parameter μ and zero-inflation parameter π. Many times, you might believe that the nonzero responses are gamma distributed. This is true for insurance claims: claim values are approximately gamma distributed but there are also zeros in the data for policies that do not have any claims. The zero-inflated gamma could handle such data directly without having to split the data into zero and nonzero responses. The parameter μ is the conditional mean based on observations coming from the gamma distribution and not the inflating zeros. The link function for μ is the logarithm. See "Statistical Details for Distributions".

Table 6.1 gives the Data Types, Modeling Types, and other requirements for Y variables assigned the various distributions.

Table 6.1 Requirements for Y for Distributions

Distribution	Data Type	Modeling Type	Other
Normal	Numeric	Continuous	
Cauchy	Numeric	Continuous	
t(5)	Numeric	Continuous	
Exponential	Numeric	Continuous	Positive
Gamma	Numeric	Continuous	Positive
Weibull	Numeric	Continuous	Positive

Table 6.1 Requirements for Y for Distributions *(Continued)*

Distribution	Data Type	Modeling Type	Other
LogNormal	Numeric	Continuous	Positive
Negative LogNormal	Numeric	Continuous	Negative
Beta	Numeric	Continuous	Between 0 and 1
Quantile Regression	Numeric	Continuous	
Cox Proportional Hazards	Numeric	Continuous	Nonnegative
Binomial, unsummarized	Any	Any	Binary
Binomial, summarized with Freq column	Any	Any	Binary
Binomial, summarized with count column entered as second Y	Numeric	Continuous	Nonnegative
Beta Binomial	Numeric	Continuous	Nonnegative
Multinomial	Any	Ordinal or Nominal	
Ordinal Logistic	Any	Ordinal	
Poisson	Numeric	Any	Nonnegative
Negative Binomial	Numeric	Any	Nonnegative
Zero-Inflated Binomial	Numeric	Any	Nonnegative
Zero-Inflated Beta Binomial	Numeric	Any	Nonnegative
Zero-Inflated Poisson	Numeric	Any	Nonnegative
Zero-Inflated Negative Binomial	Numeric	Any	Nonnegative
Zero-Inflated Gamma	Numeric	Continuous	Nonnegative

For more information about how these distributions are parameterized, see "Statistical Details for Distributions". Table 6.2 summarizes the details.

Table 6.2 Distributions, Parameters, and Link Functions

Distribution	Parameters	Mean Model Link Function
Normal	μ, σ	*Identity*(μ)
Cauchy	μ, σ	*Identity*(μ)
t(5)	μ, σ	*Identity*(μ)
Exponential	μ	*Log*(μ)
Gamma	μ, σ	*Log*(μ)
Weibull	μ, σ	*Identity*(μ)
LogNormal	μ, σ	*Identity*(μ)
Negative LogNormal	μ, σ	*Identity*(μ)
Beta	μ	*Logit*(μ)
Quantile Regression	μ	*Identity*(μ)
Cox Proportional Hazards	μ	*Log*(μ)
Binomial	*n*, *p*	*Logit*(*p*)
Beta Binomial	*n*, *p*, δ	*Logit*(*p*)
Multinomial	$n, p_1, ..., p_k$	*Multinomial Logit*(p_1, ..., p_k)
Ordinal Logistic	$p_1, ..., p_{k-1}$	*Ordinal Link*(p_1, ..., p_{k-1})
Poisson	λ	*Log*(μ)
Negative Binomial	μ, σ	*Log*(μ)
Zero-Inflated Binomial	*n*, *p*, π (zero-inflation)	*Logit*(*p*)
Zero-Inflated Beta Binomial	*n*, *p*, δ, π (zero-inflation)	*Logit*(*p*)
Zero-Inflated Poisson	λ, π (zero-inflation)	*Log*(μ)
Zero-Inflated Negative Binomial	μ, σ, π (zero-inflation)	*Log*(μ)
Zero-Inflated Gamma	μ, σ, π (zero-inflation)	*Log*(μ)

After selecting an appropriate Distribution, click **Run**. The Generalized Regression report window appears.

JMP PRO Generalized Regression Report Window

In the Fit Model launch window, when you select the Generalized Regression personality and click Run, the Generalized Regression report that appears contains the following items:

- A Model Comparison report that enables you to compare all of the models that have been fit in the report. Each time a new model is fit using the Model Launch control panel, it is added to the Model Comparison report. You can show or hide the reports for fitted models using the check boxes in the Show column. Other columns in the Model Comparison report contain information about each model that was fit, as well as fit statistics for each model. Click on the column headings to sort the models by any of the columns in the Model Comparison report. The first click sorts in ascending order; click the column heading a second time to sort in descending order.

Note: The Model Comparison report appears when one or more models have been fit.

- A Model Launch control panel for fitting models. See "Model Launch Control Panel". As you fit models, outlines are added with titles that describe the types of models that you have fit. See "Model Fit Reports" and "Model Fit Options".
  - If there are linear dependencies among the model terms, the Model Launch control panel contains a Singularity Details report that shows the linear functions that the model terms satisfy. See "Models with Linear Dependencies among Model Terms".
- A Maximum Likelihood report that shows the results of a model that was fit using maximum likelihood estimation. See "Maximum Likelihood". The Maximum Likelihood report appears only if the following conditions are met:
  - There are no linear dependencies among the predictors.
  - There are more observations than predictors.
  - There are no more than 250 predictors.

Note: When the specified Distribution is Normal, this report is labeled Standard Least Squares. When the specified Distribution is Binomial, this report is labeled Logistic Regression.

JMP PRO Generalized Regression Report Options

The Generalized Regression red triangle menu contains the following options:

Model Dialog Shows the completed Fit Model launch window for the current analysis.

Set Random Seed Sets the seed for the randomization process that is used for KFold and Holdback validation. This is useful if you want to reproduce an analysis. Set the seed to a

positive value, save the script, and the seed is automatically saved in the script. Running the script always produces the same cross validation analysis.

Save Coding Table Creates a new data table whose first columns show the JMP coding for all model parameters. The last column shows the values of the response variable. If you specified a sample size column for a binomial response, both the response and sample size columns are shown in the table. If you used two response columns to specify censoring, both response columns are shown in the table. If you used a response column that contains a Detection Limits column property to specify censoring, the response column in the coding table contains a Detection Limits column property.

Note: The coding data table contains a table variable called Original Data that gives the name of the data table that was used for the analysis. In the case where a By variable is specified, the Original Data table variable also gives the By variable and its level.

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Save By-Group Script Contains options that enable you to save a script that reproduces the platform report for all levels of a By variable to several destinations. Available only when a By variable is specified in the launch window.

JMP PRO Model Launch Control Panel

In the Generalized Regression report, the Model Launch control panel provides options to specify the model:

- “Response Distribution”
- “Estimation Method Options”
- “Advanced Controls”

- "Validation Method Options"
- "Early Stopping"
- "Go"

JMP PRO Response Distribution

In the Generalized Regression control panel, the response distributions that are available depend on the attributes of the response variable that you specified in the Fit Model launch window. Appropriate response distributions are available in the Response Distribution option. See "Specify a Distribution" for more information about the response distributions.

JMP PRO Estimation Method Options

In the Generalized Regression control panel, the estimation methods that are available can be grouped into the following techniques:

- no selection and no penalty
- step-based model selection
- penalized regression

The Maximum Likelihood, Standard Least Squares, and Logistic Regression methods fit the entire model that is specified in the Fit Model launch window. No variable selection is performed. These models can serve as baselines for comparison to other methods.

Note: Only one of Maximum Likelihood, Standard Least Squares, and Logistic Regression is available for a given report. The name of this estimation method depends on the Distribution specified in the Fit Model launch window.

The Backward Elimination, Forward Selection, Pruned Forward Selection, Best Subset, and Two Stage Forward Selection methods are based on variables entering or leaving the model at each step. However, they do not impose a penalty on the regression coefficients.

The Dantzig Selector, Lasso, Elastic Net, Ridge, and Double Lasso methods are penalized regression techniques. They shrink the size of regression coefficients and reduce the variance of the estimates, in order to improve predictive ability of the model.

Note: When your data are highly collinear, the adaptive versions of Lasso and Elastic Net might not provide good solutions. This is because the adaptive versions presume that the MLE provides a good estimate. The Adaptive option is not recommended in such cases.

Two types of penalties are used in these techniques:

- the l_1 penalty, which penalizes the sum of the *absolute values* of the regression coefficients

- the l_2 penalty, which penalizes the sum of the *squares* of the regression coefficients

The default Estimation Method for observational data is the Lasso. If the data table contains a DOE script and no singularities, the default Estimation Method is Forward Selection with the Effect Heredity option enabled. If the data table contains a DOE script and a singularity in the design matrix, the default Estimation Method is Two-Stage Forward Selection with the Effect Heredity option enabled.

The following methods are available for model fitting:

Estimation Methods with No Selection and No Penalty

Maximum Likelihood Computes maximum likelihood estimates (MLEs) for model parameters. No penalty is imposed. Maximum Likelihood is the only estimation method available for Quantile Regression. If you specified a Validation column in the Fit Model launch window, the maximum likelihood model is fit to the Training set. A maximum likelihood model report appears by default, as long as the following conditions are met:

- There are no linear dependencies among the predictors.
- There are more observations than predictors.
- There are no more than 250 predictors.

The Maximum Likelihood option gives you a way to construct classical models for the response distributions supported by the Generalized Regression personality. In addition, a model based on maximum likelihood can serve as a baseline for model comparison.

When the specified Distribution is Normal or Binomial, the Maximum Likelihood method is called Standard Least Squares or Logistic Regression, respectively.

Standard Least Squares When the Normal distribution is specified, the Maximum Likelihood estimation method is replaced with the Standard Least Squares estimation method. The default report is a Standard Least Squares report that gives the usual standard least squares results.

Logistic Regression When the Binomial distribution is specified, the Maximum Likelihood estimation method is replaced with the Logistic Regression estimation method. The default report is a Logistic Regression report. The logistic results are identical to maximum likelihood results.

Step-Based Estimation Methods

Note: Step-based estimation methods are not available when the specified Distribution is Multinomial.

Backward Elimination Computes parameter estimates using backward elimination regression. The model chosen provides the best solution relative to the selected Validation

Method. Backward elimination starts by including all parameters in the model and removing one effect at each step until reaching the intercept-only model. At each step, the Wald tests for each parameter is used to determine which parameter is removed.

Caution: The horizontal axis of the Solution Path for Backward Elimination is the reverse of the same axis in other estimation methods. Therefore, as you move left to right in the Solution Path for the Backward Elimination estimation method, terms are being removed from the model, rather than added.

Forward Selection Computes parameter estimates using forward stepwise regression. At each step, the effect with the most significant score test is added to the model. The model chosen is the one that provides the best solution relative to the selected Validation Method.

When there are interactions and the Effect Heredity option is enabled, compound effects are handled in the following manner. If the effect with the most significant score test at a given step is one that would violate effect heredity, then a compound effect is created. The compound effect contains the effect with the most significant score test as well as any other inactive effects that are needed to satisfy effect heredity. If the compound effect has the most significant score test, then all of the effects in the compound effect are added to the model.

Pruned Forward Selection Computes parameter estimates using a mixture of forward and backward steps. The algorithm starts with an intercept-only model. At the first step, the effect with the most significant score test is added to the model. After the first step, the algorithm considers the following three possibilities at each step:

1. From the effects not in the model, add the effect that has the most significant score test.
2. From the effects in the model, remove the effect that has the least significant Wald test.
3. Do both of the above actions in a single step.

To choose the action taken at each step, the algorithm uses the specified Validation Method. For example, if the Validation Method is BIC, the algorithm chooses the action that results in the smallest BIC value. When there are interactions and the Effect Heredity option is enabled, compound effects are considered for adding effects, but they are not considered for removing effects.

When the model becomes saturated, the algorithm attempts a backward step to check if that improves the model. The maximum number of steps in the algorithm is 5 times the number of parameters. The model chosen is the one that provides the best solution relative to the selected Validation Method.

Pruned Forward Selection is an alternative to the Mixed Step option in the Stepwise Regression personality. However, it does not use the *p*-value to determine which variables enter or leave the model.

Tip: The Early Stopping option is not recommended for the Pruned Forward Selection Estimation Method.

Best Subset Computes parameter estimates by increasing the number of active effects in the model at each step. In each step, the model is chosen among all possible models with a number of effects given by the step number. The values on the horizontal axes of the Solution Path plots represent the number of active effects in the model. Step 0 corresponds to the intercept-only model. Step 1 corresponds to the best model of the ones that contain only one active effect. The steps continue up to the value of Max Number of Effects specified in the Advanced Controls in the Model Launch report. See "Advanced Controls".

Tip: The Best Subset Estimation Method is computationally intensive. It is not recommended for large problems.

Two Stage Forward Selection (Available only when there are second- or higher-order effects in the model.) Computes parameter estimates in two stages. In the first stage, a forward stepwise regression model is run on the main effects to determine which to retain in the model. In the second stage, a forward stepwise regression model is run on all of the higher-order effects that are composed entirely of the main effects chosen in the first stage. This method assumes strong effect heredity.

Terms that are not retained from the first stage still appear in the Parameter Estimates reports as zeroed terms. However, they are ignored in the fitting of the second stage model. Terms that are selected in the first stage are not forced into the second stage; they are available for selection in the second stage.

SVEM Forward Selection (Not available when the specified Distribution is Cox Proportional Hazards, Binomial, Beta Binomial, Multinomial, Ordinal Logistic, ZI Binomial, or ZI Beta Binomial.) Computes parameter estimates using the self-validated ensemble modeling (SVEM) method applied to forward selection. The number of individual models in the ensemble model is specified using the Samples option in the Model Launch control panel. See "Self-Validated Ensemble Models".

Penalized Estimation Methods

Dantzig Selector (Available only when the specified Distribution is Normal and the No Intercept option is not selected.) Computes parameter estimates by applying an l_1 penalty using a linear programming approach. See Candes and Tao (2007). The Dantzig Selector is useful for analyzing the results of designed experiments. For orthogonal problems, the Dantzig Selector and Lasso give identical results. See "Dantzig Selector".

Lasso Computes parameter estimates by applying an l_1 penalty. Due to the l_1 penalty, some coefficients can be estimated as zero. Thus, variable selection is performed as part of the fitting procedure. In the ordinary Lasso, all coefficients are equally penalized.

Adaptive Lasso Computes parameter estimates by penalizing a weighted sum of the absolute values of the regression coefficients. The weights in the l_1 penalty are determined by the data in such as way as to guarantee the oracle property (Zou 2006). This option uses the MLEs to weight the l_1 penalty. MLEs cannot be computed when the number of predictors exceeds the number of observations or when there are strict linear dependencies among the predictors. If MLEs for the regression parameters cannot be computed, a generalized inverse solution or a ridge solution is used for the l_1 penalty weights. See "Adaptive Methods".

The Lasso and the adaptive Lasso options generally choose parsimonious models when predictors are highly correlated. These techniques tend to select only one of a group of correlated predictors. High-dimensional data tend to have highly correlated predictors. For this type of data, the Elastic Net might be a better choice than the Lasso. See "Lasso Regression".

Elastic Net Computes parameter estimates by applying both an l_1 penalty and an l_2 penalty. The l_1 penalty ensures that variable selection is performed. The l_2 penalty improves predictive ability by shrinking the coefficients as ridge does.

Adaptive Elastic Net Computes parameter estimates using an adaptive l_1 penalty as well as an l_2 penalty. This option uses the MLEs to weight the l_1 penalty. MLEs cannot be computed when the number of predictors exceeds the number of observations or when there are strict linear dependencies among the predictors. If MLEs for the regression parameters cannot be computed, a generalized inverse solution or a ridge solution is used for the l_1 penalty weights. You can set a value for the Elastic Net Alpha in the Advanced Controls panel. See "Adaptive Methods".

The Elastic Net tends to provide better prediction accuracy than the Lasso when predictors are highly correlated. (In fact, both Ridge and the Lasso are special cases of the Elastic Net.) In terms of predictive ability, the adaptive Elastic Net often outperforms both the Elastic Net and the adaptive Lasso. The Elastic Net has the ability to select groups of correlated predictors and to assign appropriate parameter estimates to the predictors involved. See "Elastic Net".

Note: If you select an Elastic Net fit and set the Elastic Net Alpha to missing, the algorithm computes the Lasso, Elastic Net, and Ridge fits, in that order. If a fit is time intensive, a progress bar appears. When you click Accept Current Estimates, the calculation stops and the reported parameter estimates correspond to the best model fit at that point. The progress bar indicates when the algorithm is fitting Lasso, Elastic Net, and Ridge. You can use this information to decide when to click Accept Current Estimates.

Ridge Computes parameter estimates using ridge regression. Ridge regression is a biased regression technique that applies an l_2 penalty and does not result in zero parameter estimates. It is useful when you want to retain all predictors in your model. See "Ridge Regression".

Double Lasso Computes parameter estimates in two stages. In the first stage, a Lasso model is fit to determine the terms to be used in the second stage. In the second stage, a Lasso model is fit using the terms from the first stage. The Solution Path results and the parameter estimate reports that appear are for the second-stage fit. If none of the variables enters the model in the first stage, there is no second stage, and the results of the first stage appear in the report.

The Double Lasso is especially useful when the number of observations is less than the number of predictors. By breaking the variable selection and shrinkage operations into two stages, the Lasso in the second stage is less likely to overly penalize the terms that should be included in the model. The double lasso is similar to the relaxed lasso. The relaxed lasso is described in Hastie et al. (2009, p. 91).

Adaptive Double Lasso Computes parameter estimates in two stages. In the first stage, an adaptive Lasso model is fit to determine the terms to be used in the second stage. In the second stage, an adaptive Lasso model is fit using the terms from the first stage. The second stage considers only the terms that are included in the first stage model and uses weights based on the parameter estimates in the first stage. You can choose the method of calculating the weights using the Adaptive Penalty Weights option in the Advanced Controls. See "Advanced Control Options". The results that are shown are for the second-stage fit. If none of the variables enters the model in the first stage, there is no second stage, and the results of the first stage appear in the report. See "Adaptive Methods".

SVEM Lasso (Not available when the specified Distribution is Cox Proportional Hazards, Binomial, Beta Binomial, Multinomial, Ordinal Logistic, ZI Binomial, or ZI Beta Binomial.) Computes parameter estimates using the self-validated ensemble modeling (SVEM) method applied to the Lasso model. The number of individual models in the ensemble model is specified using the Samples option in the Model Launch control panel. See "Self-Validated Ensemble Models".

JMP PRO Advanced Controls

In the Generalized Regression control panel, use the Advanced Controls options to adjust various aspects of the model fitting process. A number of controls relate to the grid for the tuning parameter.

Tuning Parameter

The solution paths for the Lasso and Ridge Estimation Methods depend on a single tuning parameter. The solution path for the Elastic Net depends on a tuning parameter for the penalty on the likelihood as well as the Elastic Net Alpha. The penalty on the likelihood for the Elastic Net is a weighted sum of the penalties associated with the Lasso and Ridge Estimation Methods. The Elastic Net Alpha determines the weights of these two penalties. See "Statistical Details for Estimation Methods" and "Statistical Details for Advanced Controls".

When the tuning parameter is zero, the solution is unpenalized and maximum likelihood estimates are obtained. As the tuning parameter increases, the penalty increases.

The solution is the set of parameter estimates that minimizes the penalized negative log-likelihood function relative to the selected validation method. The current solution is designated by the solid red vertical line in the Solution Path Plots.

Note: The value of the tuning parameter increases as the Magnitude of Scaled Parameter Estimates in the Solution Path Plot decreases. Estimates close to the MLE are associated with large magnitudes and estimates that are heavily penalized are associated with small magnitudes.

It is important to be mindful of the following:

- When the tuning parameter is too small, the data are typically overfit and result in models with high variance.
- When the tuning parameter is too large, the data are typically underfit.

The Tuning Parameter Grid

To obtain a solution, the tuning parameter is increased over a fine grid.

- For the Lasso, Elastic Net with Elastic Net Alpha specified, and Ridge, the value of the tuning parameter that gives the solution is the one that provides the best fit over the grid of tuning parameters.

  Note: Elastic Net Alpha is set to 0.99 by default.

- If you do not set a value for the Elastic Net Alpha, the value of alpha is also increased over a fine grid. For a fixed value of the tuning parameter, alpha is varied until ten consecutive values of alpha fail to improve upon the best fit as determined by the validation method. This process is repeated for the entire grid of tuning parameter values. The final values of the tuning parameter and alpha are the values that provide the best fit over the grid of tuning parameters.

The grid of tuning parameter values ranges from zero, in most cases, to the smallest value for which all of the non-intercept terms are zero. Define the smallest value of the tuning parameter for which all non-intercept terms are zero to be its *upper bound*. The *lower bound* for the tuning parameter is zero except in the following two cases where it is set to 0.0001:

- If the design matrix is singular, the maximum likelihood estimates cannot be computed. The lower bound of 0.0001 allows estimates close to the MLEs to be computed.
- If the selected distribution is binomial or multinomial, the lower bound of 0.0001 helps prevent separation.

Advanced Control Options

Enforce effect heredity Requires lower-order effects to enter the model before their related higher order effects. In most cases, this means that X^2 is not in the model unless X is in the model. For estimation methods other than Forward Selection, however, it is possible for X^2 to enter the model and X to leave the model in the same step. If the data table contains a DOE script, this option is enabled, but it is off by default.

Elastic Net Alpha Sets the α parameter for the Elastic Net. This α parameter determines the mix of the l_1 and l_2 penalty tuning parameters in estimating the Elastic Net coefficients. The default value is $\alpha = 0.99$, which sets the coefficient on the l_1 penalty to 0.99 and the coefficient on the l_2 penalty to 0.01. This option is available only when Elastic Net is selected as the Estimation Method. See "Statistical Details for Estimation Methods".

Number of Grid Points Specifies the number of grid points between the lower and upper bounds for the tuning parameter. At each grid point value, parameter estimates for that value of the tuning parameter are obtained. The default value is 150 grid points.

Minimum Penalty Fraction Indicates the minimum value for the ratio of the lower bound of the tuning parameter to its upper bound. When the lower bound for the tuning parameter is 0, the solution provides the MLE. In cases where you do not want to include the MLE or solutions very close to it, you can set the Minimum Penalty Fraction to a nonzero value. For the Double Lasso estimation method, the specified value of this option is used only in the first stage of the fit. When there is a singularity in the design matrix, the default value is 0.0001. Otherwise, the default value is 0.

Grid Scale Provides options for choosing the distribution of the grid scale. You can choose between a linear, square root, or log scale. Grid points equal in number to the specified Number of Grid Points are distributed according to the selected scale between the lower and upper bounds of the tuning parameter. The default grid scale is square root. See "Statistical Details for Advanced Controls".

First Stage Solution Provides options for choosing the solution in the first stage of the Double Lasso and Two Stage Forward Selection. By default, the solution that is the best fit according to the specified Validation Method is selected and is the solution initially shown

(Best Fit). You can choose to initially display models with larger or smaller l_1 norm values that lie in the green or yellow zones. For example, if you choose Smallest in Yellow Zone, the initially displayed solution is the model in the yellow zone that has the smallest l_1 norm. See "Comparable Model Zones".

Max Number of Effects Specifies the maximum number of effects to consider in models for the Best Subset estimation method. You can use this to limit the number of computations needed to fit the model. The default value is 10.

Initial Displayed Solution Provides options for choosing the solution that is initially displayed as the current model in the Solution Path report. The current model is identified by a solid vertical line. See "Current Model Indicator". The best fit solution is identified by a dotted vertical line. By default, the displayed solution is the one that is considered the best fit according to the specified Validation Method.

You can choose to initially display models with larger or smaller l_1 norm values that still lie in the green or yellow zones. For example, if you choose Smallest in Yellow Zone, the initially displayed solution is the model in the yellow zone that has the smallest l_1 norm. See "Comparable Model Zones".

Adaptive Penalty Weights Provides options for the calculation of the penalty weights that are used in the second stage of the Adaptive Double Lasso. By default, the Inverse Solution option is selected. This option calculates the penalty weights using the parameter estimates from the first stage fit.

The Inverse Model Average option calculates the penalty weights using the parameter estimates from a solution that is the weighted average of the AICc or BIC models. The AICc models are used if the AICc Validation Method is selected. Otherwise, the BIC models are used. If you use the Inverse Model Average option, the maximum likelihood solution, if it exists, appears as the right-most point in the Solution Path for the Adaptive Double Lasso model.

Force Terms Enables you to select which terms, if any, you want to force into the model. The terms that are forced into the model are not included in the penalty.

JMP PRO Validation Method Options

The following methods are available for validation of the Generalized Regression model fit.

KFold For each value of the tuning parameter, the following steps are conducted:

- The observations are partitioned into k subsets, or *folds*.
- In turn, each fold is used as a validation set. A model is fit to the observations *not* in the fold. The log-likelihood based on that model is calculated for the observations *in* the fold, providing a *validation* log-likelihood.

– The mean of the validation log-likelihoods for the *k* folds is calculated. This value serves as a validation log-likelihood for the value of the tuning parameter.

The value of the tuning parameter that has the maximum validation log-likelihood is used to construct the final solution. To obtain the final model, all *k* models derived for the optimal value of the tuning parameter are fit to the entire data set. Of these, the model that has the highest validation log-likelihood is selected as the final model. The training set used for that final model is designated as the Training set and the holdout fold for that model is the Validation set. These are the Training and Validation sets used in plots and in the reported results for the final solution.

Holdback Randomly selects the specified proportion of the data for a validation set, and uses the other portion of the data to fit the model. The final solution is the one that minimizes the negative log-likelihood for the validation set. This method is useful for large data sets. The random selection is based on stratified sampling across the model factors to attempt to create training and validation sets that are more balanced than ones based on simple random sampling.

Leave-One-Out Performs leave-one-out cross validation. This is equivalent to KFold, with the number of folds equal to the number of rows. This option should not be used on moderate or large data sets. It can require long processing time for even a moderate number of observations. The Training and (one-row) Validation sets used in plots and in the reported results for the final solution are determined as is done for KFold validation.

BIC Minimizes the Bayesian Information Criterion (BIC) over the solution path. See "Likelihood, AICc, and BIC".

AICc Minimizes the corrected Akaike Information Criterion (AICc) over the solution path. AICc is the default setting for Validation Method. See "Likelihood, AICc, and BIC".

Note: The AICc is not defined when the number of parameters approaches or exceeds the sample size.

ERIC Minimizes the Extended Regularized Information Criterion (ERIC) over the solution path. See "Model Fit Detail". Available only for exponential family distributions and for the Lasso and adaptive Lasso estimation methods.

None Does not use validation. Available only for the Maximum Likelihood Estimation Method and Quantile Regression.

Validation Column Uses the column specified in the Fit Model window as having the Validation role. The final solution is the one that minimizes the negative log-likelihood for the validation set. This option is not available when the specified Estimation Method is Dantzig Selector or when the specified Distribution is Quantile Regression or Cox Proportional Hazards.

Note: The only Validation Method allowed for Quantile Regression is None. The only Validation Methods allowed for the Maximum Likelihood Estimation Method are None and Validation Column. The only Validation Methods allowed for Cox Proportional Hazards are BIC, AICc, and None. The only Validation Methods allowed for the Dantzig Selector Estimation Method are BIC and AICc. The Validation Method option is not available for the SVEM Forward Selection or SVEM Lasso Estimation Methods.

JMP PRO Early Stopping

Early Stopping in the Generalized Regression control panel adds an early stopping rule:

- For Forward Selection, the algorithm terminates when 10 consecutive steps of adding variables to the model fail to improve upon the validation measure. The solution is the model at the step that precedes the 10 consecutive steps.
- For Lasso, Elastic Net, and Ridge, the algorithm terminates when 10 consecutive values of the tuning parameter fail to improve upon the best fit as determined by the validation method. The solution is the estimate corresponding to the tuning parameter value that precedes the 10 consecutive values.

Note: For the AICc and BIC validation methods, early stopping does not occur until at least four predictors have entered the model.

JMP PRO Go

When you click Go in the Generalized Regression control panel, a report opens. The title of the report specifies the response distribution, the estimation method, and the validation method that you selected. You can return to the Model Launch control panel to perform additional analyses and choose other response distributions, estimation methods, and validation methods.

JMP PRO Model Fit Reports

In the Generalized Regression control panel, a report is produced for each Estimation Method and Validation Method that you specify. The report specifies your selected Distribution, Estimation method, and Validation method in its title.

The following reports are presented by default:

- "Regression Plot"
- "Model Summary"
- "Estimation Details"
- "Solution Path"
- "Parameter Estimates for Centered and Scaled Predictors"
- "Parameter Estimates for Original Predictors"
- "Active Parameter Estimates"
- "Effect Tests"

JMP PRO Regression Plot

In the Generalized Regression report, the Regression Plot section shows a plot of the response values on the vertical axis and the continuous predictor on the horizontal axis. A regression line is shown over the points. If there is a categorical predictor in the model, each level of the categorical predictor has a separate regression line and a legend appears next to the plot. If the response is specified as the counts of the number of successes and the number of trials, the number of successes divided by the number of trials is plotted on the vertical axis.

Note: The Regression Plot report appears only when there is one continuous predictor and no more than one categorical predictor. It is not available if the Distribution option is set to Multinomial, Ordinal Logistic, or Cox Proportional Hazards. The response must be continuous.

JMP PRO Model Summary

In the Generalized Regression report, the Model Summary section describes the model that you have fit and provides summary information about the fit itself.

JMP PRO Model Description Detail

The first part of the Model Summary report gives information that describes the model that you have fit.

Response The column assigned to the Y role in the Fit Model window. When two columns are used to specify interval censoring, both column names are listed.

Distribution The Distribution selected in the Fit Model window. For Quantile Regression, the value of the specified quantile for the response is also displayed.

Estimation Method The Estimation Method selected in the Model Launch panel.

Validation Method The Validation Method selected in the Model Launch panel.

Mean Model Link The link function for the model for the mean, based on the Distribution selected in the Fit Model window.

Location Model Link The link function for the model for the location parameter, shown when either Cauchy or t(5) is selected as the Distribution in the Fit Model window.

Scale Model Link The link function for the model for the scale parameter, based on the Distribution selected in the Fit Model window.

Probability Model Link The link function for the model for the probability, based on the Distribution selected in the Fit Model window.

Dispersion Model Link The link function for the model for the dispersion parameter, based on the Distribution selected in the Fit Model window.

Zero Inflation Model Link The link function for the model for the zero inflation parameter, based on the Distribution selected in the Fit Model window.

Lower Detection Limit The lower detection limit specified in a Detection Limits column property assigned to the response column.

Upper Detection Limit The upper detection limit specified in a Detection Limits column property assigned to the response column.

Censor Column The column assigned to the Censor role in the Fit Model window.

Censor Code The value in the Censor column that designates right-censored observations. This is the value that was specified in the Censor Code list in the Fit Model window.

JMP PRO Model Fit Detail

The second part of the Model Summary report gives statistics related to the model fit. If either Holdback or Validation Column is selected as the Validation Method, these statistics are computed separately for the training and validation sets. This part of the Model Summary report is not available if either KFold or Leave-One-Out is selected as the Validation Method.

Number of rows The number of rows.

Sum of Frequencies The sum of the values of a column assigned to the Freq or Weight role in the Fit Model window.

Note: For -LogLikelihood, BIC, AICc, and ERIC, smaller is better. See "Likelihood, AICc, and BIC".

-LogLikelihood The negative of the natural logarithm of the likelihood function for the current model. See "Likelihood, AICc, and BIC".

Note: -LogLikelihood is not available for Quantile Regression.

Objective Function (Available only for Quantile Regression.) The value of the function that is minimized to fit the specified quantile regression model. The function that is minimized is the check-loss function.

Number of Parameters The number of nonzero parameters in the current model.

BIC The Bayesian Information Criterion, which is defined as follows:

$$\text{BIC} = -2LogLikelihood + k\ln(n)$$

See "Likelihood, AICc, and BIC".

AICc The corrected Akaike Information Criterion, which is defined as follows:

$$\text{AICc} = -2LogLikelihood + 2k + 2k(k+1)/(n-k-1)$$

See "Likelihood, AICc, and BIC".

ERIC (Available only for exponential family distributions and when the Lasso or adaptive Lasso estimation method is specified.) The Extended Regularization Information Criterion. See Hui et al. (2015). ERIC is defined as follows:

$$\text{ERIC} = -2LogLikelihood + (k-2)\ln(n\phi/\lambda)$$

where λ is the value of the tuning parameter and ϕ is the nuisance parameter.

Generalized RSquare (Not available for Quantile Regression.) An extension of the RSquare measure that can be applied to general regression models. Generalized RSquare compares the likelihood of the fitted model (L_M) to the likelihood of the intercept-only (constant) model (L_0). It is scaled to have a maximum of 1. For distributions other than Binomial, the Generalized RSquare is defined as follows:

$$\text{Generalized RSquare} = 1 - (L_0/L_M)^{2/n}$$

When Binomial is the specified distribution, the Generalized RSquare is defined as follows:

$$\text{Generalized RSquare} = \frac{1 - (L_0/L_M)^{2/n}}{1 - L_0^{2/n}}$$

A Generalized RSquare value of 1 indicates a perfect model; a value of 0 indicates a model that is no better than a constant model. The Generalized RSquare measure simplifies to the traditional RSquare for continuous normal responses in the standard least squares setting. Generalized RSquare is also known as the Nagelkerke or Craig and Uhler R^2, which is a normalized version of Cox and Snell's pseudo R^2. See Nagelkerke (1991).

Note: Generalized RSquare is replaced by RSquare when the Normal distribution is specified.

Caution: You should not compare Generalized RSquare values for models that use different response distributions. The comparison being made is to the intercept-only model with a given response distribution.

RSquare (Available only when the Normal distribution is specified.) Estimates the proportion of variation in the response that can be attributed to the model rather than to random error. An RSquare value of 1 indicates a perfect model; a value of 0 indicates a model that is no better than a constant model. The RSquare value is calculated as follows:

$$1 - \frac{\sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{N} (Y_i - \bar{Y})^2}$$

RSquare Adj (Available only when the Normal distribution is specified and the estimation method does not involve a penalty.) Adjusts the RSquare statistic for the number of

parameters in the model. Rsquare Adj facilitates comparisons among models with different numbers of parameters. The computation uses the degrees of freedom. The RSquare Adj value is calculated as follows:

$$1 - \frac{\frac{1}{N-(p-1)} \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2}{\frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \bar{Y})^2}$$

where N is the number of observations and p is the number of parameters.

Note: When there is a Validation set, the adjusted RSquare statistic is reported only for the Training set.

RASE (Available only when the Normal distribution is specified.) The Root Average Square Error (RASE) is the square root of the mean squared prediction error in the current model. See "RASE".

Lambda Penalty (Available only for the Dantzig Selector, Lasso, Elastic Net, Ridge, and Double Lasso estimation methods.) The value of the tuning parameter λ for the current model. See "Statistical Details for Estimation Methods".

JMP PRO Estimation Details

In the Generalized Regression report, the Estimation Details section shows the settings of the Advanced Controls for the Dantzig Selector, Lasso, Elastic Net, Ridge, and Double Lasso estimation methods. For more information about these controls, see "Advanced Controls".

JMP PRO Solution Path

In the Generalized Regression report, the Solution Path section shows two plots:

- The Solution Path Plot displays values of the estimated parameters.
- The Validation Plot displays values of the validation statistic corresponding to the selected validation method.

Note: The Solution Path report appears for all Estimation Methods except Maximum Likelihood, SVEM Forward Selection, and SVEM Lasso. The Solution Path report appears for all Distributions except Quantile Regression.

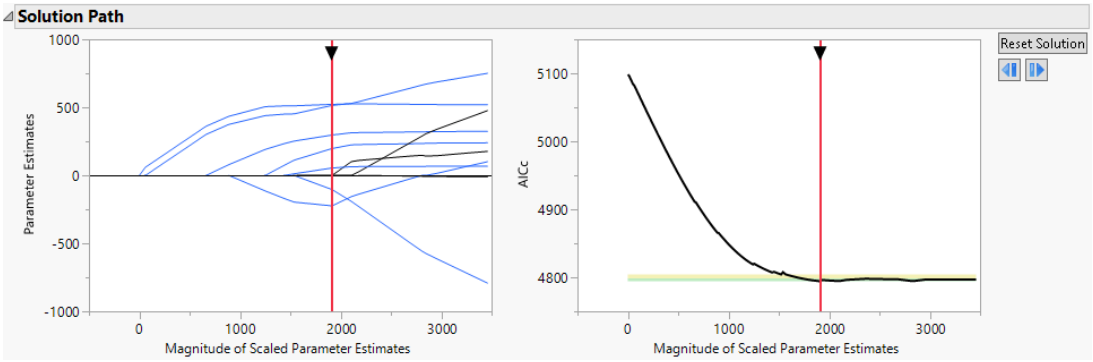
The horizontal scaling for both plots is given in terms of the Magnitude of Scaled Parameter Estimates. This is the l_1 norm, defined as the sum of the absolute values of the scaled parameter estimates for the model for the mean. (Estimates corresponding to the intercept, dispersion parameters, and zero-inflation parameters are excluded from the calculation of the l_1 norm.) Note the following:

- Estimates with large values of the l_1 norm are close to the MLE.
- Estimates with small values of the l_1 norm are heavily penalized.
- The value of the tuning parameter increases as the l_1 norm decreases.

JMP PRO Current Model Indicator

A solid vertical red line is placed in both plots at the value of the l_1 norm for the solution displayed in the Parameter Estimates for Original Predictors report. You can drag the arrow at the top of the vertical red line in either plot to change the magnitude of the penalty, indicating a new current model. In the Validation Plot, you can also click anywhere in the plot to change the model. As you drag the vertical red line to indicate a new model, the results in the report update to reflect the currently selected model. A dashed vertical line remains at the best fit model. You can click the **Reset Solution** button next to the Validation Plot to return the vertical red line and corresponding results to the initial solution. For some validation methods, the Validation Plot provides zones that identify comparable models. See "Comparable Model Zones".

Figure 6.5 Solution Path Report for Diabetes.jmp, Lasso with AICc Validation



For more information about the Solution Path Plot, see "Solution Path Plot". For more information about the Validation Plot, see "Validation Plot".

JMP PRO Solution Path Plot

You can select paths in the Solution Path Plot to highlight the corresponding terms in the Parameter Estimates reports. This action also selects the corresponding columns in the data table. Selecting rows in either of the reports highlights the corresponding rows in the other report and the corresponding paths in the Solution Path Plot. Press Shift and click to select multiple paths or rows.

The Parameter Estimates are plotted using the vertical axis of the Solution Path Plot. These are the *scaled* parameter estimates. They are derived for a model expressed in terms of centered and scaled predictors. See "Parameter Estimates for Centered and Scaled Predictors".

When the number of predictors is less than the number of observations, the Solution Path Plot usually shows the entire range of estimates from zero to the unpenalized fit given by the MLE. Otherwise, the plot extends to a magnitude that is close to the unpenalized solution. This occurs when the jump from the next-to-last grid point to the MLE solution is so large that the detail for solutions up to the next-to-last grid point is obscured. When this happens, as long as the MLE is not the final solution, the Solution Path Plot is rescaled so that the axis extends only to the next-to-last grid point.

JMP PRO The Solution ID

Internally, each solution in the Solution Path is assigned a Solution ID. When you adjust the tuning parameter to select a solution other than the one initially presented, the corresponding Solution ID appears in scripts created by the Save Script options. The Solution ID is the value N in the `Set Solution ID( N )` command. Saving the Solution ID ensures that you can re-create your selected solution when you run the script.

JMP PRO Validation Plot

The Validation Plot shows plots of statistics that describe how well models fit across the values of the tuning parameter, or equivalently, across the values of the Magnitude of the Scaled Parameter Estimates. The statistics plotted depend on the selected Validation Method. For each Validation Method, Table 6.3 lists the statistic that is plotted. For all validation methods, smaller values are better. For the KFold and Leave-One-Out validation methods, and for a Validation Column with more than three values, the statistic that is plotted is the mean of the scaled negative log-likelihood values across the folds.

The *Scaled -LogLikelihood* in Table 6.3 is the negative log-likelihood divided by the number of observations in the set for which the negative log-likelihood is computed.

Table 6.3 Validation Methods and Corresponding Validation Statistics and Zones

Validation Method	Validation Statistic	Tuning Parameter Regions
KFold	Mean of the Scaled -LogLikelihood values across the *K* folds	Two
Holdback	Scaled -LogLikelihood	None
Leave-One-Out	Mean of the Scaled -LogLikelihood values across all folds	Two
BIC	BIC for training data	Two
AICc	AICc for training data	Two
ERIC	ERIC for training data	Two
Validation Column with two or three values	Scaled -LogLikelihood	None
Validation Column with $K > 3$ values	Mean of the Scaled -LogLikelihood values across the *K* folds	Two

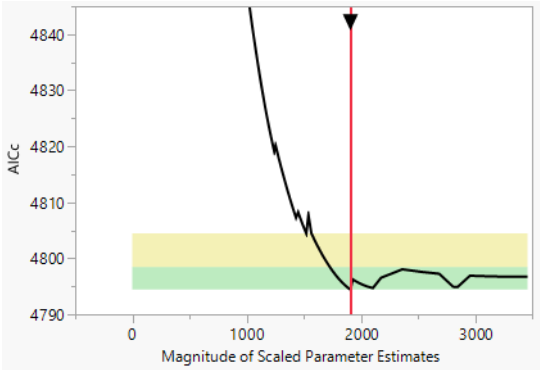
JMP PRO Comparable Model Zones

Although a model is estimated to be the best model, there can be uncertainty relative to this selection. Competing models might fit nearly as well and can contain useful information. For the AICc, BIC, ERIC, KFold, and Leave-One-Out validation methods, and for a Validation Column with more than three values, the Validation Plot provides zones that identify competing models that might deserve consideration. Models that fall outside the zones are not recommended. See Burnham and Anderson (2004) and Burnham et al. (2011).

A zone is an interval of values of the validation statistics. The zones are plotted as green or yellow rectangles that span the entire horizontal axis. A model falls in a zone if the value of its validation statistic falls in the zone. You can drag the solid vertical red line to explore solutions within the zones. See "Current Model Indicator".

Figure 6.6 shows a Validation Plot for Diabetes.jmp with the vertical axis expanded to show the two zones.

Figure 6.6 Validation Plot for Diabetes.jmp, Lasso with AICc Validation



Zones for BIC, AICc, and ERIC Validation

For these validation methods, two regions are shown in the plot. Denote the validation BIC, AICc, and ERIC values for the best solutions by V^{best}.

- The green zone identifies models for which there is strong evidence that a model is comparable to the best model. The green zone is the interval $[V^{best}, V^{best}+4]$.
- The yellow zone identifies models for which there is weak evidence that a model is comparable to the best model. The yellow zone is the interval $(V^{best}+4, V^{best}+10]$.

Zones for KFold Validation, Leave-One-Out Validation, and Validation Column with More Than Three Values

For these validation methods, two regions are shown in the plot. At the solution for the best model, the scaled negative log-likelihood functions are evaluated for each validation set. Denote the standard error of these values as L^{SE}. Denote the scaled negative log-likelihood for the best solution by L^{best}.

- The green zone identifies models for which there is strong evidence that a model is comparable to the best model. The green zone is the interval $[L^{best}, L^{best}+L^{SE}]$.
- The yellow zone identifies models for which there is weak evidence that a model is comparable to the best model. The yellow zone is the interval $(L^{best}+L^{SE}, L^{best}+2.5*L^{SE}]$.

JMP PRO Parameter Estimates for Centered and Scaled Predictors

In the Generalized Regression report, the Parameter Estimates for Centered and Scaled Predictors section gives estimates and other results for all parameters in the model. The initial table includes the coefficients for the predictors in the model. An additional table includes other model parameters such as scale, dispersion, or zero inflation parameters. See "Specify a Distribution". Both tables include the same columns of results.

Tip: You can click terms in the Parameter Estimates for Centered and Scaled Predictors report to highlight the corresponding paths in the Solution Path Plot. The corresponding columns in the data table are also selected. This is useful in terms of running further analyses. Press Shift and click the terms to select multiple rows.

For all fits in the Generalized Regression personality, every predictor is centered to have mean zero and scaled to have sum of squares equal to one:

- The mean is subtracted from each observation.
- Each difference is then divided by the square root of the sum of the squared differences from the mean.

This puts all predictors on an equal footing relative to the penalties applied.

Note: When the No Intercept option is selected in the launch window, the predictors are not centered and scaled.

The Parameter Estimates for Centered and Scaled Predictors report gives parameter estimates for the model expressed in terms of the centered and scaled predictors. The estimates are determined by the Validation Method that you specified. The estimates are depicted in the Solution Path Plots by a vertical red line.

The following information is provided:

Term A list of the model terms. "Forced in" appears next to any terms that were forced into the model using the Advanced Controls option.

Estimate The parameter estimate corresponding to the centered and scaled model term.

Std Error The standard error of the estimate. This is obtained using M-estimation and a sandwich formula (Zou 2006; Huber and Ronchetti 2009).

t **Ratio or Wald ChiSquare** The test statistic of whether the true value of each parameter is zero. If the specified Estimation Method is not a penalized regression method, the Normal distribution is specified, and no censoring is specified, the t Ratio column appears. Otherwise, the Wald ChiSquare column appears. See "Estimation Method Options".

Prob>|t| or Prob > ChiSquare The *p*-value for the test that the true parameter value is zero, against the two-sided alternative that it is not. If the specified Estimation Method is not a penalized regression method, the Normal distribution is specified, and no censoring is specified, the Prob>|t| column appears. Otherwise, the Prob > ChiSquare column appears. See "Estimation Method Options".

Lower 95% The lower bound for a 95% confidence interval for the parameter. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Upper 95% The upper bound for a 95% confidence interval for the parameter. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Singularity Details (Available only if there are linear dependencies among the model terms.) The linear function that the model term satisfies.

JMP PRO Parameter Estimates for Original Predictors

In the Generalized Regression report, the Parameter Estimates for Original Predictors section gives estimates and other results for all parameters in the model. The initial table includes the coefficients for the predictors in the model. An additional table includes other model parameters such as scale, dispersion, or zero inflation parameters. See "Specify a Distribution". Both tables include the same columns of results.

Tip: You can click terms in the Parameter Estimates for Original Predictors report to highlight the corresponding paths in the Solution Path Plot. The corresponding columns in the data table are also selected. This is useful when running further analyses. Press Shift and click the terms to select multiple rows.

The Parameter Estimates for Original Predictors report gives parameter estimates for the model expressed in terms of the original (uncentered and unscaled) predictors.

The report provides the following information:

Term A list of the model terms. "Forced in" appears next to any terms that were forced into the model using the Advanced Controls option.

Estimate The parameter estimate corresponding to the model term given in terms of the original measurements.

Std Error The standard error of the estimate. This is obtained using M-estimation and a sandwich formula (Zou 2006 and Huber and Ronchetti 2009).

t **Ratio or Wald ChiSquare** The test statistic of whether the true value of each parameter is zero. If the specified Estimation Method is not a penalized regression method, the Normal distribution is specified, and no censoring is specified, the t Ratio column appears. Otherwise, the Wald ChiSquare column appears. See "Estimation Method Options".

Prob>|t| or Prob > ChiSquare The *p*-value for the test that the true parameter value is zero, against the two-sided alternative that it is not. If the specified Estimation Method is not a penalized regression method, the Normal distribution is specified, and no censoring is specified, the Prob>|t| column appears. Otherwise, the Prob > ChiSquare column appears. See "Estimation Method Options".

Lower 95% The lower bound for a 95% confidence interval for the parameter. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Upper 95% The upper bound for a 95% confidence interval for the parameter. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Singularity Details (Available only if there are linear dependencies among the model terms.) The linear function that the model term satisfies.

VIF (Available only when the distribution is Normal. Appears only if you right-click in the parameter estimates table and select Columns > VIF.) The variance inflation factor (VIF) for each term in the model. High VIF values indicate a collinearity issue among the terms in the model.

The VIF for the i^{th} term, x_i, is calculated as follows:

$$VIF_i = \frac{1}{1 - R_i^2}$$

where R_i^2 is the RSquare for the regression of x_i as a function of the other explanatory variables.

JMP PRO Active Parameter Estimates

In the Generalized Regression report, the Active Parameter Estimates section gives a subset of the Parameter Estimates for Original Predictors report. The Active Parameter Estimates report shows only the nonzero parameters.

JMP PRO Effect Tests

In the Generalized Regression report, the Effect Tests section contains the following information about the model effects:

Source A list of the effects in the model.

Nparm The number of parameters associated with the effect.

DF The degrees of freedom for the Wald ChiSquare test. This is the number of nonzero parameter estimates associated with the effect in the model.

Wald ChiSquare The ChiSquare value for a Wald test of whether all parameters associated with the effect are zero.

Prob > ChiSquare The *p*-value for the Wald ChiSquare test.

The following columns appear in the report in place of Wald ChiSquare and Prob > ChiSquare when the Distribution is Normal and there is no penalty in the estimation method:

Sum of Squares The sum of squares for the hypothesis that the effect is zero.

F Ratio The F statistic for testing that the effect is zero. The F Ratio is the ratio of the mean square for the effect divided by the mean square for error. The mean square for the effect is the sum of squares for the effect divided by its degrees of freedom.

Prob > F The *p*-value for the effect test.

If the coefficient for an effect has been estimated as zero, then:

- If the effect has one degree of freedom, the word "Removed" appears at the far right in the row for that effect.
- If the effect has multiple degrees of freedom, the phrase "Levels removed" appears, followed by the number of levels that correspond to terms with parameter estimates of zero.

JMP PRO Model Fit Options

In the Generalized Regression report, each model fit has a red triangle menu that contains these options:

Caution: Many options in the platform are not available if you specify a column that has the Expression data type or Vector modeling type in the launch window. For SVEM model fit options, see "Model Fit Options for Self-Validated Ensemble Models".

Regression Reports Enables you to customize the reports that are shown for the specified model fit. All of the following reports are shown by default except for the Parameter Estimates for Centered and Scaled Parameter Estimates report and the Active Parameter Estimates report.

Model Summary Shows or hides the Model Summary report that includes information about the specification and goodness of fit statistics for the model. This option also displays the Estimation Details report for applicable models. See "Model Summary" and "Estimation Details".

Solution Path (Not available for Maximum Likelihood models.) Shows or hides the Solution Path and Validation Path plots. See "Solution Path".

Parameter Estimates for Centered and Scaled Predictors Shows or hides a table of centered and scaled parameter estimates. See "Parameter Estimates for Centered and Scaled Predictors".

Parameter Estimates for Original Predictors Shows or hides a table of parameter estimates in the original scale of the data. See "Parameter Estimates for Original Predictors".

Active Parameter Estimates (Not available for Maximum Likelihood or Ridge Regression models.) Shows or hides a table of active, or nonzero, parameter estimates for the currently selected model.

Show Solution Path Summary (Not available for Maximum Likelihood or Ridge Regression models.) Shows or hides a report that contains a table of fit statistics for the points on the Solution Path and Validation Path plots where the active set changes. The statistics that are available depend on the estimation method. For more information about the conditional model probabilities that are available for Normal Lasso with BIC Validation models, see Hu et al. (2019). When BIC, AICc, or ERIC Validation is specified, the cells in the BIC and AICc columns are colored in the same manner as the Validation Plot. See "Comparable Model Zones".

Effect Tests Shows or hides tests for each effect. The effect test for a given effect tests the null hypothesis that all parameters associated with that effect are zero. A nominal or ordinal effect can have several associated parameters, based on its number of levels. The effect test for such an effect tests whether all of the associated parameters are zero. When the Distribution is Multinomial, the effects are combined over the levels of the response. See "Effect Tests".

Show Prediction Expression Shows or hides the Prediction Expression report that contains the equation for the estimated model. See "Show Prediction Expression" for an example.

Select Nonzero Terms (Not available when the specified Estimation Method is Ridge Regression.) Highlights terms with nonzero coefficients in the report. Also selects all associated columns in the data table.

Select Zeroed Terms (Not available when the specified Estimation Method is Ridge Regression.) Highlights terms with zero coefficients in the report. Also selects all associated columns in the data table.

Relaunch Active Set (Not available for models that contain a predictor that has the Vector modeling type.) Contains options that open a Fit Model launch window where the Construct Model Effects list contains a set of terms based on the terms that have nonzero parameter estimates. These terms are the *active* effects. All other specifications in the launch window are those used in the original analysis.

Note: If you select any of the Relaunch Active Set options in a report that contains a By variable, the By variable is not added to the Fit Model launch window.

Relaunch with Active Effects Populates the Construct Model Effects list only with the active effects.

Relaunch Active Main Effects and Second Degree Factorial Populates the Construct Model Effects list with a second degree factorial constructed with the active effects.

Relaunch Active Main Effects and Third Degree Factorial Populates the Construct Model Effects list with a third degree factorial constructed with the active effects.

Relaunch Active Main Effects and Full Factorial Populates the Construct Model Effects list with a full factorial constructed with the active effects.

Relaunch Active Main Effects and Second Degree Polynomial Populates the Construct Model Effects list with a second degree polynomial constructed with the active effects.

Relaunch Active Main Effects and Third Degree Polynomial Populates the Construct Model Effects list with a third degree polynomial constructed with the active effects.

Relaunch Active Main Effects and Response Surface Model Populates the Construct Model Effects list with a response surface model constructed with the active effects.

Hide Inactive Paths Adjusts the transparency of the inactive paths in the Solution Path Parameter Estimates plot so that the paths that are not currently active appear faded.

Odds Ratios (Available only when the specified Distribution is Binomial and the model contains an intercept. Not available for models that contain a predictor that has the Vector modeling type.) Shows or hides a report that contains odds ratios for categorical predictors, and unit odds ratios and range odds ratios for continuous predictors. An *odds ratio* is the ratio of the odds for two events. The *odds* of an event is the probability that the event of interest occurs versus the probability that it does not occur. The event of interest is defined by the Target Level in the Fit Model launch window.

For each categorical predictor, an Odds Ratios report appears. Odds ratios are shown for all combinations of levels of a categorical model term.

If there are continuous predictors, two additional reports appear:

– Unit Odds Ratios Report. The *unit odds ratio* is calculated over a one-unit change in a continuous model term.

– Range Odds Ratios Report. The *range odds ratio* is calculated over the entire range of a continuous model term.

The confidence intervals in the Odds Ratios report are Wald-based intervals. Note that the odds ratio for a model term is meaningful only if the model term is not involved in any higher-order effects.

Note: If there are interactions in the model, you can use the Multiple Comparisons option to obtain odds ratios. See "Multiple Comparisons".

Incidence Rate Ratios (Available only when the specified Distribution is Poisson or Negative Binomial and the model contains an intercept.) Shows or hides a report that contains incidence rate ratios for categorical predictors, and unit incidence rate ratios and range incidence rate ratios for continuous predictors. An *incidence rate ratio* is the ratio of the incidence rate for two events. The *incidence rate* for a model term is the number of new events that occur over a given time period.

For each categorical predictor, an Incidence Rate Ratios report appears. Incidence rate ratios are shown for all combinations of levels of a categorical model term.

If there are continuous predictors, two additional reports appear:

– Unit Incidence Rate Ratios Report. The *unit incidence rate ratio* is calculated over a one-unit change in a continuous model term.

– Range Incidence Rate Ratios Report. The *range incidence rate ratio* is calculated over the entire range of a continuous model term.

The confidence intervals in the Incidence Rate Ratios report are Wald-based intervals. Note that the incidence rate ratio for a model term is meaningful only if the model term is not involved in any higher-order effects.

Hazard Ratios (Available only when the specified Distribution is Cox Proportional Hazards.) Shows or hides a report that contains hazard ratios for categorical predictors, and unit hazard ratios and range hazard ratios for continuous predictors. A *hazard ratio* is the ratio of the hazard rate for two events. The *hazard rate* at time t for an event is the conditional probability that the event will not survive an additional amount of time, given that it has survived to time t.

For each categorical predictor, a Hazard Ratios report appears. Hazard ratios are shown for all combinations of levels of a categorical model term.

If there are continuous predictors, two additional reports appear:

– Unit Hazard Ratios Report. The *unit hazard ratio* is calculated over a one-unit change in a continuous model term.

– Range Hazard Ratios Report. The *range hazard ratio* is calculated over the entire range of a continuous model term.

The confidence intervals in the Hazard Ratios report are Wald-based intervals. Note that the hazard ratio for a model term is meaningful only if the model term is not involved in any higher-order effects.

Covariance of Estimates Shows or hides a matrix showing the covariances of the parameter estimates. These are calculated using M-estimation and a sandwich formula (Zou 2006 and Huber and Ronchetti 2009). The covariance matrix does not contain zeroed terms.

Correlation of Estimates Shows or hides a matrix showing the correlations of the parameter estimates. These are calculated using M-estimation and a sandwich formula (Zou 2006 and Huber and Ronchetti 2009). The correlation matrix does not contain zeroed terms.

Inverse Prediction (Not available for models that contain a predictor that has the Vector modeling type.) Predicts an X value, given specific values for Y and the other X variables. This can be used to predict continuous variables only. For more information about Inverse Prediction, see "Inverse Prediction".

Multiple Comparisons (Not available for models that contain a predictor that has the Vector modeling type or for models that do not contain any categorical predictors.) Displays the Multiple Comparisons launch window. For more information about the Multiple Comparisons launch window and report, see "Multiple Comparisons". Note that the multiple comparisons are performed on the linear predictor scale. When the specified Distribution is Binomial, the multiple comparisons are performed on the odds ratios. When the specified Distribution is Poisson, the multiple comparisons are performed on the incidence rate ratios. When the specified Distribution is Cox Proportional Hazards, the multiple comparisons are performed on the hazard ratios.

Confusion Matrix (Available only when the specified Distribution is Binomial, Multinomial, or Ordinal Logistic.) Shows or hides a matrix that tabulates the actual response levels and the predicted response levels. For a good model, predicted response levels should be the same as the actual response levels. The confusion matrix enables you to assess how the predicted responses align with the actual responses. If you used validation, a confusion matrix is shown for each of the Training, Validation, and Test sets.

Set Probability Threshold (Available only when the specified Distribution is Binomial.) Specify a cutoff probability for classifying the response. By default, an observation is classified into the Target Level when its predicted probability exceeds 0.5. Change the threshold to specify a value other than 0.5 as the cutoff for classification into the Target Level. The Predicted Rate in the confusion matrix is updated to reflect classification according to the specified threshold.

If the response has a Profit Matrix column property, the initial value for the probability threshold is determined by the profit matrix.

Profilers (Not available for models that contain a predictor that has the Vector modeling type.) Provides various profilers that enable you to explore the fitted model.

Note: When the number of rows is less than or equal to 500 and the number of predictors is less than or equal to 30, the Profiler plots update continuously as you drag the current model indicator in either Solution Path plot. Otherwise, they update when you release the mouse button.

Profiler Shows or hides the Prediction Profiler. Predictors that have parameter estimates of zero and that are not involved in any interaction terms with nonzero coefficients do not appear in the profiler. For more information about the prediction profiler, see *Profilers*.

Distribution Profiler (Not available when the specified distribution is Binomial or Quantile Regression.) Shows or hides a profiler of the cumulative distribution function of the predictors and the response. The response is shown in the right-most cell.

Quantile Profiler (Not available when the specified distribution is Binomial or Quantile Regression.) Shows or hides a profiler that shows the predicted response as a function of the predictors and the quantile of the cumulative distribution function. The quantile is called Probability and is shown in the right-most cell.

Survival Profiler (Available only when the specified Distribution is Normal, Exponential, Weibull, Lognormal, or Cox Proportional Hazards.) Shows or hides a profiler that shows the survival function as a function of the predictors and the response. The response is shown in the right-most cell.

Hazard Profiler (Available only when the specified Distribution is Normal, Exponential, Weibull, Lognormal, or Cox Proportional Hazards.) Shows or hides a profiler that shows the hazard rate as a function of the predictors and the response. The response is shown in the right-most cell.

Custom Test Shows or hides a Custom Test report that enables you to test a custom hypothesis. If the model has a Solution Path, the custom test results update as you update the solution. For more information about custom tests, see "Custom Test". The Custom Test red triangle menu contains an option to remove the Custom Test report.

Diagnostic Plots Provides various plots to help assess how well the current model fits. If a Validation column is specified or if KFold, Holdback, or Leave-One-Out is selected as the Validation Method, the options below enable you to view the training, validation, and, if applicable, test sets, or they construct separate plots for these sets. If KFold or Leave-One-Out is selected, then the plots correspond to the validation set that optimizes prediction error, and its corresponding training set. See "KFold".

Note: All Diagnostic plots update continuously as you drag the current model indicator in either Solution Path plot.

Diagnostic Bundle (Not available when the specified Distribution is Binomial, Multinomial, Ordinal Logistic, or Cox Proportional Hazards.) Shows or hides a set of four graphs including a plot of residuals by predicted values, residuals by row number, a histogram of the residuals, and a histogram of the probability of observing a response larger than the observed response.

The graphs are constructed using all observations. If you used a Validation Column or if you selected KFold, Holdback, or Leave-One-Out as the Validation Method, check boxes enable you to select the Training, Validation, and, if applicable, Test sets. Rows corresponding to these sets are selected in the data table and the corresponding points and areas are highlighted in the graphs. Use this option to determine whether the model fit is similar across the sets.

The Fitted Probability of Observing a Larger Response histogram helps you assess goodness of fit of the model. Different criteria apply based on the distribution:

- For distributions other than zero-inflated distributions and quantile regression, the "correct" model should display an approximately uniform distribution of values.
- For a zero-inflated distribution, the histogram should display a point mass at zero and an approximately uniform distribution elsewhere.
- For quantile regression, the histogram should display an approximately uniform distribution of values to the left of the specified quantile and an approximately uniform distribution of slightly higher values to the right of the specified quantile.

Plot Baseline Survival and Hazard (Available only when the specified Distribution is Cox Proportional Hazards.) Shows or hides the Baseline Survival and Hazard plots, which plot the survival and hazard functions for the baseline proportional hazards function versus the response variable. Below the plots, there is a table that contains the plotted values.

Note: If the specified Distribution is Cox Proportional Hazards, the Plot Baseline Survival and Hazard option is the only available Diagnostic Plot.

ROC Curve (Available only when the specified Distribution is Binomial, Multinomial, or Ordinal Logistic.) Shows or hides the Receiver Operating Characteristic (ROC) curve. If you used validation, an ROC curve is shown for each of the Training, Validation, and Test sets.

The ROC curve measures the ability of the fitted probabilities to classify response levels correctly. The further the curve from the diagonal, the better the fit. An introduction to ROC curves is found in *Basic Analysis*.

If the response has more than two levels, the ROC Curve plot displays an ROC curve for each response level. For a given response level, this curve is the ROC curve for correct classification into that level. See *Predictive and Specialized Modeling* for more information about ROC curves.

Lift Curve (Available only when the specified Distribution is Binomial, Multinomial, or Ordinal Logistic.) Shows or hides the lift curve for the model. If you used validation, an ROC curve is shown for each of the Training, Validation, and Test sets.

A lift curve shows how effectively response levels are classified as their fitted probabilities decrease. The fitted probabilities are plotted along the horizontal axis in descending order. The vertical coordinate for a fitted probability is the proportion of correct classifications for that probability or higher, divided by the overall correct classification rate. Use the lift curve to see whether you can correctly classify a large proportion of observations if you select only those with a fitted probability that exceeds a threshold value.

If the response has more than two levels, the Lift Curve plot displays a lift curve for each response level. For a given response level, this curve is the lift curve for correct classification into that level. See *Predictive and Specialized Modeling* for more information about lift curves.

Decision Threshold (Available only for binary categorical responses.) Shows or hides Decision Thresholds reports for the training, validation, and test sets, if specified. Each report contains a graph of the distribution of fitted probabilities for each model, confusion matrices for each model, and classification graphs to compare the model fits. See *Predictive and Specialized Modeling* for more information about the Decision Thresholds report.

Plot Actual by Predicted (Not available when the specified Distribution is Binomial, Multinomial, Ordinal Logistic, or Cox Proportional Hazards.) Plots actual Y values on the vertical axis and predicted Y values on the horizontal axis. If you used validation, a plot is shown for each of the Training, Validation, and Test sets.

Plot Residual by Predicted (Not available when the specified Distribution is Binomial, Multinomial, Ordinal Logistic, or Cox Proportional Hazards.) Plots the residuals on the vertical axis and the predicted Y values on the horizontal axis. If you used validation, a plot is shown for each of the Training, Validation, and Test sets.

Plot Residual by Predictor (Not available when the specified Distribution is Binomial, Multinomial, Ordinal Logistic, or Cox Proportional Hazards. Not available for models that contain a predictor that has the Vector modeling type.) For each predictor in the model, plots the residuals on the vertical axis and the predictor values on the horizontal axis. There is a plot for each of the predictors in the model. If you used validation, a set of plots is shown for each of the Training, Validation, and Test sets.

Normal Quantile Plot (Available only when the specified Distribution is Normal and there is no censoring.) Plots normal quantiles on the vertical axis and standardized residuals on the horizontal axis. If you used validation, a plot is shown for each of the Training, Validation, and Test sets.

Save Columns Enables you to save columns based on the fitted model to the data table. See "Save Columns Options for Cox Proportional Hazards Models" for the options that are available if Cox Proportional Hazards is selected as the Distribution. For all other Distributions, the following columns can be saved to the data table:

Save Functional Prediction Formulas (Available only when the response columns contain the FDE FPC Num column property.) Saves a column for each FDE principal component response to the data table that contains a prediction formula for each functional principal component. This option also adds a model prediction formula that is a linear combination of the prediction formulas and the eigenfunction columns from the Functional Data Explorer platform. A script is added to the data table that enables you to use the model prediction formula to profile the original response, which is specified in the FDE Output column property of the FDE principal component response columns. For more information about functional principal components, see *Predictive and Specialized Modeling*.

Note: The Save Functional Prediction Formulas option saves formula columns for all FDE principal component responses in the report window. If multiple models are fit for a single response, the final model for each response is used to create the prediction formula for that response.

Save Prediction Formula Saves a column to the data table that contains the prediction formula, given in terms of the observed (unstandardized) data values. The prediction formula does not contain zeroed terms. See "Statistical Details for Distributions" for mean formulas.

When the response column is categorical, this option creates a probability column for each response level as well as a column that contains the most likely response. The Most Likely response column contains the level with the highest probability based on the model. If the Probability Threshold is a value other than 0.5, this option creates an additional column that contains the most likely response based on the probability threshold value.

Mean Confidence Interval Saves two columns to the data table that contain the lower and upper 95% confidence limits for the mean response.

Note: You can change the α level for the confidence interval in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Std Error of Predicted Saves a column to the data table that contains the standard errors of the predicted mean response.

Std Error of Predicted Formula Saves a column to the data table that contains a formula for the standard errors of the predicted mean response.

Save Residual Formula Saves a column to the data table that contains a formula for the residuals, given in the form Y minus the prediction formula. The residual formula does not contain zeroed terms. Not available if Binomial is selected as the Distribution.

Save Variance Formula Saves a column to the data table that contains a formula for the variance of the prediction. The variance of the prediction is calculated using the formula for the variance of the selected Distribution. The value of the parameter involved in the link function is estimated by applying the inverse of the link function to the estimated linear component. Other parameters are replaced by their estimates. See "Statistical Details for Distributions" for variance formulas. Not available if Binomial is selected as the Distribution.

Save Linear Predictor Saves a column to the data table that contains a formula for the product of the design matrix and the vector of parameter estimates. This is commonly referred to as $\mathbf{X}\beta$. The formula does not contain zeroed terms.

Save Validation Column (Available only if the specified Validation Method is KFold, Holdback, or Leave-One-Out.) Saves a column that describes the assignment of rows to folds. For KFold, the column lists the fold to which the row was assigned. For Holdback, each row is identified as belonging to the Training or Validation set. For Leave-One-Out, the row's value indicates its order in being left out.

Note: If you selected a Validation column in the launch window, the Save Validation Column option does not appear.

Save Distribution Formula (Not available when the specified Distribution is Binomial or Quantile Regression.) Saves a column to the data table that contains a formula for the cumulative distribution function.

Save Survival Formula (Available only when the specified Distribution is continuous.) Saves a column to the data table that contains a formula for the probability of survival at the observed time. The survival function is equal to 1 minus the cumulative distribution function.

Save Simulation Formula Saves a column to the data table that contains a formula that generates simulated values using the estimated parameters for the model that you fit. This column can be used in the Simulate utility as a Column to Switch In. See *Basic Analysis*.

Cook's D Influence (Available only if the specified Distribution is Normal and the specified Estimation Method is Standard Least Squares.) Saves a column to the data table that contains the values for Cook's *D* Influence statistic.

Hats (Available only if the specified Distribution is Normal and the specified Estimation Method is Standard Least Squares.) Saves a column to the data table that contains the diagonal elements of $\mathbf{X(X'X)^{-1}X'}$. These values are sometimes called *hat values*.

Publish Prediction Formula Creates prediction formulas and saves them as formula column scripts in the Formula Depot platform. If a Formula Depot report is not open, this option creates a Formula Depot report. See *Predictive and Specialized Modeling*.

Save Columns Options for Cox Proportional Hazards Models

Save Survival Formula Saves a column to the data table that contains a formula for the probability of survival at the observed time.

Save Cox Snell Residual Formula Saves a column to the data table that contains a formula for the Cox-Snell residuals. The Cox-Snell residuals are strictly positive. See Meeker and Escobar (1998, sec. 17.6.1) for a discussion of Cox-Snell residuals.

Save Martingale Residual Formula Saves a column to the data table that contains a formula for the martingale residuals. The martingale residual is defined as the difference between the observed number of events for an individual and a conditionally expected number of events. The martingale residuals have a mean of zero and range between negative infinity and 1. See Fleming and Harrington (1991).

Save Linear Predictor Saves a column to the data table that contains a formula for the product of the design matrix and the vector of parameter estimates. This is commonly referred to as $\mathbf{X}\beta$. The formula does not contain zeroed terms.

Remove Fit Removes the report for the fit.

Self-Validated Ensemble Models

This section contains the following information about self-validated ensemble models in the Generalized Regression platform:

- "Overview of Self-Validated Ensemble Models"
- "Reports for Self-Validated Ensemble Models"
- "Model Fit Options for Self-Validated Ensemble Models"

Overview of Self-Validated Ensemble Models

In the Generalized Regression platform, you can use the self-validated ensemble modeling (SVEM) method applied to forward selection or Lasso models. The SVEM method is a resampling method where each row is given nonzero weights in both the training and validation sets. The training and validation weights for a particular row are anti-correlated so that a row that has strong influence on the training set has weak influence on the validation set and vice versa. This approach can be useful for analyzing the results of a designed experiment.

The method can be summarized by the following steps:

1. Append a copy of the design matrix to itself vertically. The original design matrix has n rows, so the new design matrix has $2n$ rows and the same number of columns as the original design matrix.
2. Append a copy of the response vector to itself vertically. The new response vector has twice the number of rows as the original response vector.
3. Create n random values from an exponential distribution with location parameter equal to 1, where n is the number of rows in the original design matrix. These are the weights assigned to the first n rows of the design matrix.
4. Create n random values that are anti-correlated with the first n random values. These are the weights assigned to the last n rows of the design matrix.
5. Fit either a forward selection or Lasso model to the generated training and validation sets to obtain a set of parameter estimates.

6. Repeat step 3 through step 5 for each individual model. The number of individual models in the ensemble model is specified using the Samples option in the Model Launch control panel.
7. Average the parameter estimates from the individual models to form the ensemble model parameter estimates.

Note: When a Validation column is specified in the Fit Model launch window, the SVEM method is implemented on the Training set. The Validation and Test sets are held back as a test set.

For more information about self-validated ensemble modeling (SVEM) method, see Lemkus et al. (2021).

Reports for Self-Validated Ensemble Models

In the Generalized Regression report, the model fit reports for self-validated ensemble models contain sections that describe the model fit.

Parameter Estimates

The Parameter Estimates section contains estimates and other results for all parameters in the ensemble model. The initial table includes the coefficients for the predictors in the model. An additional table includes other model parameters such as scale, dispersion, or zero inflation parameters. See "Specify a Distribution". Both tables include the same columns of results.

The report provides the following information:

Term A list of the model terms. "Forced in" appears next to any terms that were forced into the model using the Advanced Controls option.

Resampling Estimate The average of the parameter estimates of the model term in the individual models. For the normal distribution, this average value is the parameter estimate in the ensemble model.

Resampling Std Dev The standard deviation of the estimates of the model term in the individual models. For the normal distribution, this value is the estimate of the parameter standard deviation in the ensemble model.

Percent Nonzero (Does not appear in the additional table in the Parameter Estimates section.) The percent of individual models that contain a nonzero estimate for each model term. Model terms that are not included in as many of the individual models are less important than model terms that are included in more of the individual models.

Sample Fit Quality

The Sample Fit Quality section contains a table of summary statistics for each of the individual models in the ensemble. The table contains the following columns:

Sample The number of the individual model.

Nonzero Predictors The number of predictors in each individual model that are nonzero.

Training MSE The mean square error for the training set in each individual model.

Validation MSE The mean square error for the validation set in each individual model.

Sample Parameter Estimates

The Sample Parameter Estimates section contains a table of the parameter estimates in the individual models. Each row of the table corresponds to an individual model in the ensemble. The first column contains the number of the individual model and the remaining columns correspond to the model terms.

Model Fit Options for Self-Validated Ensemble Models

In the Generalized Regression report, the red triangle menu in the reports for self-validated ensemble models contain the following options:

Regression Reports Enables you to customize the reports that are shown for the specified model fit.

Parameter Estimates for Original Predictors Shows or hides the Parameter Estimates report. See "Reports for Self-Validated Ensemble Models".

Covariance of Estimates Shows or hides a matrix showing the covariances of the self-validated ensemble model parameter estimates.

Correlation of Estimates Shows or hides a matrix showing the correlations of the self-validated ensemble model parameter estimates.

Profilers (Not available for models that contain a predictor that has the Vector modeling type.) Enables you to explore the self-validated ensemble model with a prediction profiler.

Profiler Shows or hides the Prediction Profiler. Predictors that have parameter estimates of zero and that are not involved in any interaction terms with nonzero coefficients do not appear in the profiler. For more information about the prediction profiler, see *Profilers*.

Diagnostic Plots Provides various plots to help assess how well the self-validated ensemble model fits.

Plot Actual by Predicted Plots actual Y values on the vertical axis and predicted Y values on the horizontal axis. If you specified a Validation column in the Fit Model specification window, a second plot labeled Test is shown for the observations held out of the SVEM procedure. This Test set corresponds to the Validation and Test sets in the Validation column.

Plot Residual by Predicted Plots the residuals on the vertical axis and the predicted Y values on the horizontal axis. If you specified a Validation column in the Fit Model specification window, a second plot labeled Test is shown for the observations held out of the SVEM procedure. This Test set corresponds to the Validation and Test sets in the Validation column.

Save Columns Enables you to save columns based on the fitted model to the data table.

Save Prediction Formula (Available only when the specified Distribution is Normal.) Saves a column to the data table that contains the prediction formula for the self-validated ensemble model. The prediction formula does not contain zeroed terms. See "Statistical Details for Distributions" for mean formulas.

Save Resample Formulas Saves multiple columns to the data table. A column group called SVEM Samples contains one formula column per individual model. These columns are saved as hidden columns. The next column is a prediction formula for the self-validated ensemble model. The next column contains the standard error formula for the self-validated ensemble model. The final column contains the median prediction from the self-validated ensemble model for each row.

Publish Prediction Formula Creates prediction formulas and saves them as formula column scripts in the Formula Depot platform. If a Formula Depot report is not open, this option creates a Formula Depot report. See *Predictive and Specialized Modeling*.

Remove Fit Removes the report for the fit.

JMP PRO Statistical Details for the Generalized Regression Personality

This section contains statistical details for the Generalized Regression personality of the Fit Model platform.

- "Statistical Details for Estimation Methods"
- "Statistical Details for Advanced Controls"
- "Statistical Details for Distributions"

JMP PRO Statistical Details for Estimation Methods

In the Generalized Regression personality of the Fit Model platform, the estimation methods include penalized regression methods, which introduce bias to the regression coefficients by penalizing them.

JMP PRO Ridge Regression

Ridge regression applies an l_2 penalty to the regression coefficients. Ridge regression coefficient estimates are defined as follows:

$$\hat{\beta}^{ridge} = \text{argmin}_{\beta}\left\{ \sum_{i=1}^{N} -LogLikelihood(\beta;y_i) + \frac{\lambda}{2}\sum_{j=1}^{p}\beta_j^2 \right\},$$

where $\sum_{j=1}^{p}\beta_j^2$ is the l_2 penalty, λ is the tuning parameter, N is the number of rows, and p is the number of variables.

JMP PRO Dantzig Selector

The Dantzig Selector method applies an l_∞ norm to the inner products of the residuals and X columns. Coefficient estimates for the Dantzig Selector satisfy the following criterion:

$$\min_{\beta}\left\|X^T(y - X\beta)\right\|_\infty \quad \text{subject to } \|\beta\|_1 \le t$$

where $\|v\|_\infty$ denotes the l_∞ norm, which is the maximum absolute value of the components of the vector v, $\|\beta\|_1 \le t$ is the l_1 penalty, and t is the tuning parameter.

JMP PRO Lasso Regression

The Lasso method applies an l_1 penalty to the regression coefficients. Coefficient estimates for the Lasso are defined as follows:

$$\hat{\beta}^{lasso} = \text{argmin}_{\beta}\left\{\frac{1}{2}\sum_{i=1}^{N} -LogLikelihood(\beta;y_i) + \lambda\sum_{j=1}^{p}|\beta_j|\right\},$$

where $\sum_{j=1}^{p}|\beta_j|$ is the l_1 penalty, λ is the tuning parameter, N is the number of rows, and p is the number of variables

JMP PRO Elastic Net

The Elastic Net method combines both l_1 and l_2 penalties. Coefficient estimates for the Elastic Net are defined as follows:

$$\hat{\beta}^{enet} = \text{argmin}_{\beta}\left\{\sum_{i=1}^{N} -LogLikelihood(\beta;y_i) + \lambda\sum_{j=1}^{p}\left(\alpha|\beta_j| + \frac{(1-\alpha)}{2}\beta_j^2\right)\right\},$$

This is the notation used in the equation:

$\sum_{j=1}^{p}|\beta_j|$ is the l_1 penalty

$\sum_{j=1}^{p}\beta_j^2$ is the l_2 penalty

λ is the tuning parameter

α is a parameter that determines the mix of the l_1 and l_2 penalties

N is the number of rows

p is the number of variables

Tip: There are two sample scripts that illustrate the shrinkage effect of varying α and λ in the Elastic Net for a single predictor. Select **Help > Sample Index**, click **Open the Sample Scripts Directory**, and select demoElasticNetAlphaLambda.jsl or demoElasticNetAlphaLambda2.jsl. Each script contains a description of how to use it and what it illustrates.

JMP PRO Adaptive Methods

The adaptive Lasso method uses weighted penalties to provide consistent estimates of coefficients. The weighted form of the l_1 penalty is defined as follows:

$$\sum_{j=1}^{p} \frac{|\beta_j|}{\tilde{\beta}_j}$$

where $\tilde{\beta}_j$ is the MLE when the MLE exists. If the MLE does not exist and the response distribution is normal, estimation is done using least squares and $\tilde{\beta}_j$ is the solution obtained using a generalized inverse. If the response distribution is not normal, $\tilde{\beta}_j$ is the ridge solution.

For the adaptive Lasso, this weighted form of the l_1 penalty is used in determining the $\hat{\beta}^{lasso}$ coefficients.

The adaptive Elastic Net uses this weighted form of the l_1 penalty and also imposes a weighted form of the l_2 penalty. The weighted form of the l_2 penalty for the adaptive Elastic Net is defined as follows:

$$\sum_{j=1}^{p} \left(\frac{\beta_j}{\tilde{\beta}_j}\right)^2$$

where $\tilde{\beta}_j$ is the MLE when the MLE exists. If the MLE does not exist and the response distribution is normal, estimation is done using least squares and $\tilde{\beta}_j$ is the solution obtained using a generalized inverse. If the response distribution is not normal, $\tilde{\beta}_j$ is the ridge solution.

JMP PRO Statistical Details for Advanced Controls

This section contains details for advanced controls in the Generalized Regression control panel.

JMP PRO Grid

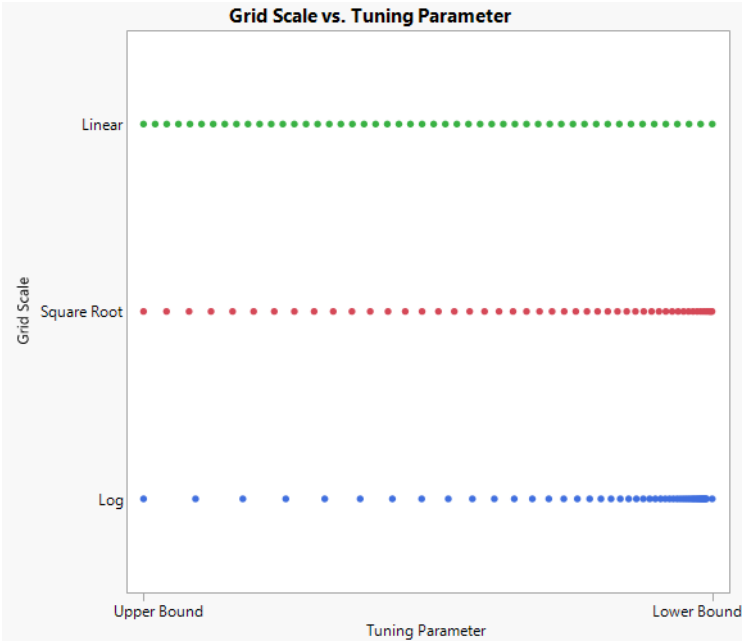
The tuning parameters for ridge regression and the Lasso that best minimize the penalized likelihood are found by searching a grid of tuning parameter values. This grid of values lies between a lower and an upper bound for the tuning parameter. You can specify the number of grid points under Advanced Controls.

The lower bound is zero except in special cases where it is set to 0.0001. See "Tuning Parameter". When the lower bound for the tuning parameter is zero, the solution is unpenalized and the coefficients are the MLEs. The upper bound is the smallest value for which all of the non-intercept terms are zero.

The grid of values between the lower and upper bounds is iteratively searched to determine the best value of the tuning parameter. The grid of possible tuning parameters can be set up in three different scales: linear, log, and square root. The default grid scale is square root.

In some cases, there is a large gap between the unpenalized estimates and the previous step. This large gap can distort the solution path. The log scale focuses its search on small tuning parameter values with few large values, whereas the linear scale evenly disperses the search from the minimum to the maximum value. The square root scale is a compromise between the other two scales. Figure 6.7 shows the different grid scales.

Figure 6.7 Options for Tuning Parameter Grid Scale



JMP PRO Statistical Details for Distributions

The distributions fit by the Generalized Regression personality are defined in terms of the parameters used in model fitting. Although it is not specifically stated as part of their descriptions, the Generalized Regression personality enables you to specify non-integer values for the discrete distributions.

JMP PRO Continuous Distributions

Normal Distribution

$$f(y|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma^2}(y-\mu)^2\right], -\infty < y < \infty$$

$$E(Y) = \mu$$

$$Var(Y) = \sigma^2$$

Cauchy Distribution

$$f(y|\mu,\sigma) = \left\{\pi\sigma\left[1+\left(\frac{y-\mu}{\sigma}\right)^2\right]\right\}^{-1}, -\infty < y < \infty$$

$$E(Y) = \text{undefined}$$

$$Var(Y) = \text{undefined}$$

t(5) Distribution

$$f(y|\mu,\sigma) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi\sigma^2}}\left[1+\frac{(x-\mu)^2}{\nu\sigma^2}\right]^{-\left(\frac{\nu+1}{2}\right)}, -\infty < y < \infty, \nu = 5$$

$$E(Y) = \mu$$

$$Var(Y) = \sigma^2\frac{\nu}{\nu-2}$$

Exponential Distribution

$$f(y|\theta) = \frac{1}{\mu}\exp\left[-\frac{y}{\mu}\right], y > 0$$

$$E(Y) = \mu$$

$$Var(Y) = \mu^2$$

Gamma Distribution

$$f(y|\mu, \sigma) = \frac{y^{(\mu/\sigma)-1}\exp[-y/\sigma]}{\Gamma[\mu/\sigma]\sigma^{\mu/\sigma}}, \; y > 0$$

$$E(Y) = \mu$$

$$Var(Y) = \mu\sigma$$

Weibull Distribution

$$f(y|\mu, \sigma) = \frac{1}{y\sigma}\exp\left[\frac{\log(y)-\mu}{\sigma}\right]\exp\left\{-\exp\left[\frac{\log(y)-\mu}{\sigma}\right]\right\}, \; y > 0$$

$$E(Y) = \exp(\mu)\Gamma[1+\sigma]$$

$$Var(Y) = \exp(2\mu)\{\Gamma[1+2\sigma] - (\Gamma[1+\sigma])^2\}$$

LogNormal Distribution

$$f(y|\mu, \sigma) = \frac{1}{y\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma^2}[\log(y)-\mu]^2\right\}, \; y > 0$$

$$E(Y) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

$$Var(Y) = [\exp(\sigma^2) - 1]\exp(2\mu + \sigma^2)$$

Negative LogNormal Distribution

$$f(y|\mu,\sigma) = \frac{-1}{y\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma^2}[\log(-y)-\mu]^2\right\},\ y<0$$

$$E(Y) = -\exp\left(\mu+\frac{\sigma^2}{2}\right)$$

$$Var(Y) = [\exp(\sigma^2)-1]\exp(2\mu+\sigma^2)$$

Beta Distribution

$$f(y|\mu,\sigma) = \frac{\Gamma[1/\sigma]}{\Gamma[\mu/\sigma]\Gamma[(1-\mu)/\sigma]}y^{\mu/\sigma-1}(1-y)^{\frac{(1-\mu)}{\sigma}-1},\ y\in(0,1)$$

$$E(Y) = \mu$$

$$Var(Y) = \mu(1-\mu)\frac{\sigma}{\sigma+1}$$

JMP PRO Discrete Distributions

Binomial Distribution

$$f(y|n,p) = \binom{n}{y}p^y(1-p)^{n-y},\ y = 0,1,2,\ldots,n$$

$$E(Y) = np$$

$$Var(Y) = np(1-p)$$

Beta Binomial Distribution

$$f(y|n,p,\delta) = \binom{n}{y}\frac{\Gamma\left[\frac{1}{\delta}-1\right]\Gamma\left[y+p\left(\frac{1}{\delta}-1\right)\right]\Gamma\left[n-y+(1-p)\left(\frac{1}{\delta}-1\right)\right]}{\Gamma\left[p\left(\frac{1}{\delta}-1\right)\right]\Gamma\left[(1-p)\left(\frac{1}{\delta}-1\right)\right]\Gamma\left[n-1+\frac{1}{\delta}\right]},\ y = 0,1,2,\ldots,n$$

$$E(Y) = np$$

$$Var(Y) = np(1-p)[1+(n-1)\delta]$$

Poisson Distribution

$$f(y|\lambda) = \frac{\lambda^y}{y!}\exp(-\lambda)\,,\ y = 0, 1, 2, \ldots$$

$$E(Y) = \lambda$$

$$Var(Y) = \lambda$$

Negative Binomial Distribution

$$f(y|\mu, \sigma) = \frac{\Gamma[y+(1/\sigma)]}{\Gamma[y+1]\Gamma[1/\sigma]}\left[\frac{(\mu\sigma)^y}{(1+\mu\sigma)^{y+(1/\sigma)}}\right],\ y = 0, 1, 2, \ldots$$

$$E(Y) = \mu$$

$$Var(Y) = \mu + \sigma\mu^2$$

JMP PRO Zero-Inflated Distributions

Zero-Inflated Binomial Distribution

$$f(y|n, p, \pi) = \begin{cases} \pi + (1-\pi)(1-p)^n, \text{ for } y = 0 \\ (1-\pi)\binom{n}{y}p^y(1-p)^{n-y}, \text{ for } y = 1, 2, \ldots, n \end{cases}$$

$$E(Y) = np(1-\pi)$$

$$Var(Y) = (1-\pi)np(1-p) + n^2p^2(1-\pi)^2$$

Zero-Inflated Beta Binomial Distribution

$$f(y|n, p, \delta, \pi) = \begin{cases} \pi + (1-\pi)\dfrac{\Gamma\left[\frac{1}{\delta}-1\right]\Gamma\left[p\left(\frac{1}{\delta}-1\right)\right]\Gamma\left[n+(1-p)\left(\frac{1}{\delta}-1\right)\right]}{\Gamma\left[p\left(\frac{1}{\delta}-1\right)\right]\Gamma\left[(1-p)\left(\frac{1}{\delta}-1\right)\right]\Gamma\left[n-1+\frac{1}{\delta}\right]}, \text{ for } y = 0 \\ (1-\pi)\binom{n}{y}\dfrac{\Gamma\left[\frac{1}{\delta}-1\right]\Gamma\left[y+p\left(\frac{1}{\delta}-1\right)\right]\Gamma\left[n-y+(1-p)\left(\frac{1}{\delta}-1\right)\right]}{\Gamma\left[p\left(\frac{1}{\delta}-1\right)\right]\Gamma\left[(1-p)\left(\frac{1}{\delta}-1\right)\right]\Gamma\left[n-1+\frac{1}{\delta}\right]}, y = 1, 2, \ldots, n \end{cases}$$

$$E(Y) = np(1-\pi)$$

$$Var(Y) = (1-\pi)np\{1+(1-p)[1+(n-1)\delta]\} - [np(1-\pi)]^2$$

Zero-Inflated Poisson Distribution

$$f(y|\lambda, \pi) = \begin{cases} \pi + (1-\pi)\exp[-\lambda], \text{ for } y = 0 \\ (1-\pi)\dfrac{\lambda^y}{y!}\exp[-\lambda], \text{ for } y = 1, 2, \ldots \end{cases}$$

$$E(Y) = (1-\pi)\lambda$$

$$Var(Y) = \lambda(1-\pi)(1+\lambda\pi)$$

Zero-Inflated Negative Binomial Distribution

$$f(y|\mu, \sigma, \pi) = \begin{cases} \pi + (1-\pi)(1+\mu\sigma)^{-(1/\sigma)}, \text{ for } y = 0 \\ (1-\pi)\dfrac{\Gamma[y+(1/\sigma)]}{\Gamma[y+1]\Gamma[1/\sigma]}\left[\dfrac{(\mu\sigma)^y}{(1+\mu\sigma)^{y+(1/\sigma)}}\right], \text{ for } y = 1, 2, \ldots \end{cases}$$

$$E(Y) = (1-\pi)\mu$$

$$Var(Y) = \mu(1-\pi)[1+\mu(\sigma+\pi)]$$

Zero-Inflated Gamma Distribution

$$f(y|\mu, \sigma, \pi) = \begin{cases} \pi \text{ , for } y = 0 \\ (1-\pi)\dfrac{\exp(-y/\sigma)}{\Gamma[\mu/\sigma]\sigma^{\mu/\sigma}y^{1-\mu/\sigma}} \text{ , for } y > 0 \end{cases}$$

$$E(Y) = \mu(1-\mu)$$

$$Var(Y) = \mu(1-\pi)(\sigma+\mu) - (1-\pi)^2\mu^2$$

Chapter 7

Generalized Regression Examples

Build Models Using Regularization Techniques

This chapter provides examples with instructional material for several models fit using the Generalized Regression personality of the Fit Model platform.

Contents

Example of Poisson Generalized Regression 349
Example of Binomial Generalized Regression 351
Example of Zero-Inflated Poisson Regression 353
Example of the Model Comparison Table in Generalized Regression 356
Example of Generalized Regression for Wide Data 358

Example of Poisson Generalized Regression

This example develops a prediction model for a count response using six predictors. The count response is modeled using a Poisson distribution.

1. Select **Help > Sample Data Folder** and open Liver Cancer.jmp.
2. Select **Analyze > Fit Model**.
3. Select Node Count from the Select Columns list and click **Y**.
4. Select BMI through Jaundice and click **Macros > Factorial to Degree**.

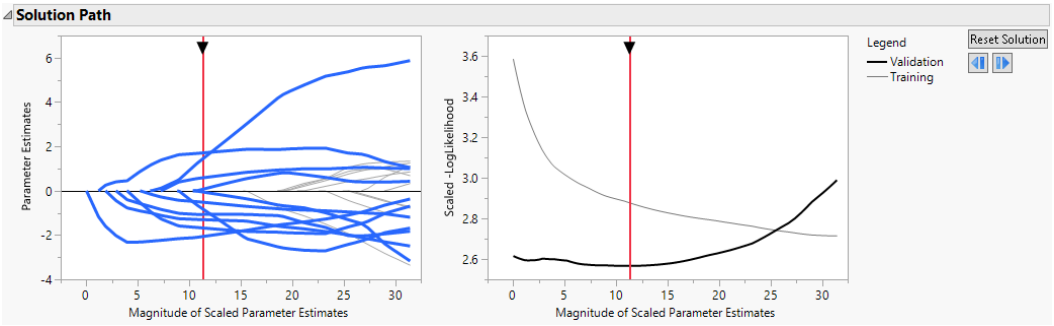
   This adds all terms up to degree 2 (the default in the **Degree** box) to the model.
5. Select Validation from the Select Columns list and click **Validation**.
6. From the Personality list, select **Generalized Regression**.
7. From the Distribution list, select **Poisson**.
8. Click **Run**.

   The Generalized Regression report that appears contains a Model Comparison report, a Model Launch control panel, and a Poisson Maximum Likelihood with Validation Column report. Note that the default estimation method is the Lasso.
9. Select the **Adaptive** box.
10. Click **Go**.
11. Click the red triangle next to Poisson Adaptive Lasso with Validation Column and select **Select Nonzero Terms**.

    The Solution Path is shown in Figure 7.1. The paths for terms that have nonzero coefficients are highlighted. Think of the solution paths as moving from right to left across the plot, as the solutions shrink farther from the MLE. A number of terms have paths that shrink them to zero fairly early.

    The vertical axis in the Solution Path Plot represents the values of the parameter estimates for the standardized predictors. The vertical red line indicates their values at the optimal shrinkage, as determined by cross validation. At this point, 11 terms have nonzero coefficients. Notice that the vertical red line indicates the minimum Scaled –LogLikelihood value in the Validation set.

Figure 7.1 Solution Path for Adaptive Lasso Fit with Nonzero Terms Highlighted



The Parameter Estimates for Original Predictors report (Figure 7.2) shows the parameter estimates for the uncentered and unscaled data. The 11 terms with nonzero parameter estimates are highlighted. These include interaction effects. In the data table, all six predictor columns are selected because every predictor column appears in a term that has a nonzero coefficient.

In the Effect Tests report, the 10 effects with zero coefficient estimates are designated as Removed. The Effect Tests report indicates that only one effect is significant at the 0.05 level: the Age*Markers interaction.

Figure 7.2 Parameter Estimates Report with Nonzero Terms Highlighted

Parameter Estimates for Original Predictors

Term	Estimate	Std Error	Wald ChiSquare	Prob > ChiSquare	Lower 95%	Upper 95%
Intercept	1.9658009	1.5268783	1.6575637	0.1979	-1.026826	4.9584273
BMI	-0.014557	0.0655911	0.049258	0.8244	-0.143114	0.1139989
Age	-0.000344	0.0052439	0.0043118	0.9476	-0.010622	0.0099335
Time	0	0	0	1.0000	0	0
Markers[0-1]	0.3853237	0.3347747	1.3247874	0.2497	-0.270823	1.0414702
Hepatitis[0-1]	0	0	0	1.0000	0	0
Jaundice[0-1]	-0.220763	0.1645355	1.8002539	0.1797	-0.543247	0.1017205
(BMI-23.1737)*(Age-56.3994)	0.0007336	0.0014188	0.2673515	0.6051	-0.002047	0.0035144
(BMI-23.1737)*(Time-9.00945)	0	0	0	1.0000	0	0
(BMI-23.1737)*Markers[0-1]	-0.066574	0.0511528	1.6938297	0.1931	-0.166832	0.0336837
(BMI-23.1737)*Hepatitis[0-1]	0.0269878	0.0827504	0.1063642	0.7443	-0.1352	0.1891755
(BMI-23.1737)*Jaundice[0-1]	0	0	0	1.0000	0	0
(Age-56.3994)*(Time-9.00945)	0	0	0	1.0000	0	0
(Age-56.3994)*Markers[0-1]	-0.023498	0.0083023	8.0106349	0.0047*	-0.03977	-0.007226
(Age-56.3994)*Hepatitis[0-1]	0	0	0	1.0000	0	0
(Age-56.3994)*Jaundice[0-1]	0	0	0	1.0000	0	0
(Time-9.00945)*Markers[0-1]	-0.007529	0.0104633	0.5178167	0.4718	-0.028037	0.0129784
(Time-9.00945)*Hepatitis[0-1]	0	0	0	1.0000	0	0
(Time-9.00945)*Jaundice[0-1]	0.0009619	0.0093476	0.0105893	0.9180	-0.017359	0.019283
Markers[0-1]*Hepatitis[0-1]	-0.422619	0.3974016	1.1309371	0.2876	-1.201512	0.356274
Markers[0-1]*Jaundice[0-1]	0	0	0	1.0000	0	0
Hepatitis[0-1]*Jaundice[0-1]	0	0	0	1.0000	0	0

12. Click the row for (Age - 56.3994)*Markers[0-1] in the Parameter Estimates for Original Predictors report.

This action highlights that effect's path in the Solution Path Plot and selects the columns Age and Markers in the data table.

13. Click the red triangle next to Poisson Adaptive Lasso with Validation Column and select **Save Columns > Save Prediction Formula** and **Save Columns > Save Variance Formula**.

    Two columns are added to the data table: Node Count Prediction Formula and Node Count Variance.

14. In the data table, right-click either column heading and select **Formula** to view the formula. Alternatively, click the plus sign to the right of the column name in the Columns panel.

    The prediction formula in the **Save Prediction Formula** column applies the exponential function to the estimated linear part of the model. The prediction variance formula in Node Count Variance is given by the identical formula, because the variance of a Poisson distribution is equal to its mean.

Example of Binomial Generalized Regression

This example shows how to develop a prediction model using the elastic net estimation method for a binomial response.

1. Select **Help > Sample Data Folder** and open Liver Cancer.jmp.
2. Select **Analyze > Fit Model**.
3. Select Severity from the Select Columns list and click **Y**.
4. Select BMI through Jaundice and click **Macros > Factorial to Degree**.

   All terms up to degree 2 (the default in the **Degree** box) are added to the model.

5. From the Personality list, select **Generalized Regression**.

   The Distribution list automatically shows the Binomial distribution. This is the only distribution available when Y is binary and has a Nominal modeling type.

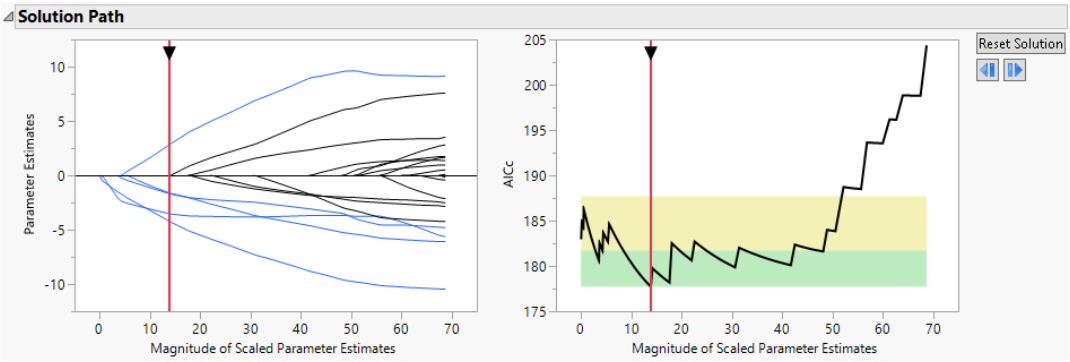
6. Click **Run**.

   The Generalized Regression report that appears contains a Model Comparison report, a Model Launch control panel, and a Logistic Regression report. Note that the default estimation method is the Lasso.

7. Select **Elastic Net** as the Estimation Method.
8. Select the **Adaptive** box.
9. Click **Go**.

   A Binomial Adaptive Elastic Net with AICc Validation report appears. The Solution Path is shown in Figure 7.3.

Figure 7.3 Solution Path Plot



The paths for terms that have nonzero coefficients are shown in blue. The optimal parameter values are substantially shrunken away from the MLE. The Validation Plot to the right indicates that several models can be considered as good as the best model. To view those models, slide the vertical red bar around in the region where the black line is in the green area.

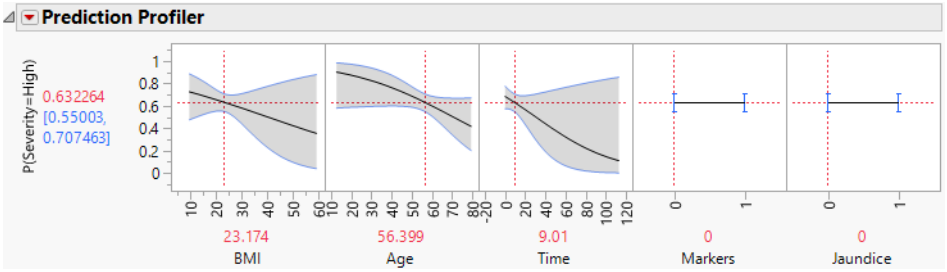
10. Click the red triangle next to Binomial Adaptive Elastic Net with AICc Validation and select the **Select Zeroed Terms** option.

    The 16 terms that have coefficient estimates of zero are highlighted in the Parameter Estimates for Original Predictors report. The Effect Tests report designates these terms as Removed.

    The Effect Tests report also shows that there are no significant terms at the 0.05 level. However, the Time*Markers interaction has a small *p*-value of 0.0626 and the Time effect has a small *p*-value of 0.1458.

11. Click the red triangle next to Binomial Adaptive Elastic Net with AICc Validation and select **Profilers > Profiler**.

Figure 7.4 Profiler for Probability That Severity = High, Time Low

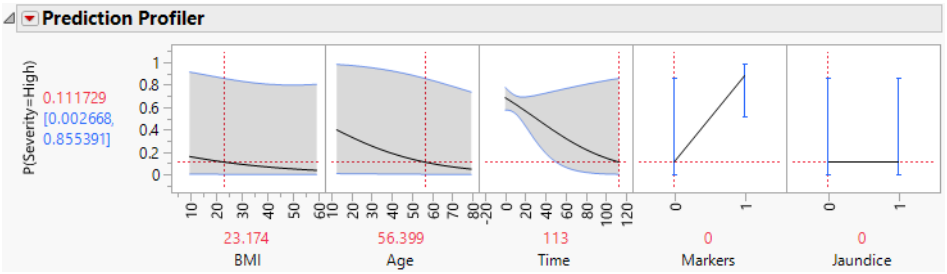


Examine the Prediction Profiler to see how Time and the Time*Markers interaction affect Severity.

Note: The predictor Hepatitis is not shown in the profiler because it does not appear in any active (nonzero) terms. Because Markers and Jaundice appear in active interaction terms, they appear in the profiler even though, as main effects, they are not active.

12. Move the red dashed line for Time from left to right to see its interaction with Markers (Figure 7.4 and Figure 7.5). For patients who enter the study with small values of Time since diagnosis, Markers have little impact on Severity. But for patients who enter the study having been diagnosed for a longer time, Markers are important. For those patients, normal markers suggest a lower probability of high Severity.

Figure 7.5 Profiler for Probability That Severity = High, Time High



Example of Zero-Inflated Poisson Regression

This example shows how to fit a zero-inflated Poisson regression model using the Generalized Regression personality of the Fit Model platform. A zero-inflated Poisson distribution is appropriate for count data where responses of zero can come from multiple sources.

In this example, you are analyzing data on the number of fist caught by groups of park visitors. The data table for this example details five factors that might affect the number of fish caught by 250 groups visiting a park.

During data collection, it was never determined whether anyone in the group had actually fished. However, the hidden Fished column is included in the table to emphasize the point that catching zero fish can happen in one of two ways: Either no one in the group fished, or everyone who fished in the group was unlucky. Therefore, zero responses can come from two sources. To address this issue, you can fit a zero-inflated distribution. Because a Poisson distribution is appropriate for the count data resulting from people who fished, you fit a zero-inflated Poisson distribution.

1. Select **Help > Sample Data Folder** and open Fishing.jmp.
2. Select **Analyze > Fit Model**.
3. Select Fish Caught from the Select Columns list and click **Y**.

4. Select Live Bait through Children and click **Macros > Factorial to Degree**.

   Terms up to degree 2 (the default in the **Degree** box) are added to the model.
5. Select Validation from the Select Columns list and click **Validation**.
6. From the Personality list, select **Generalized Regression**.
7. From the Distribution list, select **ZI Poisson**.
8. Click **Run**.

   The Generalized Regression report that appears contains a Model Comparison report, a Model Launch control panel, and a ZI Poisson Maximum Likelihood with Validation Column report. Note that the default estimation method is the Lasso.
9. From the Estimation Method List, select **Elastic Net**.
10. Click **Go**.

    A ZI Poisson Elastic Net with Validation Column report appears. The Solution Path, the Parameter Estimates for Original Predictors report, and the Effect Tests report indicate that one term is zeroed. The Zero Inflation parameter, whose estimate is shown on the last line of both Parameter Estimates reports, is highly significant. This indicates that some of the variation in the response, Fish Caught, might be due to the fact that some groups did not fish.

Figure 7.6 Parameter Estimates for Original Predictors Report

Parameter Estimates for Original Predictors

Term	Estimate	Std Error	Wald ChiSquare	Prob > ChiSquare	Lower 95%	Upper 95%
Intercept	1.3599777	0.3237335	17.647702	<.0001*	0.7254716	1.9944837
Live Bait[0-1]	-0.379555	0.1919937	3.9081951	0.0481*	-0.755856	-0.003255
Fishing Poles	0.3034685	0.0465239	42.547553	<.0001*	0.2122833	0.3946537
Camper[0-1]	-0.363094	0.1899392	3.6543491	0.0559	-0.735368	0.0091795
People	0.0776017	0.121377	0.4087617	0.5226	-0.160293	0.3154962
Children	-0.192656	0.1028943	3.5057658	0.0612	-0.394325	0.009013
Live Bait[0-1]*(Fishing Poles-0.856)	-0.026766	0.0668764	0.160184	0.6890	-0.157841	0.1043095
Live Bait[0-1]*Camper[0-1]	-0.336382	0.48759	0.4759452	0.4903	-1.292041	0.6192764
Live Bait[0-1]*(People-2.516)	-0.164164	0.1994702	0.6773258	0.4105	-0.555118	0.2267908
Live Bait[0-1]*(Children-0.936)	0.1025463	0.2362161	0.1884607	0.6642	-0.360429	0.5655213
(Fishing Poles-0.856)*Camper[0-1]	-0.446199	0.1120088	15.869111	<.0001*	-0.665732	-0.226666
(Fishing Poles-0.856)*(People-2.516)	-0.00611	0.0853344	0.0051267	0.9429	-0.173362	0.1611423
(Fishing Poles-0.856)*(Children-0.936)	-0.17587	0.080167	4.8127459	0.0282*	-0.332995	-0.018746
Camper[0-1]*(People-2.516)	-0.195261	0.3035369	0.4138148	0.5200	-0.790182	0.3996608
Camper[0-1]*(Children-0.936)	0.1899805	0.2998548	0.4014171	0.5264	-0.397724	0.7776852
(People-2.516)*(Children-0.936)	0	0	0	1.0000	0	0

ZI Poisson Distribution Parameters	Estimate	Std Error	Wald ChiSquare	Prob > ChiSquare	Lower 95%	Upper 95%
Zero Inflation	0.7815222	0.0354433	486.19905	<.0001*	0.7120545	0.8509898

The Effect Tests report indicates that four terms are significant at the 0.05 level: Live Bait, Fishing Poles, Fishing Poles*Camper, and Fishing Poles*Children.

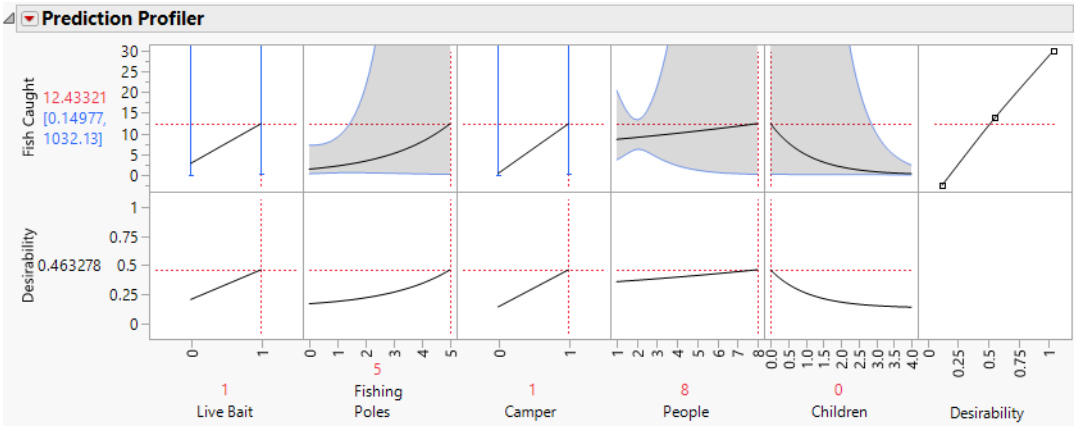
11. Click the red triangle next to ZI Poisson Elastic Net with Validation Column and select **Profilers > Profiler**.

12. Click the Prediction Profiler red triangle and select **Optimization and Desirability > Desirability Functions**.

    A function is imposed on the response, which indicates that maximizing the number of Fish Caught is desirable. For more information about desirability functions, see *Profilers*.

13. Click the Prediction Profiler red triangle and select **Optimization and Desirability > Maximize Desirability**.

Figure 7.7 Prediction Profiler with Fish Caught Maximized



You can vary the settings of the predictors to see the impact of the significant effects: Live Bait, Fishing Poles, Fishing Poles*Camper, and Fishing Poles*Children. For example, Live Bait is associated with more fish; a Camper tends to bring more fishing poles than someone who is not camping and therefore catches more fish.

14. Click the red triangle next to ZI Poisson Elastic Net with Validation Column and select **Save Columns > Save Prediction Formula** and **Save Columns > Save Variance Formula**.

    Two columns are added to the data table: Fish Caught Prediction Formula and Fish Caught Variance.

15. In the data table, right-click either column heading and select **Formula** to view the formula. Alternatively, click the plus sign to the right of the column name in the Columns panel. Note the appearance of the estimated zero-inflation parameter, 0.781522155, in both of these formulas.

Example of the Model Comparison Table in Generalized Regression

This example develops multiple prediction models for a count response that has six predictors. The count response is modeled using a Poisson distribution. The four prediction models are the Lasso, the Elastic Net, the Adaptive Lasso, and the Adaptive Elastic Net. Use the Model Comparison report in Generalized Regression to compare the four prediction models with each other, as well as with the maximum likelihood model, to choose a final model.

1. Select **Help > Sample Data Folder** and open Liver Cancer.jmp.
2. Select **Analyze > Fit Model**.
3. Select Node Count from the Select Columns list and click **Y**.
4. Select BMI through Jaundice and click **Macros > Factorial to Degree**.

   This adds all terms up to degree 2 (the default in the **Degree** box) to the model.
5. Select Validation from the Select Columns list and click **Validation**.
6. From the Personality list, select **Generalized Regression**.
7. From the Distribution list, select **Poisson**.
8. Click **Run**.

   The Generalized Regression report that appears contains a Model Comparison report, a Model Launch control panel, and a Poisson Maximum Likelihood with Validation Column report. Note that the default estimation method is the Lasso.

Fit the Lasso Model

9. Click **Go**.

Fit the Elastic Net Model

10. Scroll to the top of the report window and open the Model Launch outline.
11. Select **Elastic Net** as the Estimation Method.
12. Click **Go**.

Fit the Adaptive Lasso Model

13. Scroll to the top of the report window and open the Model Launch outline.
14. Select **Lasso** as the Estimation Method.
15. Select the **Adaptive** box.
16. Click **Go**.

Fit the Adaptive Elastic Net Model

17. Scroll to the top of the report window and open the Model Launch outline.
18. Select **Elastic Net** as the Estimation Method.

Note: Confirm that the Adaptive box is still selected from the previous model.

19. Click **Go**.

Compare the Models

20. Scroll to the top of the report window.
21. Click the Validation Generalized RSquare column heading in the Model Comparison table.

Figure 7.8 Model Comparison Report

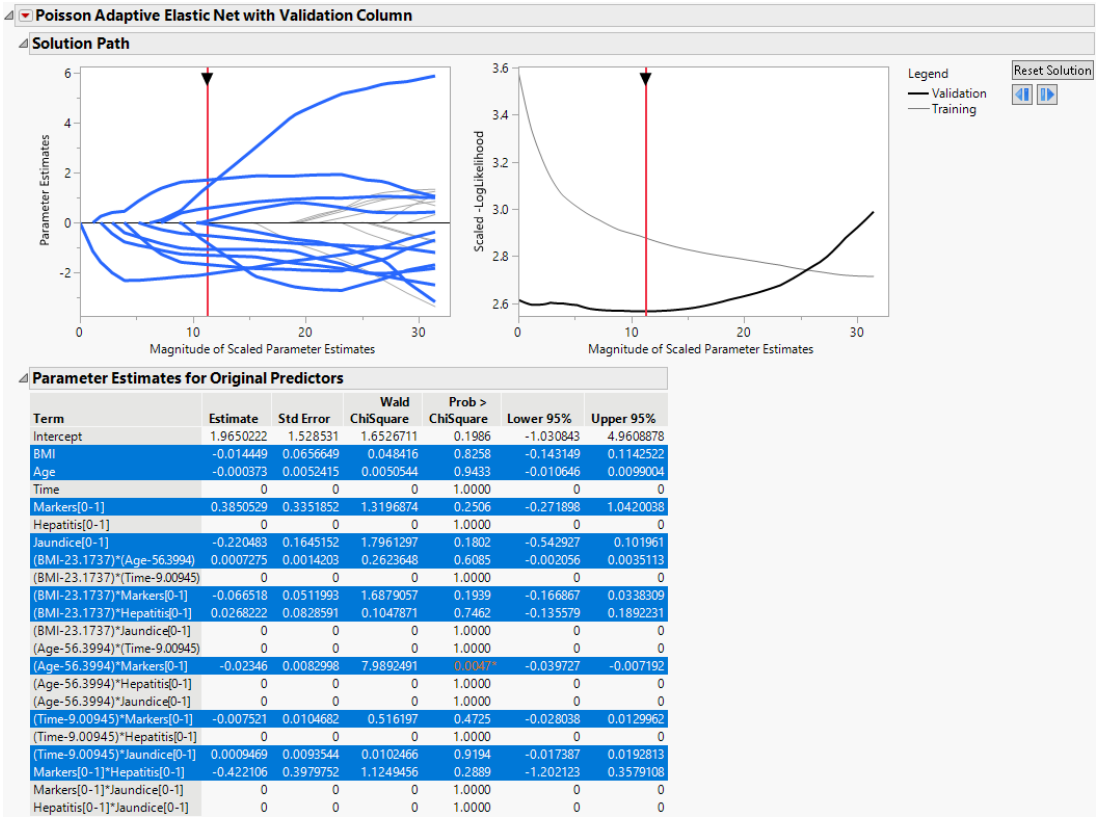
Model Comparison

Show	Response Distribution	Estimation Method	Validation Method	Nonzero Parameters	AICc	BIC	Generalized RSquare	Validation Generalized RSquare
☑	Poisson	Maximum Likelihood	Validation Column	22	568.58535	610.28431	0.8242066	-1.22458
☑	Poisson	Elastic Net	Validation Column	3	613.41549	620.77871	0.5076069	0.0052459
☑	Poisson	Lasso	Validation Column	3	612.80144	620.16465	0.5108129	0.005532
☑	Poisson	Adaptive Lasso	Validation Column	12	568.76457	595.43225	0.7566166	0.0478662
☑	Poisson	Adaptive Elastic Net	Validation Column	12	568.82707	595.49476	0.7564547	0.0479201

The Model Comparison table is now sorted by the Validation Generalized RSquare values in ascending order. These RSquare values represent how well the models fit the validation set. The negative Validation Generalized RSquare value for the Maximum Likelihood model indicates that this model is over-fitting. The penalized methods all fit the validation data better than the Maximum Likelihood model. While none of the penalized models fit the validation data particularly well, the adaptive methods fit better than the non-adaptive methods. Since the Adaptive Elastic Net model performs best on the validation data, you decide to use it for prediction.

22. Uncheck all the boxes under Show except for the one in the Adaptive Elastic Net row.
23. Click the red triangle next to Poisson Adaptive Elastic Net with Validation Column and select **Select Nonzero Terms**.

Figure 7.9 Solution Path for Adaptive Elastic Net Fit with Nonzero Terms Highlighted



Term	Estimate	Std Error	Wald ChiSquare	Prob > ChiSquare	Lower 95%	Upper 95%
Intercept	1.9650222	1.528531	1.6526711	0.1986	-1.030843	4.9608878
BMI	-0.014449	0.0656649	0.048416	0.8258	-0.143149	0.1142522
Age	-0.000373	0.0052415	0.0050544	0.9433	-0.010646	0.0099004
Time	0	0	0	1.0000	0	0
Markers[0-1]	0.3850529	0.3351852	1.3196874	0.2506	-0.271898	1.0420038
Hepatitis[0-1]	0	0	0	1.0000	0	0
Jaundice[0-1]	-0.220483	0.1645152	1.7961297	0.1802	-0.542927	0.101961
(BMI-23.1737)*(Age-56.3994)	0.0007275	0.0014203	0.2623648	0.6085	-0.002056	0.0035113
(BMI-23.1737)*(Time-9.00945)	0	0	0	1.0000	0	0
(BMI-23.1737)*Markers[0-1]	-0.066518	0.0511993	1.6879057	0.1939	-0.166867	0.0338309
(BMI-23.1737)*Hepatitis[0-1]	0.0268222	0.0828591	0.1047871	0.7462	-0.135579	0.1892231
(BMI-23.1737)*Jaundice[0-1]	0	0	0	1.0000	0	0
(Age-56.3994)*(Time-9.00945)	0	0	0	1.0000	0	0
(Age-56.3994)*Markers[0-1]	-0.02346	0.0082998	7.9892491	0.0047*	-0.039727	-0.007192
(Age-56.3994)*Hepatitis[0-1]	0	0	0	1.0000	0	0
(Age-56.3994)*Jaundice[0-1]	0	0	0	1.0000	0	0
(Time-9.00945)*Markers[0-1]	-0.007521	0.0104682	0.516197	0.4725	-0.028038	0.0129962
(Time-9.00945)*Hepatitis[0-1]	0	0	0	1.0000	0	0
(Time-9.00945)*Jaundice[0-1]	0.0009469	0.0093544	0.0102466	0.9194	-0.017387	0.0192813
Markers[0-1]*Hepatitis[0-1]	-0.422106	0.3979752	1.1249456	0.2889	-1.202123	0.3579108
Markers[0-1]*Jaundice[0-1]	0	0	0	1.0000	0	0
Hepatitis[0-1]*Jaundice[0-1]	0	0	0	1.0000	0	0

The nonzero terms in the adaptive elastic net model are selected in the Solution Path and in the Parameter Estimates table.

At this point, you can use this model for prediction or open the Prediction Profiler to further explore the effects of the parameters on the response variable.

Example of Generalized Regression for Wide Data

Wide data is the term used to describe a data set where there are more predictors than there are observations. For wide data, traditional regression methods are not practical. Regression methods that incorporate variable selection enable you to fit a regression model in these situations. In this example, you compare three models with varying degrees of variable selection.

1. Select **Help > Sample Data Folder** and open Prostate Cancer.jmp.
2. Select **Analyze > Fit Model**.

3. Select Status from the Select Columns list and click **Y**.

   Because this is a Nominal response column, the Personality changes to Nominal Logistic and the Target Level option appears. The default value for this option is CCD, because that is the value specified in the Target Level column property in the data table.

4. From the Personality list, select **Generalized Regression**.

   The Distribution list automatically shows the Binomial distribution. This is the only distribution available when Y is binary and has a Nominal modeling type.

5. Select the Proteins column group from the Select Columns list and click **Add**.

   This adds all 667 columns in the column group to the model.

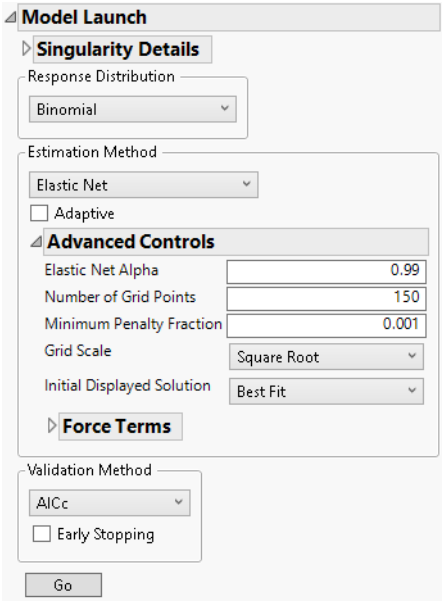
6. Click **Run**.

   The Generalized Regression report that appears contains a Model Launch control panel. There is no initial Logistic Regression model fit because the number of predictors is greater than the number of observations.

7. Select **Elastic Net** as the Estimation Method.

8. Click the gray disclosure icon next to Advanced Controls.

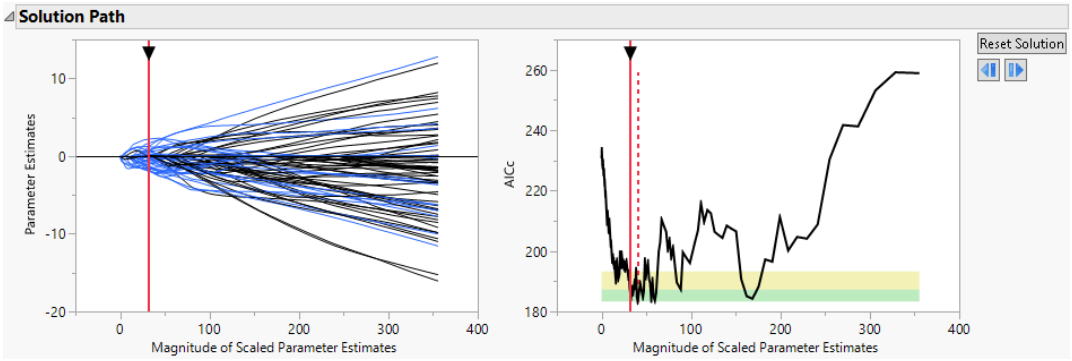
Figure 7.10 Advanced Controls



9. Select **Smallest in Green Zone** as the Initial Displayed Solution.

10. Click **Go**.

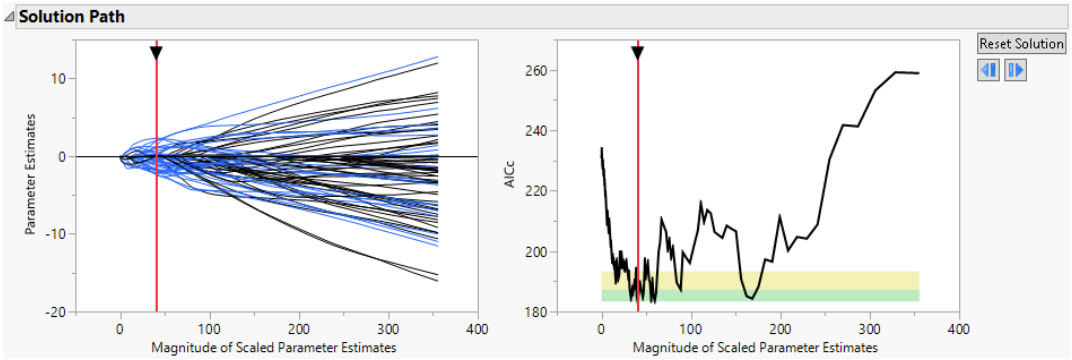
Figure 7.11 Smallest in Green Zone Model



The Solution Path shows the smallest model that is considered comparable to the minimum AICc model, where smallest means the one with the fewest parameters.

11. Click the gray disclosure icon next to Binomial Elastic Net with AICc Validation.
12. Click the gray disclosure icon next to Model Launch.
13. Select **Best Fit** as the Initial Displayed Solution.
14. Click **Go**.

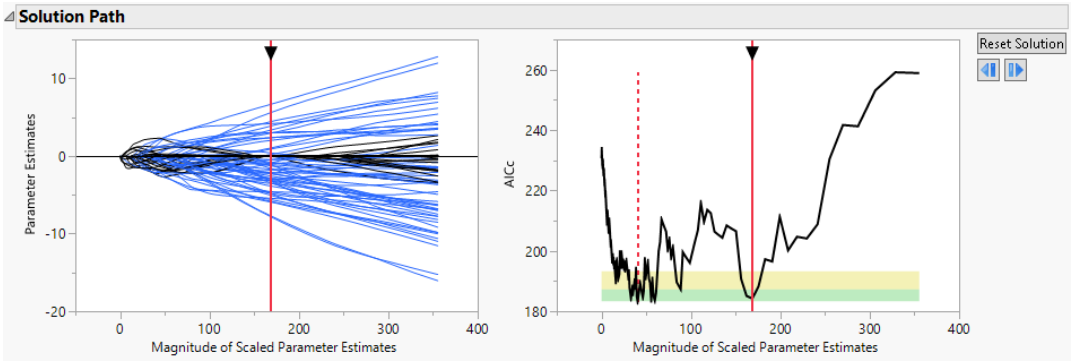
Figure 7.12 Best Fit Model



The Solution Path shows the best fit model, where best fit means the one with the minimum AICc value.

15. Click the gray disclosure icon next to Binomial Elastic Net with AICc Validation.
16. Click the gray disclosure icon next to Model Launch.
17. Select **Biggest in Green Zone** as the Initial Displayed Solution.
18. Click **Go**.

Figure 7.13 Biggest in Green Zone Model



The Solution Path shows the largest model that is considered comparable to the minimum AICc model, where largest means the one with the most parameters.

19. Click the gray disclosure icon next to Binomial Elastic Net with AICc Validation.

Figure 7.14 Model Comparison Table

Model Comparison

Show	Response Distribution	Estimation Method	Validation Method	Nonzero Parameters	AICc	BIC	Generalized RSquare
☑	Binomial	Elastic Net	AICc	36	186.07102	277.07256	0.7466199
☑	Binomial	Elastic Net	AICc	40	183.29544	281.08164	0.8021925
☑	Binomial	Elastic Net	AICc	57	184.25399	299.49849	0.9824565

The Model Comparison report shows the three models. You can identify the size of each model using the Nonzero Parameters column. As the number of parameters in the models increases, the Generalized RSquare values increase. Because these models are all in the green zone, there is strong evidence that any of these models are comparable to the best model.

Chapter 8

JMP PRO Mixed Models

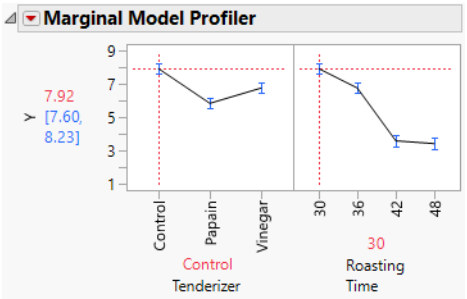
Jointly Model the Mean and Covariance

The Mixed Model personality of the Fit Model platform is available only in JMP Pro.

The Mixed Model personality fits a wide variety of linear models for continuous responses with complex covariance structures. These models include random coefficients, repeated measures, spatial data, and data with multiple correlated responses. Use the Mixed Model personality to specify linear mixed models and their covariance structures conveniently using an intuitive interface, and to fit these models using maximum likelihood methods.

The modeling results are supported by interactive visualization tools such as profilers, surface plots, and contour plots. You can use these tools to complement your understanding of the model.

Figure 8.1 Marginal Model Profiler for a Split Plot Experiment



Contents

Overview of the Mixed Model Personality . . . 365
Example Using the Mixed Model Personality . . . 366
Launch the Mixed Model Personality . . . 370
Fit Model Launch Window . . . 370
Data Format . . . 376
Mixed Model Report and Options . . . 376
Random Effects Covariance Parameter Estimates . . . 383
Fixed Effects Parameter Estimates . . . 385
Repeated Effects Covariance Parameter Estimates . . . 386
Random Coefficients . . . 386
Random Effects Predictions . . . 386
Fixed Effects Tests . . . 387
Sequential Tests . . . 387
Multiple Comparisons . . . 388
Compare Slopes . . . 389
Marginal Model Inference . . . 389
Actual by Predicted Plot . . . 389
Residual Plots . . . 389
Marginal Model Profiler . . . 390
Variogram . . . 391
Conditional Model Inference . . . 392
Actual by Conditional Predicted Plot . . . 392
Conditional Residual Plots . . . 392
Conditional Profilers . . . 393
Additional Examples of the Mixed Model Personality . . . 394
Example of Repeated Measures . . . 394
Example of a Split Plot Experiment . . . 409
Example of a Uniformity Trial . . . 414
Example of a Correlated Response . . . 424
Statistical Details for the Mixed Model Personality . . . 431
Statistical Details for the Convergence Score Test . . . 431
Statistical Details for the Random Coefficient Model . . . 432
Statistical Details for Repeated Measures . . . 434
Statistical Details for Repeated Covariance Structures . . . 434
Statistical Details for Spatial and Temporal Variability . . . 440
Statistical Details for the Kackar-Harville Correction . . . 442

JMP PRO Overview of the Mixed Model Personality

The Mixed Model personality of the Fit Model platform enables you to analyze models with complex covariance structures. The situations that can be analyzed include:

- Split plot experiments
- Random coefficients models
- Repeated measures designs
- Spatial data
- Correlated response data

Split plot experiments are experiments with two or more levels, or sizes, of experimental units resulting in multiple error terms. Such designs are often necessary when some factors are easy to vary and others are more difficult to vary. See the *Design of Experiments Guide*.

Random coefficients models are also known as hierarchical or multilevel models (Singer 1998; Sullivan et al. 1999). These models are used when batches or subjects are thought to differ randomly in intercept and slope. Drug stability trials in the pharmaceutical industry and individual growth studies in educational research often require random coefficient models.

Repeated measures designs, spatial data, and correlated response data share the property that observations are not independent, requiring that you model their correlation structure.

- Repeated measures designs, also known as within-subject designs, model changes in a response over time or space while allowing errors to be correlated.
- Spatial data are measurements made in two or more dimensions, typically latitude and longitude. Spatial measurements are often correlated as a function of their spatial proximity.
- Correlated response data result from making several measurements on the same experimental unit. For example, height, weight, and blood pressure readings taken on individuals in a medical study, or hardness, strength, and elasticity measured on a manufactured item, are likely to be correlated. Although these measurements can be studied individually, treating them as correlated responses can lead to useful insights.

Failure to account for correlation between observations can result in incorrect conclusions about treatment effects. However, estimating covariance structure parameters uses information in the data. The number of parameters being estimated impacts power and the Type I error rate. For this reason, you must choose covariance models judiciously. See "Example of Repeated Measures".

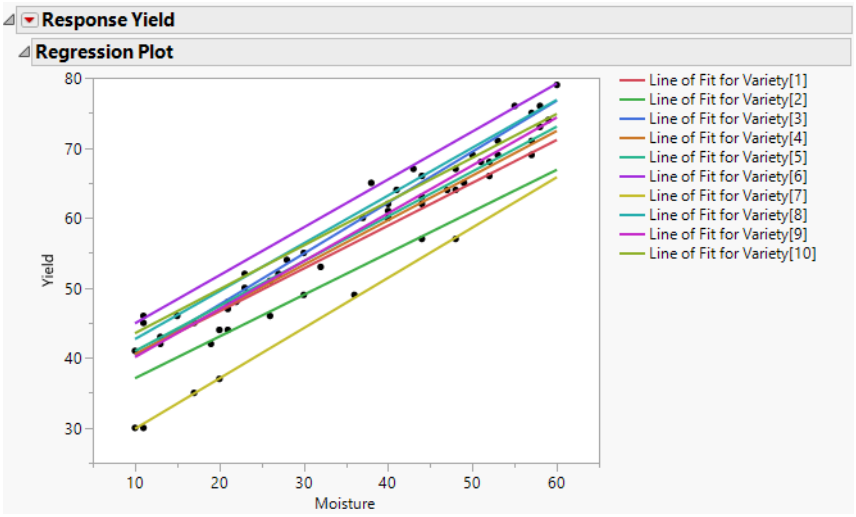
JMP PRO Example Using the Mixed Model Personality

This example illustrates using the Mixed Model personality of the Fit Model platform to fit a model for the correlation between the intercept and the slope. In a study of wheat yield, 10 varieties of wheat are randomly selected from the population of varieties of hard red winter wheat adapted to dry climate conditions. These are randomly assigned to six one-acre plots of land. The preplanting moisture content of the plots could influence the germination rate and hence the eventual yield of the plots. Thus, the amount of preplanting moisture in the top 36 inches of soil is determined for each plot. You are interested in determining if the moisture content affects yield.

Because the varieties are randomly selected, the regression model for each variety is a random model selected from the population of variety models. The intercept and slope are random for each variety and might be correlated. The random coefficients are centered at the fixed effects. The fixed effects are the population intercept and the slope, which are the expected values of the population of the intercepts and slopes of the varieties. This example is taken from Littell et al. (2006, p. 320).

Fitting the model using REML in the Standard Least Squares personality lets you view the variation in intercepts and slopes (Figure 8.2). Note that the slopes do not have much variability, but the intercepts have quite a bit. The intercept and slope might be negatively correlated; varieties with lower intercepts seem to have higher slopes.

Figure 8.2 Standard Least Squares Regression

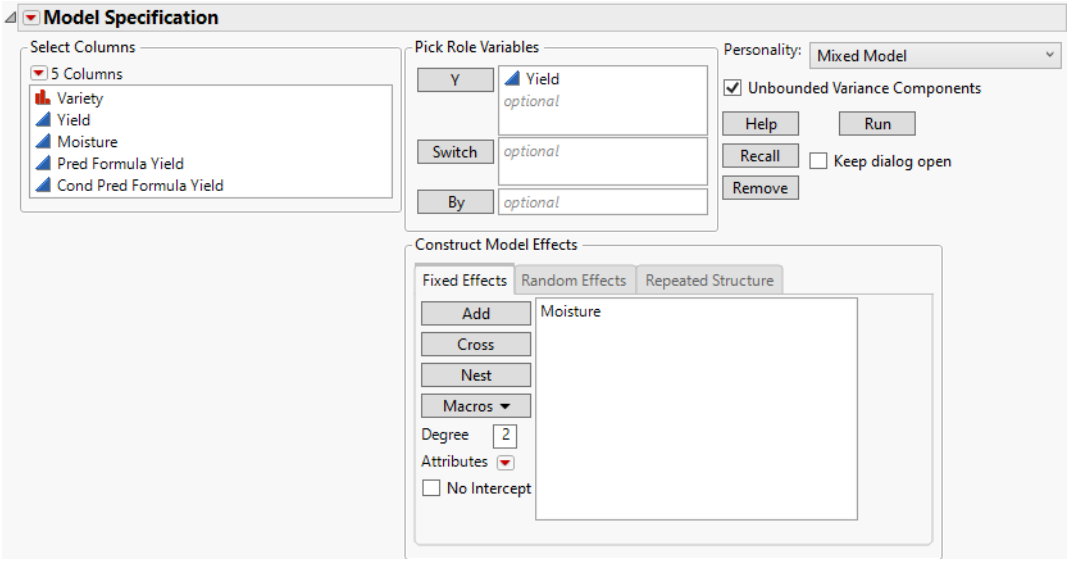


To model the correlation between the intercept and the slope, use the Mixed Model personality. You are interested in determining the population regression equation as well as variety-specific equations.

1. Select **Help > Sample Data Folder** and open Wheat.jmp.
2. Select **Analyze > Fit Model**.
3. Select Yield and click **Y**.

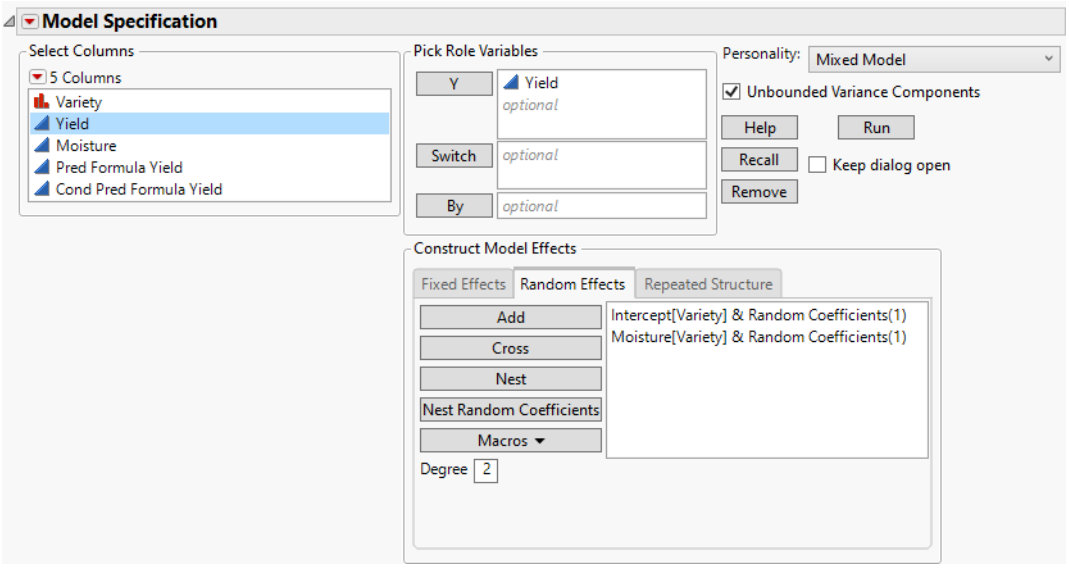
   When you add this column as Y, the fitting Personality becomes Standard Least Squares.
4. Select **Mixed Model** from the Personality list. Alternatively, you can select the Mixed Model personality first, and then click **Y** to add Yield.
5. Select Moisture and click **Add** on the Fixed Effects tab.

Figure 8.3 Completed Fit Model Launch Window Showing Fixed Effects



6. Select the **Random Effects** tab.
7. Select Moisture and click **Add**.
8. Select Variety from the Select Columns list, select Moisture from the Random Effects tab, and then click **Nest Random Coefficients**.

Figure 8.4 Completed Fit Model Launch Window Showing Random Effects Tab



Random effects are grouped by variety, and the intercept is included as a random component.

9. (Optional.) Click the Model Specification red triangle and check the setting of the **Center Polynomials** option.

   Even if the Center Polynomials option is selected, the Moisture effect will not be centered at its mean because it is involved in a random effect.

10. Click **Run**.

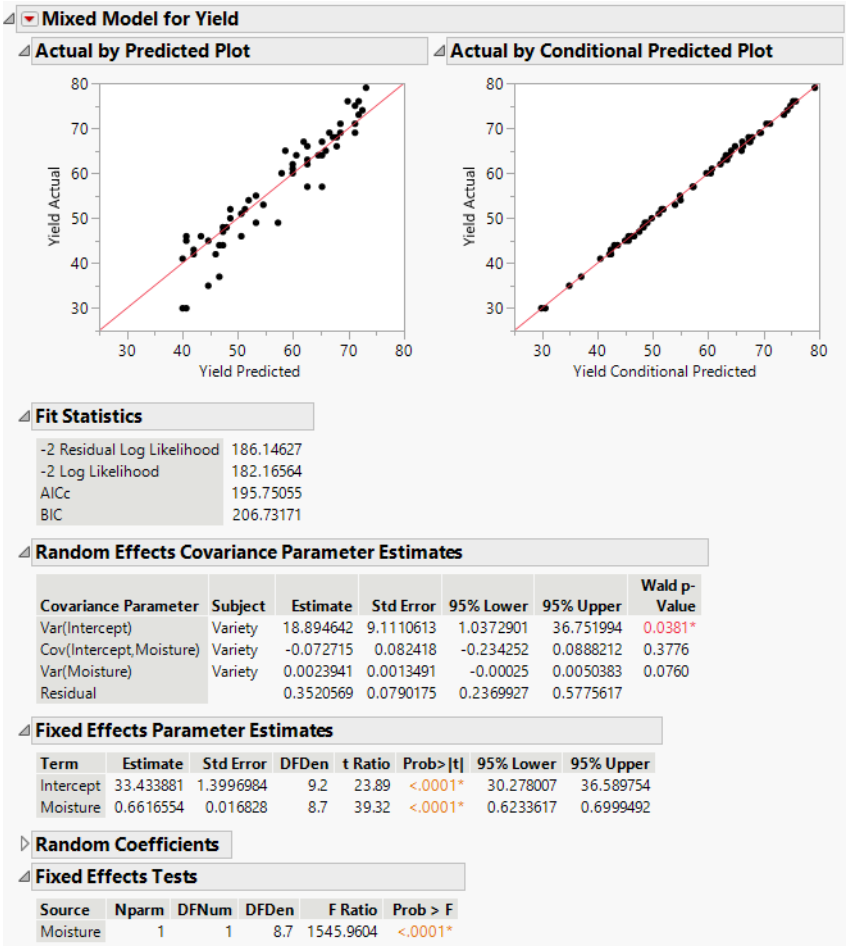
    The Mixed Model report is shown in Figure 8.5. The Actual by Predicted plot shows no discrepancy in terms of model fit and underlying assumptions.

    Because there are no apparent problems with the model fit, you can now interpret the statistical tests and obtain the regression equation. The effect of moisture upon yield is significant, as shown in the Fixed Effects Tests report. The estimates given in the Fixed Effects Parameter Estimates indicate that the following equation is the estimated population regression equation:

    $$Yield = 33.43 + 0.66 * Moisture$$

    The Random Effects Covariance Parameter Estimates report gives estimates of the variance of the varieties' intercepts, Var(Intercept), and slopes, Var(Moisture), and their covariance, Cov(Moisture, Intercept). In this case, the intercept and slope are not significantly correlated, because the confidence interval for the estimate includes zero. The report also gives an estimate of the residual variance.

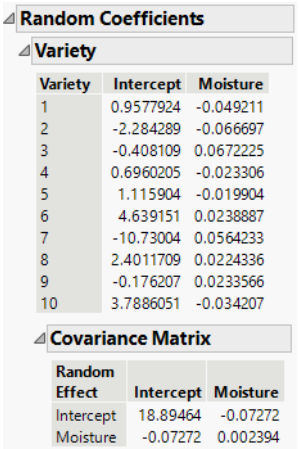
Figure 8.5 Mixed Model Report



Although you have an estimate of the population regression equation, you are also interested in Variety 2's estimated yield.

11. Open the Random Coefficients report to see the estimates of the variety effects for Intercept and Moisture. These coefficients estimate how each variety differs from the population.

Figure 8.6 Random Coefficients Report



Random Coefficients

Variety

Variety	Intercept	Moisture
1	0.9577924	-0.049211
2	-2.284289	-0.066697
3	-0.408109	0.0672225
4	0.6960205	-0.023306
5	1.115904	-0.019904
6	4.639151	0.0238887
7	-10.73004	0.0564233
8	2.4011709	0.0224336
9	-0.176207	0.0233566
10	3.7886051	-0.034207

Covariance Matrix

Random Effect	Intercept	Moisture
Intercept	18.89464	-0.07272
Moisture	-0.07272	0.002394

From the Fixed Effects Parameter Estimates and Random Coefficients reports, you obtain the following prediction equation for Variety 2:

$$Yield = 33.433 + 0.662 * Moisture - 2.284 - 0.067 * Moisture$$

$$Yield = 31.149 + 0.595 * Moisture$$

Variety 2 starts with a lower yield than the population average and increases with Moisture at a slower rate than the population average.

JMP PRO Launch the Mixed Model Personality

Launch the Mixed Model personality by selecting **Analyze > Fit Model** and selecting **Mixed Model** from the **Personality** menu. Note that when you enter a continuous variable in the **Y** list *before* selecting a Personality, the Personality defaults to Standard Least Squares.

JMP PRO Fit Model Launch Window

You can specify models with fixed effects, random effects, a repeated structure or a combination of those. The options differ based on the nature of the model that you specify. For more information about the options in the Select Columns red triangle menu, see *Using JMP*.

When fitting models using the Mixed Model personality, you can allow unbounded variance components. This means that variance components that have negative estimates are not reported as zero. This option is selected by default. It should remain selected if you are interested in fixed effects, because bounding the variance estimates at zero leads to bias in the tests for fixed effects. See "Negative Variances" for more information about the Unbounded Variance Components option.

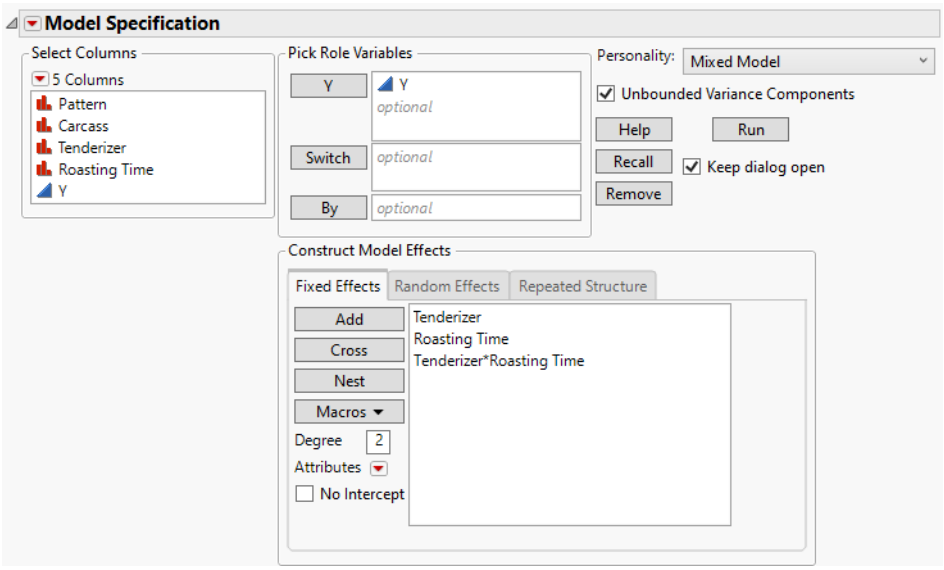
JMP PRO Fixed Effects Tab

Add all fixed effects on the Fixed Effects tab. Use the Add, Cross, Nest, Macros, and Attributes options as needed. For more information about these options, see "Model Specification".

Note: If a continuous column is involved in a random effect, that column is not centered, even if the Center Polynomials option in the Model Specifications red triangle menu is selected.

The fixed effects for analysis of the Split Plot.jmp sample data table appear in Figure 8.7. Note that it is possible to have no fixed effects in the model. For an example, see "Example of a Uniformity Trial".

Figure 8.7 Fit Model Launch Window Showing Completed Fixed Effects



JMP PRO Random Effects Tab

Specify traditional variance component models and random coefficients models using the Random Effects tab.

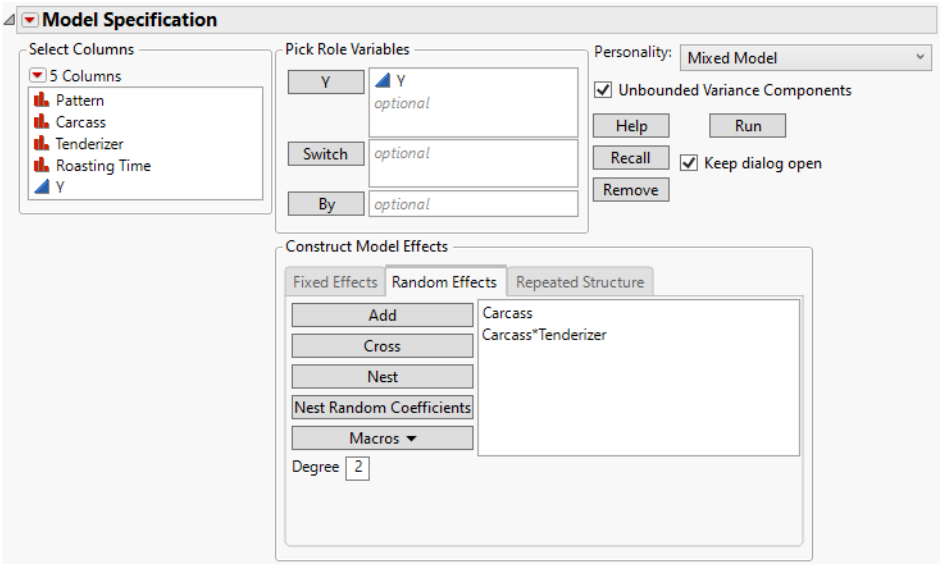
Note: If a continuous column is involved in a random effect, that column is not centered, even if the Center Polynomials option in the Model Specifications red triangle menu is selected.

Variance Components

For a traditional variance component model, specify terms such as random blocks, whole plot error terms, and subplot error terms using the Add, Cross, or Nest options. For more information about these options, see "Model Specification".

Figure 8.8 shows the random effects specification for the Split Plot.jmp sample data where Carcass is a random block. "Example of a Split Plot Experiment" describes the example in detail.

Figure 8.8 Fit Model Launch Window Showing Completed Random Effects Tab



Random Coefficients

To construct random coefficients models, use the Nest Random Coefficients button to create groups of random coefficients.

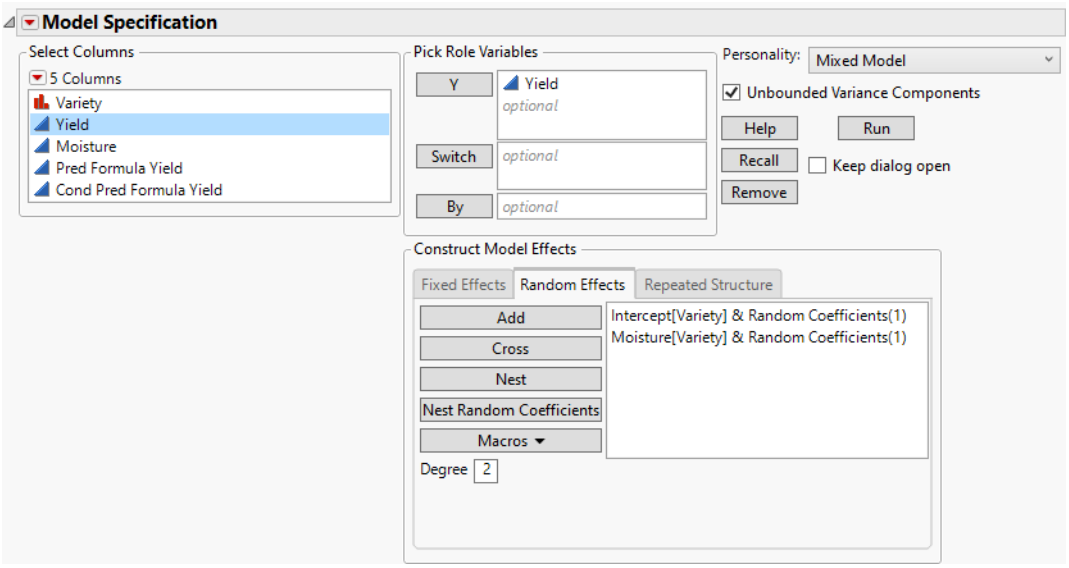
1. Select the continuous columns from the Select Columns list that are predictors.
2. Select the **Random Effects** tab and then **Add**.
3. Select these effects in the Random Effects tab. Also select the column that contains the random effect whose levels define the individual regression models. This column is essentially the subject in a random statement in SAS PROC MIXED.
4. Click the **Nest Random Coefficients** button.

This last step creates random intercept and random slope effects that are correlated within the levels of the random effect. The subject is nested within the other effects due to the variability among subjects. If you believed that the intercept might be fixed for all groups, you would select Intercept[<*group*>]&Random Coefficients(1) and then click **Remove**.

You can define multiple groups of random coefficients in this fashion, as in hierarchical linear models. This might be necessary when you have both a random batch effect and a random batch by treatment effect on the slope and intercept coefficients. This might also be necessary in a hierarchical linear model: when you have a random student effect and random school effect on achievement scores and students are nested within school.

Random coefficients are modeled using an unstructured covariance structure. Figure 8.9 shows the random coefficients specification for the Wheat.jmp sample data. See also "Example Using the Mixed Model Personality".

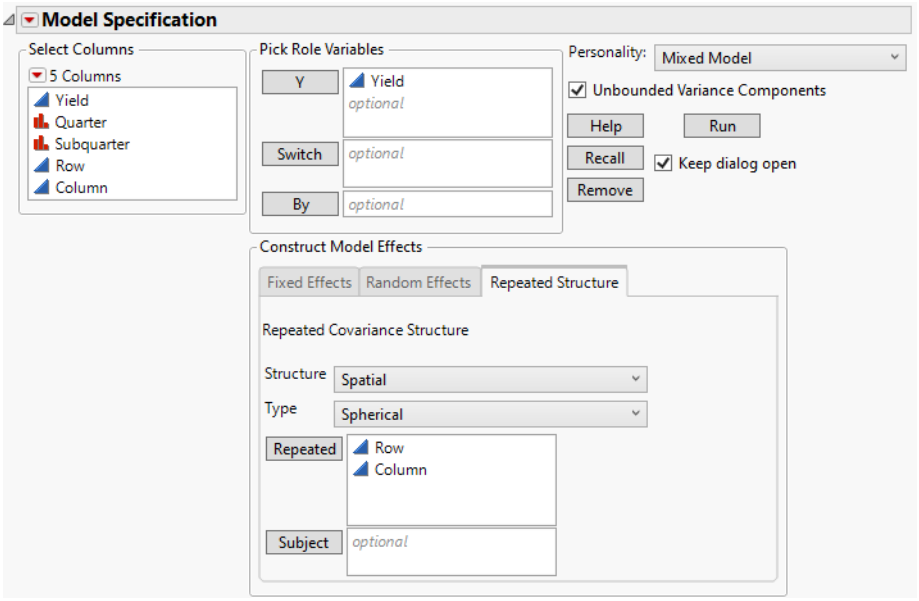
Figure 8.9 Completed Fit Model Launch Window Showing Random Coefficients



JMP PRO Repeated Structure Tab

Use the Repeated Structure tab to select a covariance structure for repeated effects in the model.

Figure 8.10 Completed Fit Model Launch Window Showing Repeated Structure Tab



Structure

The repeated structure is set to Residual by default. The Residual structure specifies that there is no covariance between observations, namely, the errors are independent. Besides the Residual and Unequal Variances structures, all other covariance structures model covariance between observations. For more information about the structures, see "Statistical Details for Repeated Measures" and "Statistical Details for Spatial and Temporal Variability" in the Statistical Details section.

Table 8.1 lists the covariance structures available, the requirements for using each structure, and the number of covariance parameters for the given structure. The number of observation times is denoted by *J*.

Table 8.1 Repeated Covariance Structure Requirements

Structure	Repeated Column Type	Required Number of Repeated Columns	Subject	Number of Parameters
Residual	not applicable	0	not applicable	0
Unequal Variances	categorical	1	optional	*J*
Unstructured	categorical	1	required	*J*(*J*+1)/2
AR(1)	continuous	1	optional	2
Compound Symmetry	categorical	1	required	2
Antedependent Equal Variance	categorical		required	*J*
Toeplitz	categorical	1	required	*J*
Compound Symmetry Unequal Variances	categorical	1	required	*J*+1
Antedependent	categorical		required	2*J*-1
Toeplitz Unequal Variances	categorical	1	required	2*J*-1
Spatial	continuous	2+	optional	
Spatial Anisotropic	continuous	2+	optional	
Spatial with Nugget	continuous	2+	optional	
Spatial Anisotropic with Nugget	continuous	2+	optional	

If you enter a Repeated or Subject column with the Residual structure, those columns are ignored. This alert appears: "Repeated columns and subject columns are ignored when the Residual covariance structure is selected."

Type

When you select one of the spatial covariance structures, a Type list appears from which you select a type of spatial structure. Four Types are available: Power, Exponential, Gaussian, and Spherical. Figure 8.10 shows the Spatial Spherical selection for the Uniformity Trial.jmp sample data.

Repeated

Enter columns that define the repeated measures structure. The modeling types of Repeated columns depend on the covariance structure. See Table 8.1 for more information about the requirements for each repeated measures covariance structure.

Subject

Enter one or more columns that define the Subject. Subject columns must be categorical.

JMP PRO Data Format

The Mixed Model personality of the Fit Model platform requires that all response measurements be contained in one response column. Repeated measures data are sometimes recorded in multiple columns, where each row is a subject and the repeated measurements are recorded in separate response columns. Data that are in this format must be stacked before running the Mixed Model personality. The Cholesterol.jmp and Cholesterol Stacked.jmp sample data tables illustrate the wide format and the stacked format, respectively. Notice that each row in the wide table corresponds to one level of Patient in the stacked table.

JMP PRO Mixed Model Report and Options

The Mixed Model red triangle menu contains the following options:

Model Reports Produces reports that relate to the mixed model fit. These reports give estimates and tests for model parameters, as well as fit statistics.

Fit Statistics Shows or hides a report for model fit statistics. See "Fit Statistics".

Random Effects Covariance Parameter Estimates (Available only when there are random effects specified in the launch window.) Shows or hides a report of random effects covariance parameter estimates. See "Random Effects Covariance Parameter Estimates".

Fixed Effects Parameter Estimates Shows or hides a report of fixed effects parameter estimates. See "Fixed Effects Parameter Estimates".

Repeated Effects Covariance Parameter Estimates (Available only when there are repeated effects specified in the launch window.) Shows or hides a report of repeated effects covariance parameter estimates. See "Repeated Effects Covariance Parameter Estimates".

Random Coefficients (Available only when there are random effects specified in the launch window.) Shows or hides a report of random coefficients. See "Random Coefficients".

Random Effects Predictions (Available only when there are random effects specified in the launch window.) Shows or hides a report of random effect predictions. See "Random Effects Predictions".

Indicator Parameterization Estimates (Available only when there are nominal columns among the fixed effects and the model contains an intercept.) Shows or hides the Indicator Function Parameterization report. This report gives parameter estimates for the fixed effects based on a model where nominal fixed effect columns are coded using indicator (SAS GLM) parameterization and are treated as continuous. Ordinal columns remain coded using the usual JMP coding scheme. The SAS GLM and JMP coding schemes are described in "The Factor Models".

Caution: Standard errors, t-ratios, and other results given in the Indicator Function Parameterization report differ from those in the Parameter Estimates report. This is because the estimates are estimating different parameters.

Fixed Effects Test (Available only for models that contain at least one fixed effect.) Shows or hides the tests of fixed effects. See "Fixed Effects Tests".

Sequential Tests (Available only for models that contain at least one fixed effect.) Shows or hides the Sequential (Type 1) Tests report that contains the sums of squares as effects are added to the model sequentially. Conducts *F* tests based on the sequential sums of squares. See "Sequential Tests".

Multiple Comparisons Opens the Multiple Comparisons launch window where you can select one or more effects and initial comparisons. This report is available for categorical fixed effects. See "Multiple Comparisons".

Linear Combination of Variance Components (Not available when there are no G-side effects.) Shows a report that enables you to compute confidence intervals for linear combinations of variance components. Initially, the report contains an editable text box and a table of variance components in the model. Use the text box to label the linear combination. Enter values in the cells in the right column of the table to specify the linear functions for your confidence intervals. After you specify a linear combination of parameters and click Done, a table appears that contains confidence intervals for the specified linear combination.

The table contains an estimate and standard error, as well as two types of confidence intervals (Satterthwaite and Wald) and a Wald *p*-value. The Wald *p*-value corresponds to a hypothesis test that the estimate differs from zero.

Tip: The Satterthwaite confidence interval is restricted to positive values, so it is not recommended for cases where the specified coefficients are negative. If the estimate is negative, the Satterthwaite confidence interval cannot be constructed and is reported as missing.

Compare Slopes (Available only when there is one nominal term, one continuous term, and their interaction effect for the fixed effects.) Shows or hides a report that enables you to compare the slopes of each level of the interaction effect in an analysis of covariance (ANCOVA) model. See "Compare Slopes".

Inverse Prediction (Available only when there is at least one continuous fixed effect term and there is a residual variance term.) For one or more values of the response, predicts values of explanatory variables. See "Inverse Prediction".

Marginal Model Inference Shows or hides plots that are based on marginal predicted values and marginal residuals. These plots display the variation due to random effects.

Actual by Predicted Plot Plots actual values versus values predicted by the model, but without accounting for the random effects. The Actual by Predicted Plot appears by default. See "Actual by Predicted Plot".

Residual Plots Provides residual plots that assess model fit, without accounting for the random effects. See "Residual Plots".

Profiler, Contour Profiler, Mixture Profiler, Surface Profiler Provides profilers to examine the relationship between the response and the model terms, without accounting for random effects. See "Marginal Model Profiler".

Variogram Provides a variogram plot that shows the change in covariance as the distance between observations increases. When the Residual structure is selected, you can select the columns to use as temporal or spatial coordinates. See "Variogram".

Conditional Model Inference Shows or hides plots that are based on conditional predicted values and conditional residuals. These plots display the variation that remains, once random effects are accounted for.

Actual by Conditional Predicted Plot Plots actual values versus values predicted by the model, accounting for the random effects. When there are random effects, the Actual by Conditional Predicted Plot appears by default. See "Actual by Conditional Predicted Plot".

Conditional Residual Plots Provides residual plots that assess model fit, accounting for the random effects. See "Conditional Residual Plots".

Conditional Profiler, Conditional Contour Profiler, Conditional Mixture Profiler, Conditional Surface Profiler Provides profilers to examine the relationship between the response and the model terms, accounting for random effects. See "Conditional Profilers".

Covariance and Correlation Matrices Contains options to view the covariance and correlation matrices that are associated with the model.

Covariance of Fixed Effects Shows or hides the covariance matrix for the fixed effects in the model.

Covariance of Covariance Parameters Shows or hides the covariance matrix for the random effects in the model. The effects in the matrix are ordered as follows: G-side random effects, R-side random effects, and residual effects.

Covariance of All Parameters Shows or hides the covariance matrix for all effects in the model. The effects in the matrix are ordered as follows: fixed effects, G-side random effects, R-side random effects, and residual effects.

Correlation of Fixed Effects Shows or hides the correlation matrix for the fixed effects in the model.

Repeated Measures Covariance Diagnostics (Available only for models that specify an unstructured repeated covariance structure.) Shows or hides a report that contains diagnostic tools to help determine candidate covariance structures for the repeated measures analysis. The report contains the covariance matrix and correlation matrix of the repeated measures parameters. The report also contains a heat map of the correlations. The scale of the heat map is determined by the range of the correlations. If all the correlations are positive, the scale is 0 to 1; otherwise, the scale is -1 to 1.

Save Columns Contains options to save various model results as columns in the data table.

Prediction Formula Creates a new column called Pred Formula <colname> that contains both the formula and the marginal mean predicted values. A Predicting column property is added, noting the source of the prediction. See "Marginal Model Inference".

Standard Error of Predicted Creates a new column called StdErr Pred <colname> that contains standard errors for the predicted marginal mean responses.

Mean Confidence Interval Creates two new columns called Lower 95% Mean <colname> and Upper 95% Mean <colname>. These columns contain the lower and upper 95% confidence limits for the mean response. These intervals include the variation in the estimation, but not in the response. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Indiv Confidence Interval (Available for models that contain only G-side effects.) Creates two new columns called Lower 95% Indiv <colname> and Upper 95% Indiv <colname>. These columns contain lower and upper 95% confidence limits for individual response values. These intervals include the variation in both the response and its estimation. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Residuals Creates a new column called Residual <colname> that contains the observed response values minus their marginal mean predicted values. See "Marginal Model Inference".

Conditional Prediction Formula Creates a new column called Cond Pred Formula <colname> that contains both the formula and the conditional mean predicted values. A Predicting column property is added, noting the source of the prediction. See "Conditional Profilers".

Standard Error of Conditional Predicted Creates a new column called StdErr Cond Pred <colname> that contains standard errors for the predicted conditional mean responses.

Conditional Mean CI (Available for models that contain a G-side effect.) Creates two new columns called Lower 95% Cond Mean <colname> and Upper 95% Cond Mean <colname>. These columns contain the lower and upper 95% confidence limits for the expected value from conditional prediction. The confidence intervals include random effect estimates for models with random effects. See "Conditional Model Inference". You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Conditional Residuals Creates a new column called Cond Residual <colname> that contains the observed response values minus their conditional mean predicted values. See "Conditional Model Inference".

Save Simulation Formula (Available only for variance component and random coefficient models. Not available when a By variable is specified in the Fit Model launch window.) Saves a column to the data table that contains a formula that generates simulated values using the estimated parameters for the model that you fit. This column can be used in the Simulate utility as a Column to Switch In. See *Basic Analysis*.

Model Dialog Opens the completed Fit Model launch window for the current analysis. See "Fit Model Launch Window".

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Save By-Group Script Contains options that enable you to save a script that reproduces the platform report for all levels of a By variable to several destinations. Available only when a By variable is specified in the launch window.

JMP PRO Fit Statistics

The Fit Statistics report in the Mixed Model personality provides statistics used for model comparison. For all fit statistics, smaller is better. A likelihood ratio test between two models can be performed if one model is contained within the other. If not, a cautious comparison of likelihoods can be informative. For an example, see "Fit a Spatial Structure Model".

"Description of the Fit Statistics Report" uses the following notation:

- Specify the mixed model:

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{Z}\gamma + \varepsilon$$

  Here $\mathbf{y}$ is the *nx1* vector of observations, β is a vector of fixed-effect parameters, γ is a vector of random-effect parameters, and ε is a vector of errors.

- The vectors γ and ε are assumed to have a multivariate normal distribution where

$$E\begin{bmatrix}\gamma\\ \varepsilon\end{bmatrix} = \begin{bmatrix}\mathbf{0}\\ \mathbf{0}\end{bmatrix}$$

and

$$Var\begin{bmatrix}\gamma\\ \varepsilon\end{bmatrix} = \begin{bmatrix}\mathbf{G} & \mathbf{0}\\ \mathbf{0} & \mathbf{R}\end{bmatrix}$$

- With these assumptions, the variance of **y** is calculated as follows:

$$\mathbf{V} = \mathbf{Z}\mathbf{G}\mathbf{Z}' + \mathbf{R}$$

Description of the Fit Statistics Report

-2 Residual Log Likelihood The final evaluation of twice the negative residual log-likelihood, the objective function.

$$-2log\,likelihood_{\mathbf{R}}(\mathbf{G},\mathbf{R}) = \log|\mathbf{V}| + \log|\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}| + \mathbf{r}'\mathbf{V}^{-1}\mathbf{r} + (n-p)\log(2\pi)$$

where

$$\mathbf{r} = \mathbf{y} - \mathbf{X}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{y})$$

and p is the rank of X. Use the residual likelihood only for model comparisons where the fixed effects portion of the model is identical. See "Likelihood, AICc, and BIC".

-2 Log Likelihood The evaluation of twice the negative log-likelihood function. See "Likelihood, AICc, and BIC".

Use the log-likelihood for model comparisons in which the fixed, random, and repeated effects differ in any of the models.

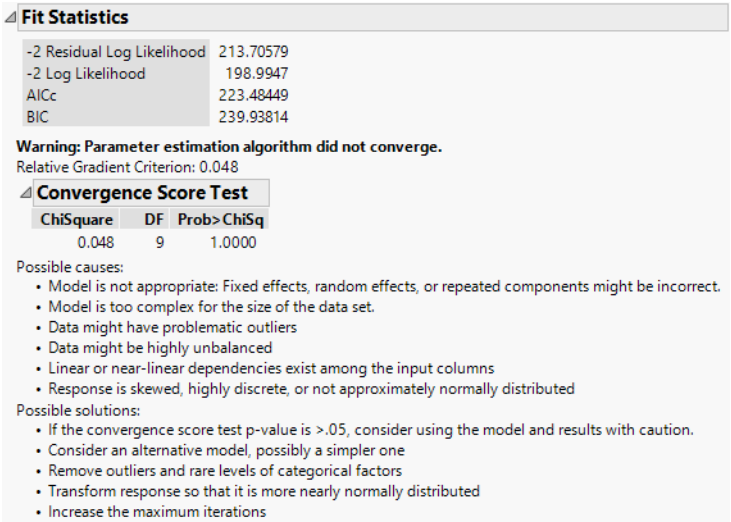
AICc Corrected Akaike's Information Criterion. See "Likelihood, AICc, and BIC".

BIC Bayesian Information Criterion. See "Likelihood, AICc, and BIC".

JMP PRO Convergence Score Test

If there are problems with model convergence, a warning message is displayed below the fit statistics. Figure 8.11 shows the warning that suggests the cause and possible solutions to the convergence issue. It also includes a test of the relative gradient at the final iteration. If this test is non-significant, the model might be correct but not fully reaching the convergence criteria. In this case, consider using the model and results with caution. See "Statistical Details for the Convergence Score Test".

Figure 8.11 Convergence Score Test



JMP PRO Random Effects Covariance Parameter Estimates

The Random Effects Covariance Parameter Estimates report in the Mixed Model personality provides details for the covariance parameters of the random effects that you specified in the model.

Covariance Parameter Lists all the covariance parameters of the random effects that you specified in the model.

> **Note:** This column is labeled Variance Component when the model contains only variance components.

Subject Lists the subject from which the block diagonal covariance matrix was formed.

Estimate Gives the estimated variance or covariance component for the effect.

Note: When the model is equivalent to a REML model, a row for a Total covariance parameter is added to the table. The estimate for the Total covariance component is the sum of the positive variance components only.

Std Error Gives the standard error for the covariance component estimate.

95% Lower Gives the lower 95% confidence limit for the covariance component. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu. See "Confidence Intervals for Variance Components".

95% Upper Gives the upper 95% confidence limit for the covariance component. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu. See "Confidence Intervals for Variance Components".

Wald p-Value Gives the *p*-value for the test that the covariance parameter is equal to zero. This column appears only when you have selected Unbounded Variance Components in the Fit Model launch window.

Var Ratio Gives the ratio of the variance component for the effect to the variance component for the residual. It compares the effect's estimated variance to the model's estimated error variance.

Pct Total Variance Gives the ratio of the variance component for the effect to the variance component for the total as a percentage.

JMP PRO Confidence Intervals for Variance Components

The method used to calculate the confidence limits depends on whether you have selected Unbounded Variance Components in the Fit Model launch window. Note that the Unbounded Variance Components is selected by default.

- If Unbounded Variance Components is selected, Wald-based confidence intervals are computed. These intervals are valid asymptotically, but note that they can be unreliable with small samples. The intervals are wider, which might lead you to mistakenly believe that an estimate is not significantly different from zero.
- If Unbounded Variance Components is not selected, meaning that the parameters have a lower boundary constraint of zero, a Satterthwaite approximation is used (Satterthwaite 1946). The confidence intervals are also bounded at zero.

JMP PRO Fixed Effects Parameter Estimates

The Fixed Effects Parameter Estimates report in the Mixed Model personality provides details for the fixed effect parameters specified in the model. For each parameter, the report provides the following details:

- the estimate
- the standard error (Std Error)
- a *t* test for the hypothesis that the estimate equals zero
- a 95% confidence interval on the estimate

The Fixed Effects Parameter Estimates report contains the following columns:

Term Gives the model term corresponding to the estimated parameter. The first term is always the intercept, unless you selected the No Intercept option in the Fit Model launch window. Continuous columns that are part of higher order terms are centered by default. Nominal or ordinal effects appear with values of levels in brackets. See "The Factor Models" for information about the coding of nominal and ordinal terms.

Note: If a continuous column is involved in a random effect, that column is not centered, even if the Center Polynomials option in the Model Specifications red triangle menu was selected.

Estimate Gives the parameter estimate for each term. This is the estimate of the term's coefficient in the model.

Std Error Gives an estimate of the standard error for the parameter estimate.

DFDen Gives the denominator degrees of freedom, that is, the degrees of freedom for error, for the effect test. DFDen is calculated using the Kenward-Roger first order approximation. See "Statistical Details for the Kackar-Harville Correction".

t Ratio Tests whether the true value of the parameter is zero. The *t* Ratio is the ratio of the estimate to its standard error. Given the usual assumptions about the model, the *t* Ratio has a Student's *t* distribution under the null hypothesis.

Prob>|t| Lists the *p*-value for a two-sided test of the *t* Ratio.

95% Lower Shows the lower 95% confidence limit for the parameter. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

95% Upper Shows the upper 95% confidence limit for the parameter. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

JMP PRO Repeated Effects Covariance Parameter Estimates

The Repeated Effects Covariance Parameter Estimates report in the Mixed Model personality provides details for the covariance parameters of the repeated effects that you specified in the model. It includes the Estimate, Standard Error, and 95% confidence bounds for each parameter. For isotropic spatial models, the covariance parameter estimates have interpretations in terms of range, nugget, and sill. See "Variogram".

Note: Variances are covariances of effects with themselves.

The Repeated Effects Covariance Parameter Estimates table contains the following columns:

Covariance Parameter Lists all the covariance parameters for the repeated effects in the model.

Estimate Gives the estimated variance or covariance component for the effect.

Std Error Gives the standard error for the variance or covariance component estimate.

95% Lower Gives the lower 95% confidence limit for the covariance component. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu. See "Confidence Intervals for Variance Components".

95% Upper Gives the upper 95% confidence limit for the covariance component. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu. See "Confidence Intervals for Variance Components".

JMP PRO Random Coefficients

For each random effect, the Mixed Model personality provides a report showing estimated coefficients and a report showing the matrix of covariance estimates. Each row of the coefficients report corresponds to one level of the random effect. The row shows all coefficient estimates associated with that level of the random effect. The random coefficient estimates are used in conjunction with fixed effect estimates to create predictions for any specific level of the random effect.

JMP PRO Random Effects Predictions

For each random effect in the model, the Random Effects Predictions report in the Mixed Model personality provides an estimate known as the *best linear unbiased predictor* (BLUP), its standard error, degrees of freedom, and a Satterthwaite-based confidence interval.

Estimation of the standard errors requires calculation of the BLUP covariance matrix, which can be time-intensive. If the calculation time is noticeable, a progress bar appears.

JMP PRO Fixed Effects Tests

The Fixed Effects Tests report in the Mixed Model personality provides a significance test for each fixed effect in the model. The test for a given effect tests the null hypothesis that all parameters associated with that effect are zero. An effect might have only one parameter as for a single continuous explanatory variable. In this case, the test is equivalent to the *t* test for that term in the Fixed Effects Parameter Estimates report. A nominal or ordinal effect can have several associated parameters, based on its number of levels. The effect test for such an effect tests whether all of the associated parameters are zero.

The Fixed Effects Tests report contains the following columns:

Source Lists the fixed effects in the model.

Nparm Shows the number of parameters associated with the effect. A continuous effect has one parameter. The number of parameters for a nominal or ordinal effect is one less than its number of levels. The number of parameters for a crossed effect is the product of the number of parameters for each individual effect.

DFNum Shows the numerator degrees of freedom for the effect test.

DFDen Shows the denominator degrees of freedom for the effect test (the degrees of freedom for error). DFDen is calculated using the Kenward-Roger first order approximation. See "Statistical Details for the Kackar-Harville Correction".

F Ratio Gives the computed *F* ratio for testing that the effect is zero.

Prob > F Gives the *p*-value for the effect test.

JMP PRO Sequential Tests

The Sequential Tests report in the Mixed Model personality provides sequential (type I) tests of the fixed effects. The report contains the sums of squares as effects are added to the model sequentially. The order of entry is defined by the order of effects as they appear in the Fit Model launch window's Construct Model Effects list.

The sums of squares that form the basis for sequential tests are also called *Type I Sums of Squares*. They are computed by fitting models in steps following the specified entry order of effects. Consider a specific effect. Compute the model sum of squares for a model containing all effects entered *prior* to that effect. Then compute the model sum of squares for a model containing those effects *and* the specified effect. The sequential sum of squares for the specified effect is the increase in the model sum of squares.

Sequential tests are considered appropriate in the following situations:

- balanced analysis of variance models specified in proper sequence (that is, two-way interactions follow main effects in the effects list, and so on)

- purely nested models specified in the proper sequence
- polynomial regression models specified in the proper sequence.

The Sequential (Type I) Tests report contains the following columns:

Source Lists the fixed effects in the model.

Nparm Shows the number of parameters associated with the effect. A continuous effect has one parameter. The number of parameters for a nominal or ordinal effect is one less than its number of levels. The number of parameters for a crossed effect is the product of the number of parameters for each individual effect.

DFNum Shows the numerator degrees of freedom for the effect test.

DFDen Shows the denominator degrees of freedom for the effect test (the degrees of freedom for error). DFDen is calculated using the Kenward-Roger first order approximation. See "Statistical Details for the Kackar-Harville Correction".

F Ratio Gives the computed *F* ratio for testing that the effect is zero.

Prob > F Gives the *p*-value for the effect test.

JMP PRO Multiple Comparisons

In the Mixed Model personality of the Fit Model platform, the Multiple Comparisons option provides various methods for comparing least squares means of main effects and interaction effects. For more information about the multiple comparisons options, see "Multiple Comparisons". For mixed model examples, see "Compare All Treatments in June" and "Example of a Split Plot Experiment".

Only the fixed effect portion of the model is used in the multiple comparisons. The Multiple Comparisons report shows estimates of the least squares means, standard error, a *t* test of no effect, and a 95% confidence interval. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu. This report is followed by the multiple comparisons test that you select. The All Pairwise Comparisons report provides equivalence tests.

JMP PRO Compare Slopes

In the Mixed Model personality of the Fit Model platform, the Compare Slopes option appears when there is one nominal term, one continuous term, and their interaction effect for the fixed effects. This option produces a report that enables you to compare the slopes in an analysis of covariance (ANCOVA) model. The report compares the slopes of each level of the interaction effect to the overall slope. The comparison uses analysis of means (ANOM) with the overall average. For more information about the analysis of means (ANOM) report, see "Comparisons with Overall Average".

The overall average slope is a weighted average of the slopes, where the weights are inversely proportional to the variances of the slope estimates. These variances are the squared values of the Std Error column in the Differences from Overall Average Slope table.

JMP PRO Marginal Model Inference

In the Mixed Model report, the marginal model plots are based on marginal predicted values and marginal residuals.

JMP PRO Actual by Predicted Plot

In the Mixed Model report, the Actual by Predicted plot appears by default. It provides a visual assessment of model fit that reflects variation due to random effects. It plots the observed values of Y against the marginal predicted values of Y. These are the predicted values obtained if you select Save Columns > Prediction Formula.

Denote the linear mixed model by $E[\mathbf{Y}|\gamma] = \mathbf{X}\beta + \mathbf{Z}\gamma$. Here β is the vector of fixed effect coefficients and γ is the vector of random effect coefficients. The *marginal predictions* are the predictions from the fixed effects part of the predictive model, given by $\mathbf{X}\hat{\beta}$.

JMP PRO Residual Plots

In the Mixed Model personality of the Fit Model platform, marginal residuals reflect the prediction error based only on the fit of fixed effects. Marginal residuals are the differences between actual values and the predicted values obtained if you select Save Columns > Prediction Formula.

Denote the linear mixed model by $E[\mathbf{Y}|\gamma] = \mathbf{X}\beta + \mathbf{Z}\gamma$. Here β is the vector of fixed effect coefficients and γ is the vector of random effect coefficients. The *marginal residuals* are the residuals from the fixed effects part of the predictive model:

$$\mathbf{r} = \mathbf{Y} - \mathbf{X}\hat{\beta}$$

The Residual Plots option provides three visual methods to assess model fit:

Residual by Predicted Plot Shows the residuals plotted against the predicted values of Y. You typically want to see the residual values scattered randomly about zero.

Residual Quantile Plot Shows the quantiles of the residuals plotted against the quantiles of a standard normal distribution. Also shown is a bar chart of the residuals. If the residuals are normally distributed, the points on the normal quantile plot should approximately fall along the red diagonal line. This type of plot is also called a quantile-quantile plot, or Q-Q plot. The normal quantile plot also shows Lilliefors confidence bounds (Conover 1999).

Residual by Row Plot Shows residuals plotted against row numbers. This plot can help you detect patterns that result from the row ordering of the observations.

JMP PRO Marginal Model Profiler

In the Mixed Model report, the Marginal Model Profiler plots are based on marginal predicted values. These are the predicted values obtained if you select Save Columns > Prediction Formula.

Denote the linear mixed model by $E[\mathbf{Y}|\gamma] = \mathbf{X}\beta + \mathbf{Z}\gamma$. Here β is the vector of fixed effect coefficients and γ is the vector of random effect coefficients. The *marginal predictions* are the predictions from the fixed effects part of the predictive model, given by $\mathbf{X}\hat{\beta}$.

Note: Marginal model profiler plots show predictions for distinct settings of the fixed effects only. The Profiler shows only cells corresponding to fixed effects. In other profilers, where random effects can be displayed, only the settings of the fixed effects determine the predicted values.

Four types of profilers are provided:

- Profiler
- Contour Profiler
- Mixture Profiler
- Surface Profiler

Options that are appropriate for the model that you are fitting are enabled. See Figure 8.21 for an example of a profiler. See Figure 8.42 for an example of a Surface Profiler. For more information about the surface profiler, see *Profilers*.

JMP PRO Variogram

In the Mixed Model report, a Variogram plot describes the spatial or temporal correlation of observations in terms of their distance. The plot shows the semivariance as a function of distance or time. The theoretical semivariance is one half of the variance of the difference between response values at locations that are a given distance apart. Note that semivariance and correlation at a given distance are inversely related. If the correlation between values at a given distance is small, the semivariance between observations at that distance is large.

When you specify any isotropic Repeated Structure (AR(1), Spatial, or Spatial with Nugget) in the Fit Model window, a Variogram plot is shown by default. If you specify the Residual structure, selecting the Variogram option in the red triangle menu enables you to select the continuous columns to be used in calculating the variogram. You can include any number of columns that describe the spatial or temporal structure of your data.

The initial Variogram report shows a plot of the empirical semivariance against distance. For additional background and more information, see "Antedependent Covariance Structure".

Semivariance Curves for Isotropic Structures

Semivariance curves are provided for the isotropic covariance structures: AR(1), Power, Exponential, Gaussian, and Spherical. For the spatial structures, curves are provided for models with and without nuggets.

The curves for the theoretical models are fit using the covariance parameter estimates. For the underlying formulas, see Chilès and Delfiner (2012) and Cressie (1993).

Use the theoretical models to determine whether your data conform to your selected isotropic structure. If you have selected the Residual structure, you can use the empirical variogram to determine whether your data exhibit some temporal or spatial structure. If the points seem follow a horizontal line, this suggests that the correlation does not change with distance and that the Residual structure is appropriate. If the points show a pattern, fitting various isotropic models might suggest an appropriate Repeated structure with which to refit your model.

JMP PRO Nugget

The *nugget* is the vertical jump from the value of 0 at the origin of the variogram to the value of the semivariance at a very small separation distance. A variogram model with a nugget has a discontinuity at the origin. The value of the theoretical curve for distances just above 0 is the nugget.

JMP PRO Variogram Options

AR(1) Plots a variogram for an AR(1) covariance structure.

Spatial Plots a variogram for an Exponential, Gaussian, Power, or Spherical covariance structure.

Spatial with Nugget Plots a variogram for an Exponential, Gaussian, Power, or Spherical covariance structure with nugget.

JMP PRO Conditional Model Inference

In the Mixed Model report, the conditional model diagnostic plots are based on conditional residuals. Conditional residuals reflect the prediction error once both fixed and random effects have been fit.

JMP PRO Actual by Conditional Predicted Plot

In the Mixed Model report, the Actual by Conditional Predicted plot appears by default. It provides a visual assessment of model fit that accounts for variation due to random effects. It plots the observed values of Y against the conditional predicted values of Y. These are the predicted values obtained if you select Save Columns > Conditional Prediction Formula.

Denote the linear mixed model by $E[\mathbf{Y}|\gamma] = \mathbf{X}\beta + \mathbf{Z}\gamma$. Here β is the vector of fixed effect coefficients and γ is the vector of random effect coefficients. The *conditional predictions* are the predictions obtained from the model given by $\mathbf{X}\hat{\beta} + \mathbf{Z}\hat{\gamma}$.

JMP PRO Conditional Residual Plots

In the Mixed Model report, the conditional residual plots reflect the prediction error based on fitting both fixed and random effects. Conditional residuals are the differences between actual values and the conditional predicted values obtained if you select Save Columns > Conditional Prediction Formula.

Denote the linear mixed model by $E[\mathbf{Y}|\gamma] = \mathbf{X}\beta + \mathbf{Z}\gamma$. Here β is the vector of fixed effect coefficients and γ is the vector of random effect coefficients. The *conditional residuals* are calculated as follows:

$$\mathbf{r} = \mathbf{Y} - (\mathbf{X}\hat{\beta} + \mathbf{Z}\hat{\gamma})$$

The Conditional Residual Plots option provides three visual methods to assess model fit:

Conditional Residual by Predicted Plot Shows the conditional residuals plotted against the conditional predicted values of *Y*. You typically want to see the conditional residual scattered randomly about zero.

Conditional Residual Quantile Plot Shows the quantiles of the conditional residuals plotted against the quantiles of a standard normal distribution. Also shown is a bar chart of the conditional residuals. If the conditional residuals are normally distributed, the points on the normal quantile plot should approximately fall along the red diagonal line. This type of plot is also called a quantile-quantile plot, or Q-Q plot. The normal quantile plot also shows Lilliefors confidence bounds (Conover 1999).

Conditional Residual by Row Plot Shows conditional residuals plotted against row numbers. This plot can help you detect patterns that result from the row ordering of the observations.

JMP PRO Conditional Profilers

In the Mixed Model report, the conditional model profiler plots are based on conditional predicted values. These are the predicted values obtained if you select Save Columns > Conditional Prediction Formula.

Denote the linear mixed model by $E[\mathbf{Y}|\gamma] = \mathbf{X}\beta + \mathbf{Z}\gamma$. Here β is the vector of fixed effect coefficients and γ is the vector of random effect coefficients. The *conditional predictions* are the predictions obtained from the model given by $\mathbf{X}\hat{\beta} + \mathbf{Z}\hat{\gamma}$.

Four types of profilers are provided:

- Conditional Profiler
- Conditional Contour Profiler
- Conditional Mixture Profiler
- Conditional Surface Profiler

Options that are appropriate for the model that you are fitting are enabled. See Figure 8.21 for an example of a Profiler. See Figure 8.42 for an example of a Surface Profiler. For more information about the profiler, see *Profilers*.

JMP PRO Additional Examples of the Mixed Model Personality

This section contains examples using the Mixed Model personality of the Fit Model platform.

- "Example of Repeated Measures"
- "Example of a Split Plot Experiment"
- "Example of a Uniformity Trial"
- "Example of a Correlated Response"

JMP PRO Example of Repeated Measures

You are interested in using the Fit Model platform to fit a repeated measures mixed model to determine whether either of two new drugs is effective at lowering cholesterol and whether time and the treatment interact. A study was performed to test two new cholesterol drugs against a control drug. Twenty patients with high cholesterol were randomly assigned to each of four treatments (the two experimental drugs, the control, and a placebo). Each patient's total cholesterol was measured at six times during the study: the first day in April, May, and June in the morning and afternoon.

JMP PRO Background

Two methods have historically been used to analyze such a design:

- Multivariate analysis of variance (MANOVA)
- A split-plot in time univariate analysis of variance (ANOVA) with either the Huynh-Feldt (1976) or Greenhouse-Geisser (1959) correction

Both of these options are available using the MANOVA personality in Fit Model. These two options are the two extremes for modeling the covariance structure. The MANOVA analysis assumes an unstructured covariance structure where all variances and covariances are estimated individually. The independent split-plot in time analysis assumes that all errors are independent. In the Gaussian data case, this is equivalent to assuming a compound symmetry covariance structure.

These two models can result in vastly different conclusions about the treatment effects. When you assume a complex covariance structure, information in the data is used to estimate the covariance parameters. If you fit too many covariance parameters, you run the risk of overfitting your model. When you model repeated measures data, you must find a covariance structure that balances these issues.

- When the model is overfit, the power to detect differences is smaller than if you were to assume a less complex covariance structure.
- When the model is underfit, Type I error control is lost. In some cases, this leads to inflated rejection rates. In other cases, decreased rejection rates occur due to inflated variance.

JMP PRO Covariance Structures

The Mixed Model personality fits a variety of covariance structures. For repeated measures in time, both the Toeplitz covariance structure and the first-order autoregressive (AR(1)) covariance structures often provide appropriate correlation structures. These structures allow for correlated observations without overfitting the model. The AR(1) assumes a common variance parameter, whereas the Toeplitz covariance matrix with unequal variances estimates unique variances for each unit of the repeated measure variable. See "Repeated Covariance Structure Requirements".

In this example, you fit the four covariance structures. The number of observation times, J, is equal to six.

- "Covariance Structure: Unstructured". The Unstructured model fits all covariance parameters, $J(J+1)/2$ in total. In this example, the model fits 21 variances.
- "Covariance Structure: Residual". The Residual model is equivalent to the usual variance components structure. In this example, the model fits two variances.
- "Covariance Structure: Toeplitz". The Toeplitz model fits $2J$-1 covariance parameters. In this example, the model fits 11 variances.
- "Covariance Structure: AR(1)". This model fits two covariance parameters. One parameter determines the variance and the other determines how the covariance changes with time.

You use AICc to evaluate model fits. The BIC criterion can also be used. In this case, the same model is chosen by both criteria. You select a best covariance structure and then continue to do additional analysis:

- "Further Analysis Using AR(1) Structure"
- "Regression Model for AR(1) Model Example"

Tip: Leave the Fit Model launch window open as you work through this example.

JMP PRO Data Structure

The Cholesterol.jmp data table is in a format that is typically used for recording repeated measures data. To use the Mixed Model personality to analyze these data, each cholesterol measurement needs to be in its own row, as in Cholesterol Stacked.jmp. To construct Cholesterol Stacked.jmp, the data in Cholesterol.jmp were stacked using Tables > Stack.

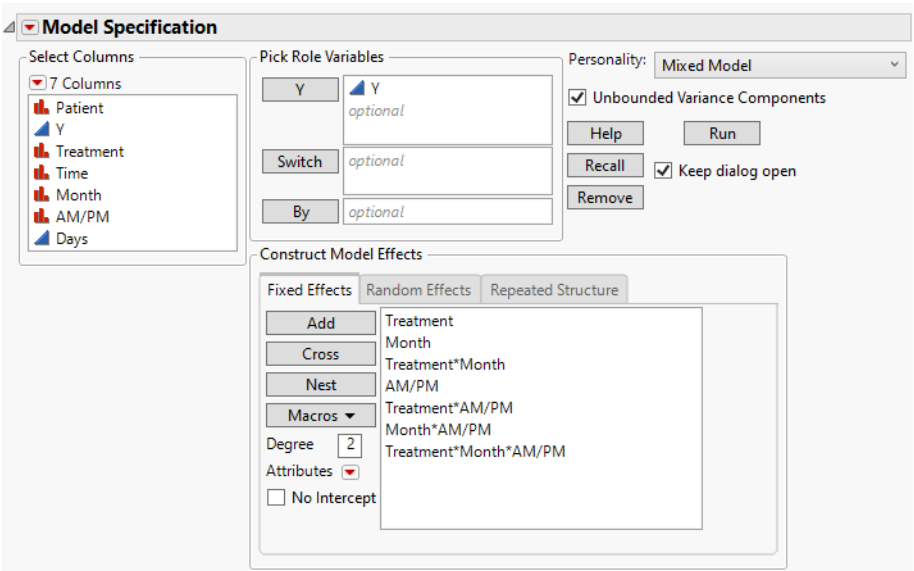
The Days column in the stacked table was constructed using a formula. The Days column gives the number days into the study when the cholesterol measurement was taken. Its modeling type is continuous. This is necessary because the AR(1) covariance structure requires the repeated effect be continuous.

JMP PRO Covariance Structure: Unstructured

Begin by fitting a model using an Unstructured covariance structure.

1. Select **Help > Sample Data Folder** and open Cholesterol Stacked.jmp.
2. Select **Analyze > Fit Model**.
3. Select **Keep dialog open** so that you can return to the launch window in the next example.
4. Select Y and click **Y**.
5. Select **Mixed Model** from the Personality list.
6. Select Treatment, Month, and AM/PM, and then select **Macros > Full Factorial**.

Figure 8.12 Fit Model Launch Window Showing Completed Fixed Effects Tab

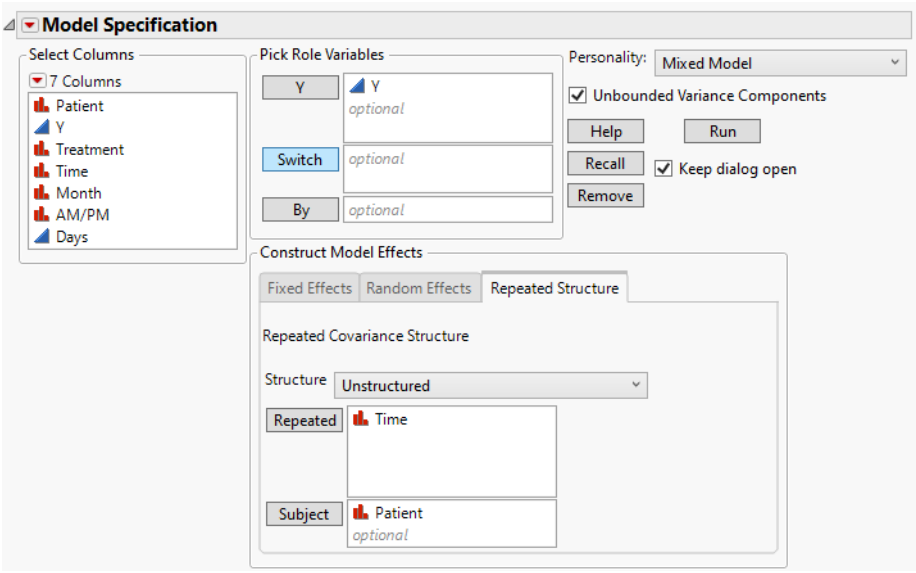


7. Select the **Repeated Structure** tab.

8. Select **Unstructured** from the Structure list.
9. Select Time and click **Repeated**. The **Repeated** column defines the repeated measures within a subject.
10. Select Patient and click **Subject**.

Note: The Unstructured covariance model does not allow the repeated structure variables to assume duplicate values. Suppose that, in this example, the subject was nested within treatment, and that the patients had been numbered using the values 1, 2, 3, 4, and 5 within each treatment. A warning would be given when you run this analysis. You would need to renumber the patients to have different identifiers for each value of the Repeated variable. Or you would need to create a column in the data table that represents nesting within treatment and enter this effect as Subject.

Figure 8.13 Fit Model Launch Window Showing Completed Repeated Structure Tab



11. Click **Run**.

The Mixed Model report is shown in Figure 8.14. Because you want to compare your three models using AICc or BIC, you are interested in the Fit Statistics report. The AICc for the unstructured model is 703.84.

The Repeated Effects Covariance Parameter Estimates report shows estimates of all 21 covariance parameters. As you would expect, observations taken closer in time have higher covariance than those farther apart. Also, variance increases with time.

Figure 8.14 Mixed Model Report for Unstructured Covariance Structure

Mixed Model for Y

Actual by Predicted Plot

Fit Statistics

-2 Residual Log Likelihood	554.98818
-2 Log Likelihood	557.89101
AICc	703.83696
BIC	773.32814

Repeated Effects Covariance Parameter Estimates

Repeated Effect: Time
Subject: Patient

Covariance Parameter	Estimate	Std Error	95% Lower	95% Upper
Var(April AM)	18.725	6.6202872	5.7494754	31.700525
Cov(April PM,April AM)	18.354884	6.6035621	5.4121399	31.297628
Var(April PM)	19.268932	6.8125963	5.9164888	32.621376
Cov(May AM,April AM)	9.2756709	8.4629182	-7.311344	25.862686
Cov(May AM,April PM)	5.5074058	8.3704001	-10.89828	21.913089
Var(May AM)	56.603347	20.012305	17.379949	95.826744
Cov(May PM,April AM)	9.4147226	8.5038584	-7.252534	26.081979
Cov(May PM,April PM)	6.6230805	8.4532303	-9.944946	23.191108
Cov(May PM,May AM)	55.365523	19.83529	16.489068	94.241978
Var(May PM)	57.058089	20.173081	17.519577	96.596602
Cov(June AM,April AM)	1.1945478	8.6266098	-15.7133	18.102392
Cov(June AM,April PM)	0.3183447	8.7461245	-16.82374	17.460434
Cov(June AM,May AM)	1.106725	14.992149	-28.27735	30.490796
Cov(June AM,May PM)	0.6455101	15.050552	-28.85303	30.14405
Var(June AM)	63.512277	22.454981	19.501323	107.52323
Cov(June PM,April AM)	1.5810194	8.7687993	-15.60551	18.76755
Cov(June PM,April PM)	0.7647113	8.8882627	-16.65596	18.185386
Cov(June PM,May AM)	0.9262595	15.232066	-28.92804	30.780561
Cov(June PM,May PM)	0.6543895	15.292238	-29.31785	30.626625
Cov(June PM,June AM)	63.878686	22.700344	19.386829	108.37054
Var(June PM)	65.568482	23.181959	20.132677	111.00429

Fixed Effects Parameter Estimates

Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Treatment	3	3	16.0	274.96713	<.0001*
Month	2	2	15.0	340.48166	<.0001*
Treatment*Month	6	6	18.3	123.47461	<.0001*
AM/PM	1	1	16.0	360.93593	<.0001*
Treatment*AM/PM	3	3	16.0	0.6339843	0.6038
Month*AM/PM	2	2	15.0	1.1988247	0.3289
Treatment*Month*AM/PM	6	6	18.3	1.1642781	0.3671

JMP PRO Covariance Structure: Residual

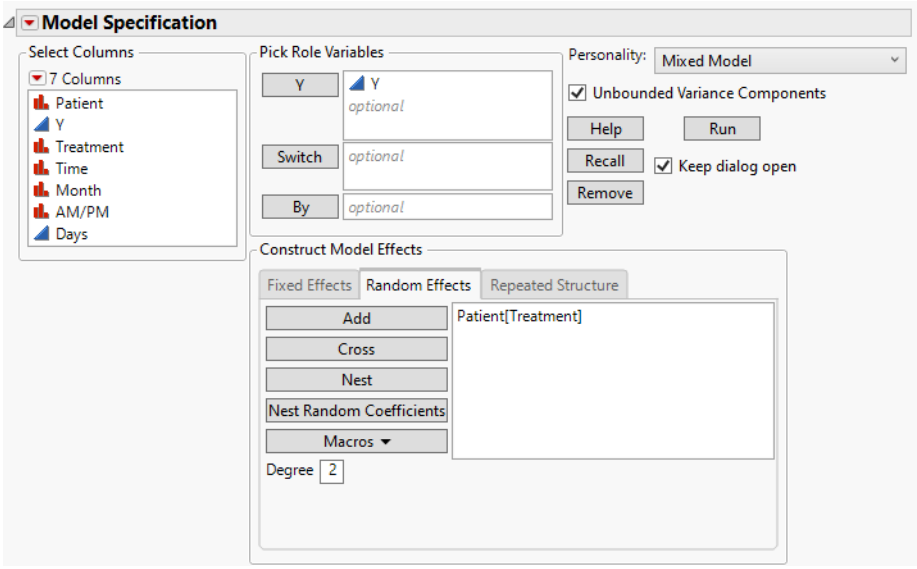
The Residual covariance structure is appropriate when you fit a split-plot model.

1. Complete step 1 through step 7 in "Covariance Structure: Unstructured".
2. On the **Repeated Structure** tab, select **Residual** from the Structure list.
3. If you are continuing from the previous example, remove Time and Patient.

   Otherwise, a warning appears: "Repeated columns and subject columns are ignored when the Residual covariance structure is selected." You are given the option to click **OK** to continue the analysis.

4. Select the **Random Effects** tab.
5. Select Patient and click **Add**.
6. Select Patient in the Random Effects area, select the Treatment column, and then click **Nest**.

Figure 8.15 Fit Model Launch Window Showing Completed Random Effects Tab



7. Click **Run**.

   The Mixed Model report is shown in Figure 8.16. The Fit Statistics report shows that the AICc for the Residual model is 832.55, as compared to 703.84 for the Unstructured model.

   The estimates of the two covariance parameters are shown in the Random Effects Covariance Parameter Estimates report. These are estimates of the variance of Patient within Treatment, and of the Residual variance.

Figure 8.16 Mixed Model Report for Residual Error Covariance Structure

Mixed Model for Y

Actual by Predicted Plot **Actual by Conditional Predicted Plot**

Fit Statistics

-2 Residual Log Likelihood	721.03949
-2 Log Likelihood	765.45515
AICc	832.55192
BIC	889.92993

Random Effects Covariance Parameter Estimates

Variance Component	Var Ratio	Estimate	Std Error	95% Lower	95% Upper	Wald p-Value	Pct of Total
Patient[Treatment]	0.33372	11.707432	6.2749012	-0.591148	24.006012	0.0621	25.022
Residual		35.081923	5.546939	26.320843	49.105827		74.978
Total		46.789355	7.7386523	34.678063	66.607248		100.000

Fixed Effects Parameter Estimates

Random Coefficients

Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Treatment	3	3	16.0	274.96713	<.0001*
Month	2	2	80.0	622.88066	<.0001*
Treatment*Month	6	6	80.0	226.49464	<.0001*
AM/PM	1	1	80.0	13.700114	0.0004*
Treatment*AM/PM	3	3	80.0	0.0240643	0.9949
Month*AM/PM	2	2	80.0	0.0324977	0.9680
Treatment*Month*AM/PM	6	6	80.0	0.030708	0.9999

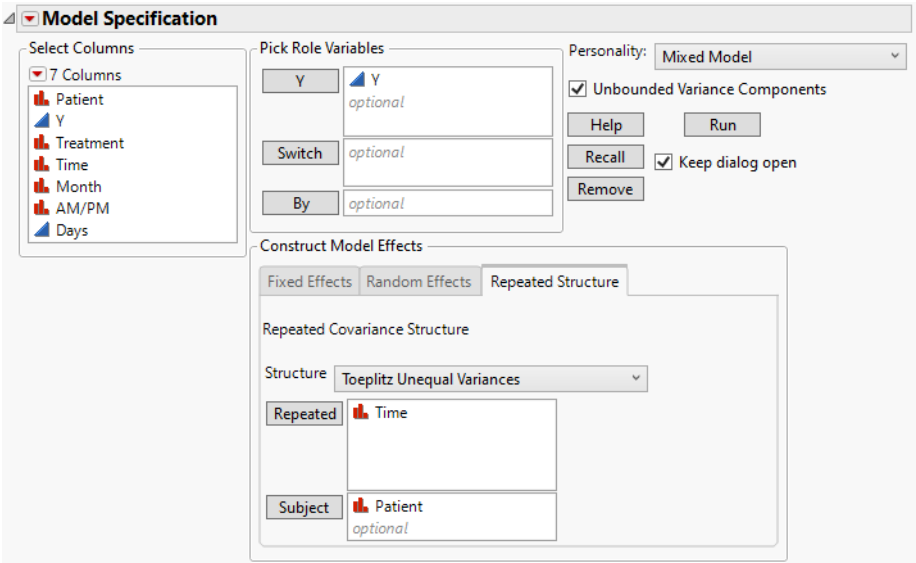
JMP PRO Covariance Structure: Toeplitz

Fit the model using the Toeplitz Unequal Variances structure.

1. Complete step 1 through step 6 in "Covariance Structure: Unstructured".
2. If you are continuing from the previous example, select Patient[Treatment] on the **Random Effects** tab and then click **Remove**.

   If you include both random effects and repeated effects, there is often insufficient data to estimate both effects.
3. Select the **Repeated Structure** tab.
4. Select **Toeplitz Unequal Variances** from the Structure list.
5. Select Time and click **Repeated**.
6. Select Patient and click **Subject**.

Figure 8.17 Fit Model Launch Window Showing Completed Repeated Structure Tab



7. Click **Run**.

Figure 8.18 Mixed Model Report for Toeplitz Unequal Variances Structure

Mixed Model for Y

Actual by Predicted Plot

Fit Statistics

-2 Residual Log Likelihood	659.09915
-2 Log Likelihood	688.02972
AICc	788.02972
BIC	855.59193

Repeated Effects Covariance Parameter Estimates

Subject: Patient

Covariance Parameter	Estimate	Std Error	95% Lower	95% Upper
Toeplitz Correlation(1)	0.6230225	0.0856377	0.4551756	0.7908693
Toeplitz Correlation(2)	0.2555783	0.1463264	-0.031216	0.5423728
Toeplitz Correlation(3)	0.0962006	0.1887883	-0.273818	0.466219
Toeplitz Correlation(4)	-0.1112	0.2248683	-0.551934	0.3295338
Toeplitz Correlation(5)	0.2427218	0.1955609	-0.140571	0.6260141
Variance(April AM)	17.552396	6.9782256	3.8753253	31.229467
Variance(April PM)	24.652754	8.3215651	8.3427857	40.962721
Variance(May AM)	73.950898	25.930509	23.128035	124.77376
Variance(May PM)	63.668022	21.766899	21.005684	106.33036
Variance(June AM)	69.45679	22.578117	25.204495	113.70909
Variance(June PM)	52.484909	19.171565	14.909332	90.060486

Fixed Effects Parameter Estimates

Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Treatment	3	3	14.1	230.25344	<.0001*
Month	2	2	23.7	363.8599	<.0001*
Treatment*Month	6	6	26.2	134.01838	<.0001*
AM/PM	1	1	5.2	106.73459	0.0001*
Treatment*AM/PM	3	3	5.2	0.1874794	0.9007
Month*AM/PM	2	2	26.9	0.036706	0.9640
Treatment*Month*AM/PM	6	6	34.6	0.0347301	0.9998

Note: The Mixed Model personality in JMP reports correlations, whereas PROC MIXED in SAS reports covariances.

The Fit Statistics report shows that the AICc for the Toeplitz with Unequal Variances model is 788.03. Compare this number to 832.55 for the Residual Model and 703.84 for the Unstructured model.

The Toeplitz Unequal Variances structure requires the estimation of eleven covariance parameters. These estimates are shown in the Repeated Effects Covariance Parameter Estimates report. The Toeplitz correlation estimates are shown, followed by the variance estimates for each time point. See "Statistical Details for Repeated Measures" for information about how this matrix is parameterized.

JMP PRO Covariance Structure: AR(1)

Finally, fit the AR(1) structure.

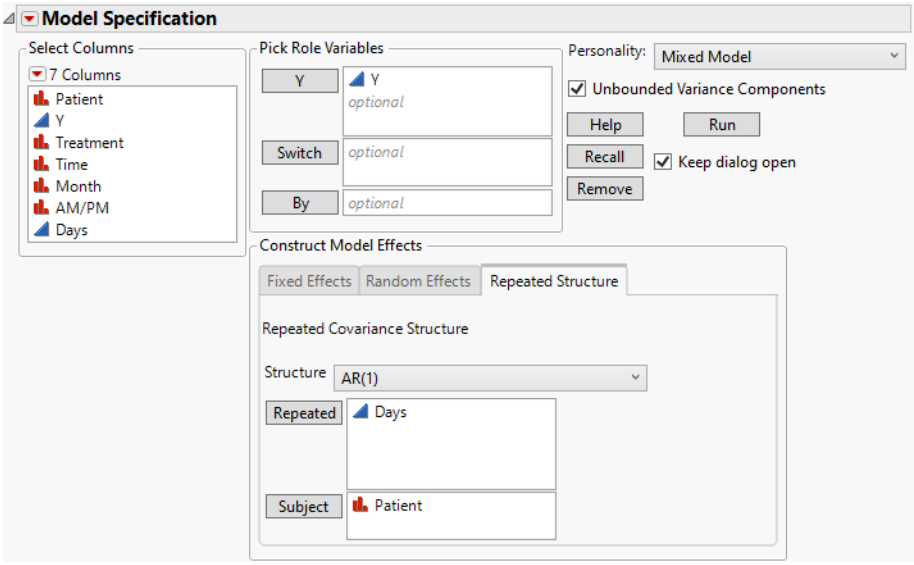
1. Complete step 1 through step 6 in "Covariance Structure: Unstructured".

2. If you are continuing from the previous example, select Time in the **Repeated** box and then click **Remove**.

   AR(1) requires a continuous variable for the repeated value.

3. Select **AR(1)** from the Structure list.
4. Select Days and click **Repeated**.

Figure 8.19 Fit Model Launch Window Showing Completed Repeated Structure Tab



5. Click **Run**.

   The Mixed Model report is shown in Figure 8.20. The Fit Statistics report shows that the AICc for the AR(1) model is 652.63. Compare this number to 832.55 for the Residual model, 703.84 for the Unstructured model, and 788.03 for the Toeplitz Unequal Variances model. Based on the AICc criterion, the AR(1) model is the best of the four models.

   The AR(1) structure requires the estimation of two covariance parameters. These estimates are shown in the Repeated Effects Covariance Parameter Estimates report. The AR(1) Days parameter estimate is an estimate of ρ, the correlation parameter in the AR(1) structure.

   The Variogram plot shows the empirical semivariances and the curve for the AR(1) model. Since there are only five nonzero values for Days, only four distance classes are possible and only four points are shown. The AR(1) structure seems appropriate. To explore other structures, select options from the Variogram red triangle menu. For more information about Variogram options, see "Variogram".

Figure 8.20 Mixed Model Report for AR(1) Covariance Structure

Mixed Model for Y

Actual by Predicted Plot

Fit Statistics

-2 Residual Log Likelihood	577.09954
-2 Log Likelihood	585.53021
AICc	652.62698
BIC	710.00499

Repeated Effects Covariance Parameter Estimates

Subject: Patient

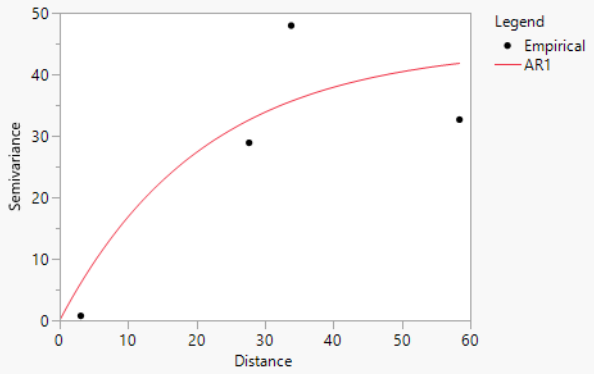
Covariance Parameter	Estimate	Std Error	95% Lower	95% Upper
AR(1) Days	0.9536532	0.0120225	0.9300895	0.9772168
Residual	44.579921	9.1281274	30.970215	69.685214

Fixed Effects Parameter Estimates

Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Treatment	3	3	22.8	242.61165	<.0001*
Month	2	2	80.8	266.42271	<.0001*
Treatment*Month	6	6	82.7	98.224978	<.0001*
AM/PM	1	1	57.6	461.72978	<.0001*
Treatment*AM/PM	3	3	57.6	0.8110288	0.4930
Month*AM/PM	2	2	58.0	1.0871135	0.3440
Treatment*Month*AM/PM	6	6	58.0	1.0275594	0.4168

Variogram



JMP PRO Further Analysis Using AR(1) Structure

Because the AR(1) model gives the best fit, you adopt it as your model and proceed with your analysis. The Fixed Effects Tests report indicates that there is a significant interaction between Treatment and Month as well as a main effect of AM/PM. Here, we explore these significant effects.

1. Click the Mixed Model red triangle and select **Marginal Model Inference > Profiler**.

   The Marginal Model Profiler report (Figure 8.21) enables you to see the effect on cholesterol levels (Y) for various settings of Treatment, Month, and AM/PM.

2. In the plot for Month, drag the vertical dotted red line from April to May and then to June.

Notice that the predicted AM measurements for Y decrease over the three months from a mean of 277.4 in April to a mean of 177.7 in June.

3. In the plot for Treatment, drag the vertical dotted red line from A to B.

   By dragging the line in the plot for Month from April to June, you see that, for Treatment B, the predicted AM mean for Y decreases from 276.8 in April to 191.2 in June.

4. In the plot for Treatment, drag the vertical dotted line to Control and then to Placebo.

   Notice that when you set Treatment to Control or Placebo, you see virtually no change over the three months (Figure 8.22).

   Next, you explore the effect of AM/PM.

5. Set Treatment and Month to all twelve combinations of their levels by dragging the vertical red lines.

   For all twelve combinations, the predicted cholesterol level is consistently higher in the afternoon than in the morning, demonstrating the main effect.

Note that Treatment A seems to result in lower cholesterol readings in May than Treatment B does. If this effect is significant, it might indicate that Treatment A acts more quickly than B. The next section, "Compare All Treatments in June", shows you how to evaluate the treatments.

Figure 8.21 Marginal Profiler Plot for Treatment A

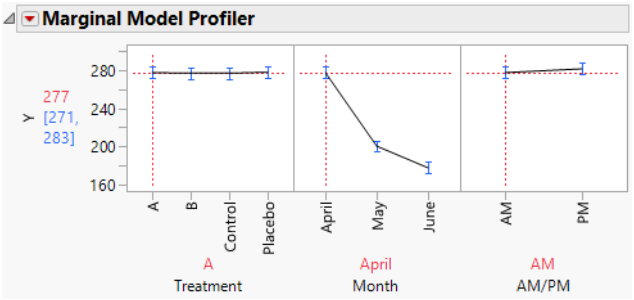
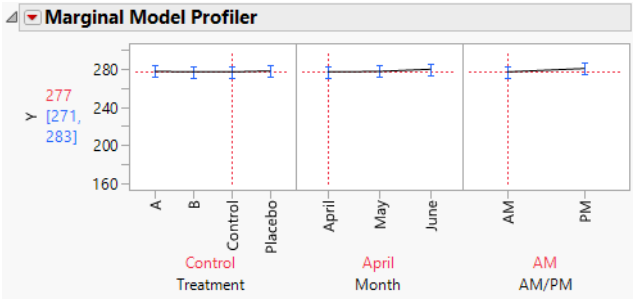


Figure 8.22 Marginal Profiler Plot for Control

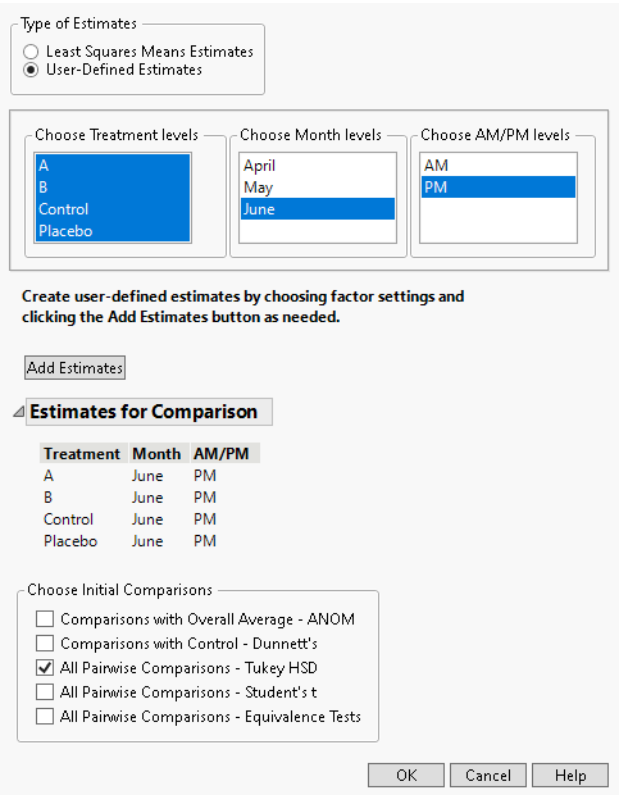


Compare All Treatments in June

The study is conducted over the months of April, May, and June. You are interested in which treatments differ on the PM measurement in June.

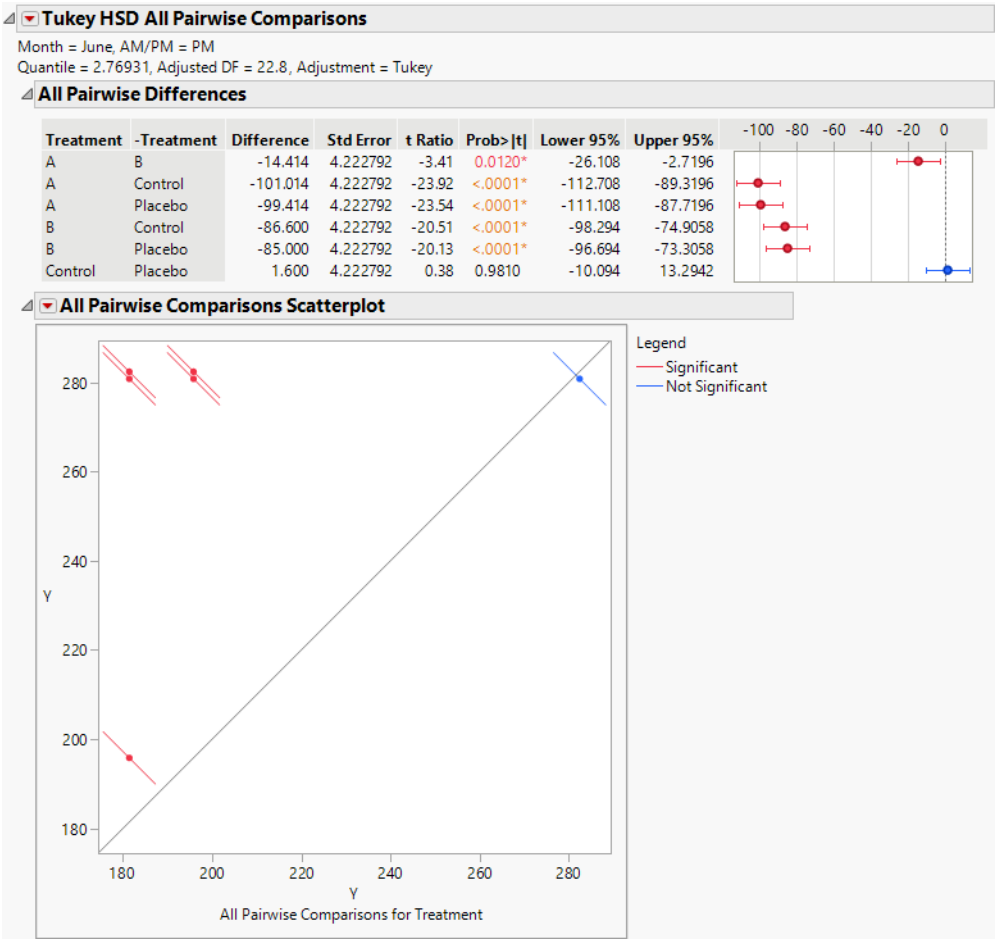
1. Click the Mixed Model red triangle and select **Multiple Comparisons**.
2. Under Types of Estimates, select **User-Defined Estimates**.
3. From the Choose Treatment levels panel, select all four treatment types.
4. From the Choose Month levels panel, select June.
5. From the Choose AM/PM levels panel, select PM.
6. Click Add Estimates.
7. From the Choose Initial Comparisons list, select **All Pairwise Comparisons - Tukey HSD**.

Figure 8.23 Completed Multiple Comparisons Window



8. Click **OK**.

Figure 8.24 Tukey HSD All Pairwise Comparisons Report for All Treatments for June PM



The Tukey HSD All Pairwise Comparisons report shows an All Pairwise Differences report and an All Pairwise Comparisons Scatterplot. All treatments other than the Control and Placebo differ significantly on the June PM measurements.

Consider the difference between treatments A and B. The difference in means is -14.414 and the confidence interval ranges from -26.108 to -2.7196. You conclude that the reduction in cholesterol measurements due to treatment A exceeds the reduction by treatment B by somewhere between 2.7 and 26.1 points. Both treatments A and B are highly effective compared to the Control and Placebo.

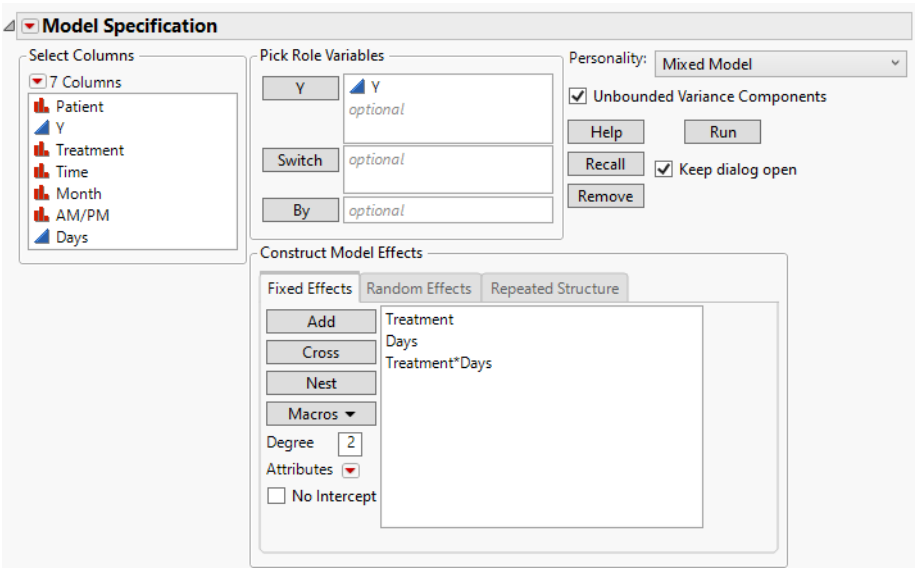
JMP PRO Regression Model for AR(1) Model Example

Using the Month and AM/PM categorical effects, you have compared four covariance structures for the cholesterol data. (Note that a categorical effect was required for the Unstructured fit.) You have decided to use an AR(1) covariance structure.

Suppose now that you want to model the effect of treatment in terms of the continuous effect Days instead of the categorical effects. You can then predict cholesterol levels at arbitrary time during treatment.

1. After following step 1 to step 4 in "Covariance Structure: AR(1)", return to the Fit Model launch window.
2. On the Fixed Effects tab, select the existing fixed effects and click **Remove**.
3. Select Treatment and Days then select **Macros > Full Factorial**.

Figure 8.25 Fit Model Launch Window Showing Fixed Effects Tab



4. Click the Model Specification red triangle and deselect **Center Polynomials**.

Note: In the default setting, the continuous effects used in interaction terms are centered. By turning off the Center Polynomials option, the continuous effects used in interaction terms are not centered.

5. Click **Run**.

The Mixed Model report is shown in Figure 8.26. You see that the interaction of Treatment and Days is highly significant indicating different regressions for the drugs.

Note: To predict outcomes for the drugs at different days, use the profiler. See *Profilers*.

Figure 8.26 Mixed Model Report for AR(1) Covariance Structure with Continuous Fixed Effect

Mixed Model for Y

Fit Statistics

-2 Residual Log Likelihood	791.54856
-2 Log Likelihood	789.76821
AICc	811.78655
BIC	837.64312

Repeated Effects Covariance Parameter Estimates

Subject: Patient

Covariance Parameter	Estimate	Std Error	95% Lower	95% Upper
AR(1) Days	0.7983061	0.0473103	0.7055796	0.8910327
Residual	100.5539	18.519406	72.239307	149.60244

Fixed Effects Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|	95% Lower	95% Upper
Intercept	276.17698	1.9946144	52.7	138.46	<.0001*	272.17582	280.17815
Treatment[A]	-6.13599	3.4547735	52.7	-1.78	0.0815	-13.06621	0.7942322
Treatment[B]	-0.352827	3.4547735	52.7	-0.10	0.9190	-7.283049	6.5773954
Treatment[Control]	1.9453265	3.4547735	52.7	0.56	0.5758	-4.984896	8.8755487
Days	-0.735803	0.0505395	53.3	-14.56	<.0001*	-0.83716	-0.634446
Treatment[A]*Days	-0.877856	0.087537	53.3	-10.03	<.0001*	-1.053411	-0.7023
Treatment[B]*Days	-0.643666	0.087537	53.3	-7.35	<.0001*	-0.819221	-0.46811
Treatment[Control]*Days	0.788161	0.087537	53.3	9.00	<.0001*	0.6126056	0.9637164

Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Treatment	3	3	52.7	1.302892	0.2832
Days	1	1	53.3	211.9633	<.0001*
Treatment*Days	3	3	53.3	76.472494	<.0001*

JMP PRO Example of a Split Plot Experiment

In this example, use the Mixed Model personality in the Fit Model platform to specify and analyze a split-plot experiment with two factors. When you specify the Mixed Model personality, the Fit Model launch window provides tabs for specifying effects. The resulting analysis is targeted to random effects. Note, however, that split-plot experiments can also be analyzed using the Standard Least Squares personality.

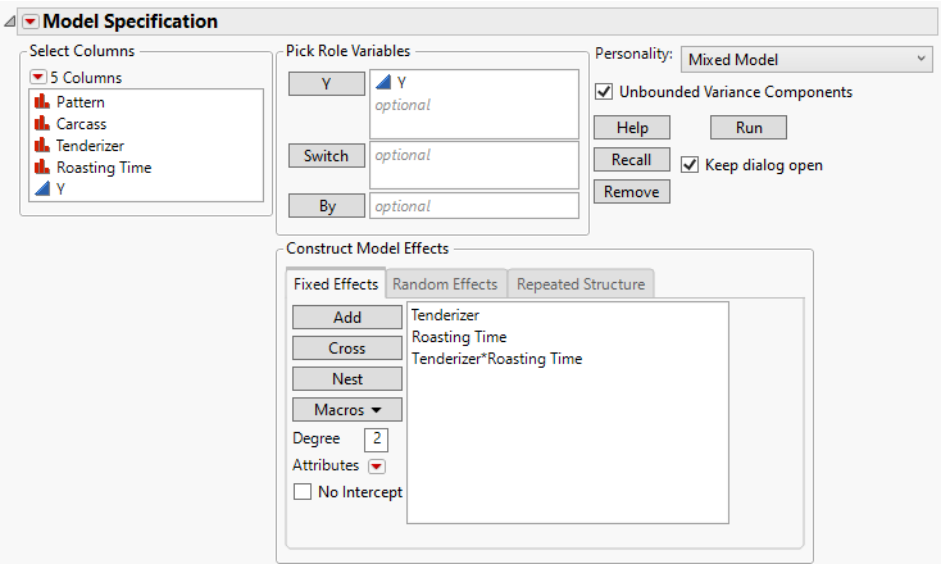
The data in this example come from a study of the effects of tenderizer and length of cooking time on meat. Six beef carcasses were randomly selected from carcasses at a meat packaging plant. From the right rib-eye muscle of each carcass, three rolled roasts were prepared under uniform conditions. Each of these three roasts was assigned a tenderizing treatment at random. After treatment, a coring device was used to mark four cores of meat near the center of each.

The three roasts from the same carcass were placed together in a preheated oven and allowed to cook. After 30 minutes, one of the cores was taken at random from each roast. Cores were removed in this fashion again after 36 minutes, 42 minutes, and 48 minutes. As each set cooled to serving temperature, the cores were measured for tenderness using the Warner-Bratzler device. Larger measurements indicate tougher meat.

Your interest centers on the effects of tenderizer, roasting time, and especially whether there is an interaction between tenderizer and roasting time. This design addresses that goal.

1. Select **Help > Sample Data Folder** and open Split Plot.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Select **Mixed Model** from the Personality list.
5. Select Tenderizer and Roasting Time, and then select **Macros > Full Factorial**.

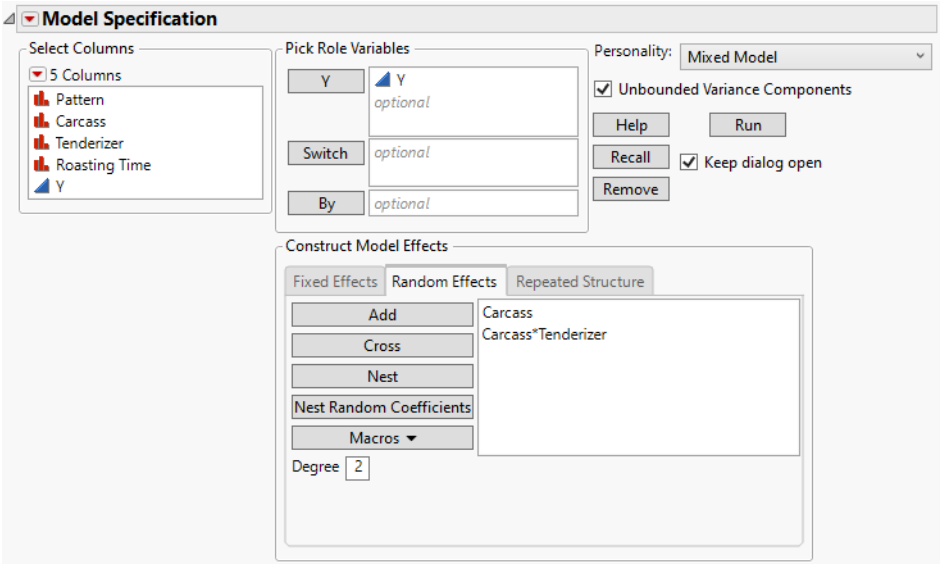
Figure 8.27 Fit Model Launch Window Showing Completed Fixed Effects Tab



6. Select the **Random Effects** tab.
7. Select Carcass and click **Add** to create the random carcass effect.
8. Select Carcass and Tenderizer and click **Cross**.

   The Carcass*Tenderizer interaction is the error term for the whole plot factor, Tenderizer. This is equivalent to the Carcass*Tenderizer&Random term in Standard Least Squares.

Figure 8.28 Fit Model Launch Window Showing Completed Random Effects Tab

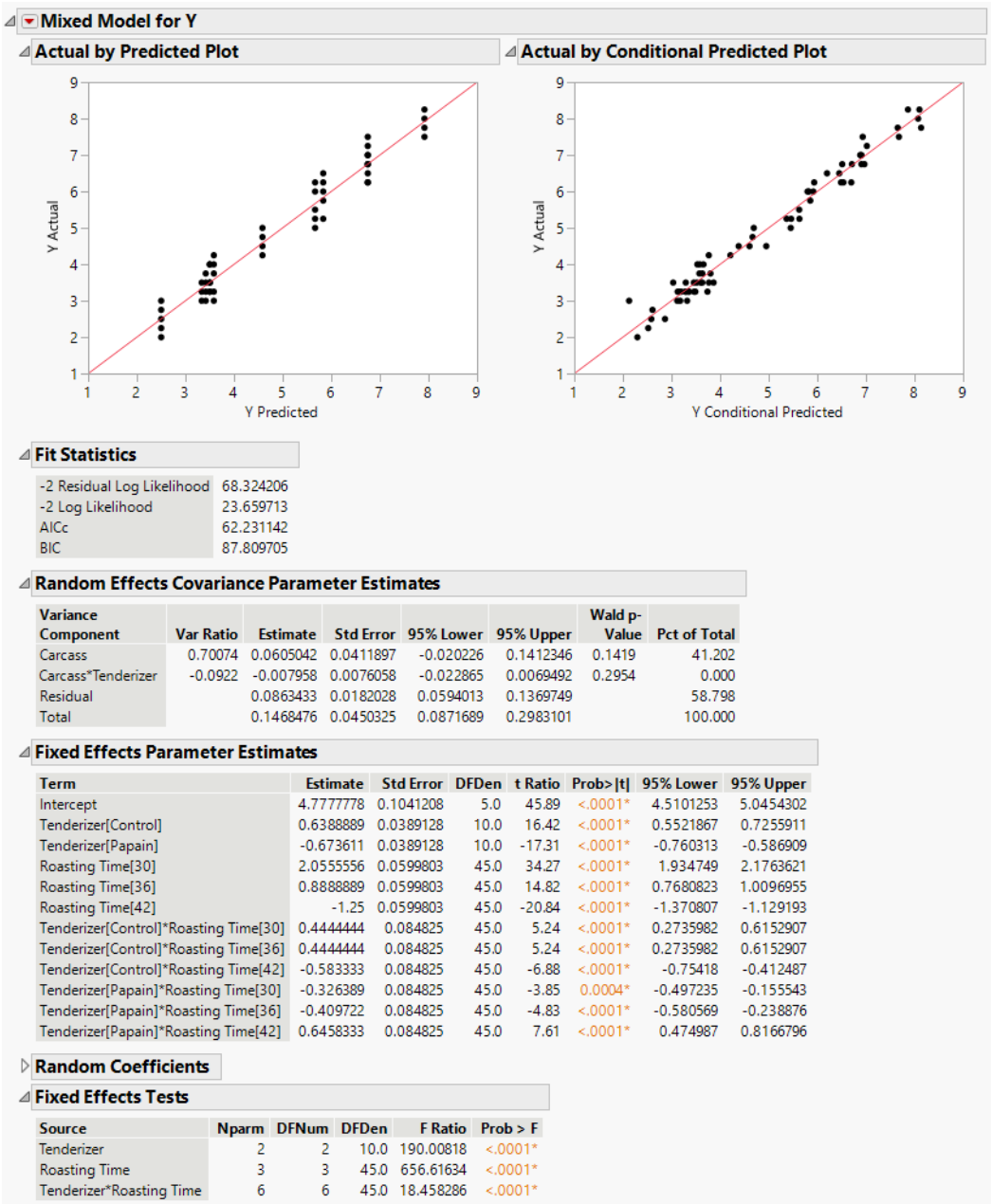


9. Click **Run**.

   The Mixed Model report is shown in Figure 8.29.

   The Actual by Predicted Plot and the Actual by Conditional Predicted Plot show no issues with model fit, so you can proceed to interpret the results. The Fixed Effects Tests report indicates that there is a significant interaction between tenderizer and roasting time.

Figure 8.29 Mixed Model Report



Fit Statistics

-2 Residual Log Likelihood	68.324206
-2 Log Likelihood	23.659713
AICc	62.231142
BIC	87.809705

Random Effects Covariance Parameter Estimates

Variance Component	Var Ratio	Estimate	Std Error	95% Lower	95% Upper	Wald p-Value	Pct of Total
Carcass	0.70074	0.0605042	0.0411897	-0.020226	0.1412346	0.1419	41.202
Carcass*Tenderizer	-0.0922	-0.007958	0.0076058	-0.022865	0.0069492	0.2954	0.000
Residual		0.0863433	0.0182028	0.0594013	0.1369749		58.798
Total		0.1468476	0.0450325	0.0871689	0.2983101		100.000

Fixed Effects Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|	95% Lower	95% Upper
Intercept	4.7777778	0.1041208	5.0	45.89	<.0001*	4.5101253	5.0454302
Tenderizer[Control]	0.6388889	0.0389128	10.0	16.42	<.0001*	0.5521867	0.7255911
Tenderizer[Papain]	-0.673611	0.0389128	10.0	-17.31	<.0001*	-0.760313	-0.586909
Roasting Time[30]	2.0555556	0.0599803	45.0	34.27	<.0001*	1.934749	2.1763621
Roasting Time[36]	0.8888889	0.0599803	45.0	14.82	<.0001*	0.7680823	1.0096955
Roasting Time[42]	-1.25	0.0599803	45.0	-20.84	<.0001*	-1.370807	-1.129193
Tenderizer[Control]*Roasting Time[30]	0.4444444	0.084825	45.0	5.24	<.0001*	0.2735982	0.6152907
Tenderizer[Control]*Roasting Time[36]	0.4444444	0.084825	45.0	5.24	<.0001*	0.2735982	0.6152907
Tenderizer[Control]*Roasting Time[42]	-0.583333	0.084825	45.0	-6.88	<.0001*	-0.75418	-0.412487
Tenderizer[Papain]*Roasting Time[30]	-0.326389	0.084825	45.0	-3.85	0.0004*	-0.497235	-0.155543
Tenderizer[Papain]*Roasting Time[36]	-0.409722	0.084825	45.0	-4.83	<.0001*	-0.580569	-0.238876
Tenderizer[Papain]*Roasting Time[42]	0.6458333	0.084825	45.0	7.61	<.0001*	0.474987	0.8166796

Random Coefficients

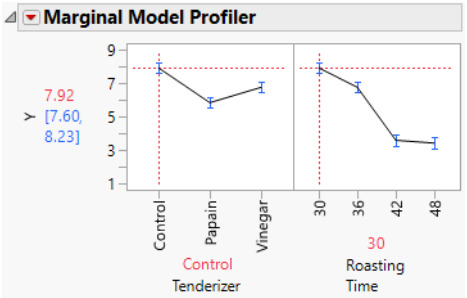
Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Tenderizer	2	2	10.0	190.00818	<.0001*
Roasting Time	3	3	45.0	656.61634	<.0001*
Tenderizer*Roasting Time	6	6	45.0	18.458286	<.0001*

Explore the Interaction between Tenderizer and Roasting Time

1. Click the Mixed Model red triangle and select **Marginal Model Inference > Profiler**.

Figure 8.30 Marginal Model Profiler with Roasting Time Set to 30 Minutes



2. Move the red dashed vertical line in the Roasting Time panel to 36, 42, and 48.

   In Figure 8.30, notice that both the papain and vinegar tenderizers result in significantly lower tenderness scores than the control when roasting time is either 30 or 36 minutes. However, at 42 minutes, there are no significant differences. At 48 minutes, papain gives a value lower than the control, but vinegar does not. Papain gives lower tenderness scores than does vinegar at all times except 42 minutes.

3. Click the Mixed Model red triangle and select **Multiple Comparisons**.
4. Select **Tenderizer*Roasting Time**.
5. Select **All Pairwise Comparisons - Tukey HSD** and then click **OK**.

   Figure 8.31 shows a partial list of pairwise comparisons. Most of the differences between papain and vinegar that you observed in the profiler are statistically significant. Therefore, it appears that papain is the better tenderizer.

Figure 8.31 Multiple Comparisons, Partial View

Mixed Model for Y

Multiple Comparisons for Tenderizer*Roasting Time

Least Squares Means Estimates

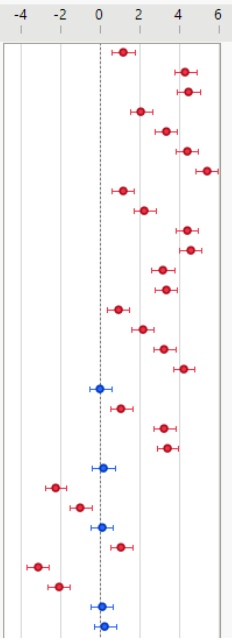
Tenderizer	Roasting Time	Estimate	Std Error	DF	Lower 95%	Upper 95%
Control	30	7.9166667	0.15214556	20.355	7.5996515	8.2336819
Control	36	6.7500000	0.15214556	20.355	6.4329848	7.0670152
Control	42	3.5833333	0.15214556	20.355	3.2663181	3.9003485
Control	48	3.4166667	0.15214556	20.355	3.0996515	3.7336819
Papain	30	5.8333333	0.15214556	20.355	5.5163181	6.1503485
Papain	36	4.5833333	0.15214556	20.355	4.2663181	4.9003485
Papain	42	3.5000000	0.15214556	20.355	3.1829848	3.8170152
Papain	48	2.5000000	0.15214556	20.355	2.1829848	2.8170152
Vinegar	30	6.7500000	0.15214556	20.355	6.4329848	7.0670152
Vinegar	36	5.6666667	0.15214556	20.355	5.3496515	5.9836819
Vinegar	42	3.5000000	0.15214556	20.355	3.1829848	3.8170152
Vinegar	48	3.3333333	0.15214556	20.355	3.0163181	3.6503485

Tukey HSD All Pairwise Comparisons

Quantile = 3.44501, Adjusted DF = 45.0, Adjustment = Tukey-Kramer

All Pairwise Differences

Tenderizer	Roasting Time	-Tenderizer	-Roasting Time	Difference	Std Error	t Ratio	Prob>|t|	Lower 95%	Upper 95%
Control	30	Control	36	1.16667	0.1696500	6.88	<.0001*	0.58222	1.75111
Control	30	Control	42	4.33333	0.1696500	25.54	<.0001*	3.74889	4.91778
Control	30	Control	48	4.50000	0.1696500	26.53	<.0001*	3.91555	5.08445
Control	30	Papain	30	2.08333	0.1616430	12.89	<.0001*	1.52647	2.64019
Control	30	Papain	36	3.33333	0.1616430	20.62	<.0001*	2.77647	3.89019
Control	30	Papain	42	4.41667	0.1616430	27.32	<.0001*	3.85981	4.97353
Control	30	Papain	48	5.41667	0.1616430	33.51	<.0001*	4.85981	5.97353
Control	30	Vinegar	30	1.16667	0.1616430	7.22	<.0001*	0.60981	1.72353
Control	30	Vinegar	36	2.25000	0.1616430	13.92	<.0001*	1.69314	2.80686
Control	30	Vinegar	42	4.41667	0.1616430	27.32	<.0001*	3.85981	4.97353
Control	30	Vinegar	48	4.58333	0.1616430	28.35	<.0001*	4.02647	5.14019
Control	36	Control	42	3.16667	0.1696500	18.67	<.0001*	2.58222	3.75111
Control	36	Control	48	3.33333	0.1696500	19.65	<.0001*	2.74889	3.91778
Control	36	Papain	30	0.91667	0.1616430	5.67	<.0001*	0.35981	1.47353
Control	36	Papain	36	2.16667	0.1616430	13.40	<.0001*	1.60981	2.72353
Control	36	Papain	42	3.25000	0.1616430	20.11	<.0001*	2.69314	3.80686
Control	36	Papain	48	4.25000	0.1616430	26.29	<.0001*	3.69314	4.80686
Control	36	Vinegar	30	8.882e-16	0.1616430	0.00	1.0000	-0.55686	0.55686
Control	36	Vinegar	36	1.08333	0.1616430	6.70	<.0001*	0.52647	1.64019
Control	36	Vinegar	42	3.25000	0.1616430	20.11	<.0001*	2.69314	3.80686
Control	36	Vinegar	48	3.41667	0.1616430	21.14	<.0001*	2.85981	3.97353
Control	42	Control	48	0.16667	0.1696500	0.98	0.9974	-0.41778	0.75111
Control	42	Papain	30	-2.25000	0.1616430	-13.92	<.0001*	-2.80686	-1.69314
Control	42	Papain	36	-1.00000	0.1616430	-6.19	<.0001*	-1.55686	-0.44314
Control	42	Papain	42	0.08333	0.1616430	0.52	1.0000	-0.47353	0.64019
Control	42	Papain	48	1.08333	0.1616430	6.70	<.0001*	0.52647	1.64019
Control	42	Vinegar	30	-3.16667	0.1616430	-19.59	<.0001*	-3.72353	-2.60981
Control	42	Vinegar	36	-2.08333	0.1616430	-12.89	<.0001*	-2.64019	-1.52647
Control	42	Vinegar	42	0.08333	0.1616430	0.52	1.0000	-0.47353	0.64019
Control	42	Vinegar	48	0.25000	0.1616430	1.55	0.9187	-0.30686	0.80686



JMP PRO Example of a Uniformity Trial

Use the Mixed Model personality of the Fit Model platform to analyze a uniformity trial. You are interested in analyzing an agronomic uniformity trial that was conducted on an 8x8 grid of plots. In a uniformity trial, a test crop is grown on a field with no experimental treatments applied. The response variable, often yield, is measured. The idea is to characterize variability in the field as background for planning a designed experiment to be conducted on that field.

Your objective is to use the information from these data to design a yield trial with 16 treatments. Specifically, you want to decide whether to conduct future experiments on the field:

- a complete block design with 4 blocks (denoted Quarter in the data)
- an incomplete block design with 16 blocks (denoted Subquarter in the data)
- a completely randomized design with spatially correlated errors

With this objective, spatial data can be treated as repeated measures with two or more dimensions as repeated effects. So, you can compare and choose an appropriate model using the values in the Fit Statistics report. You start by determining if there is significant spatial variability, then you determine whether there is a nugget effect.

Once you have established whether there is a nugget effect, you determine the best fitting spatial covariance structure. Finally, you fit the blocking models and compare these to the best spatial structure. In this example, both AICc and BIC are used to select a best model. "Spatial Correlation Structure" provides more information about nugget effects and other spatial terminology.

Tip: This section walks you through many aspects of fitting spatial data (from fitting the model to deciding on the best covariance structure). Leave the Fit Model launch window open as you work through each example.

JMP PRO Fit a Spatial Structure Model

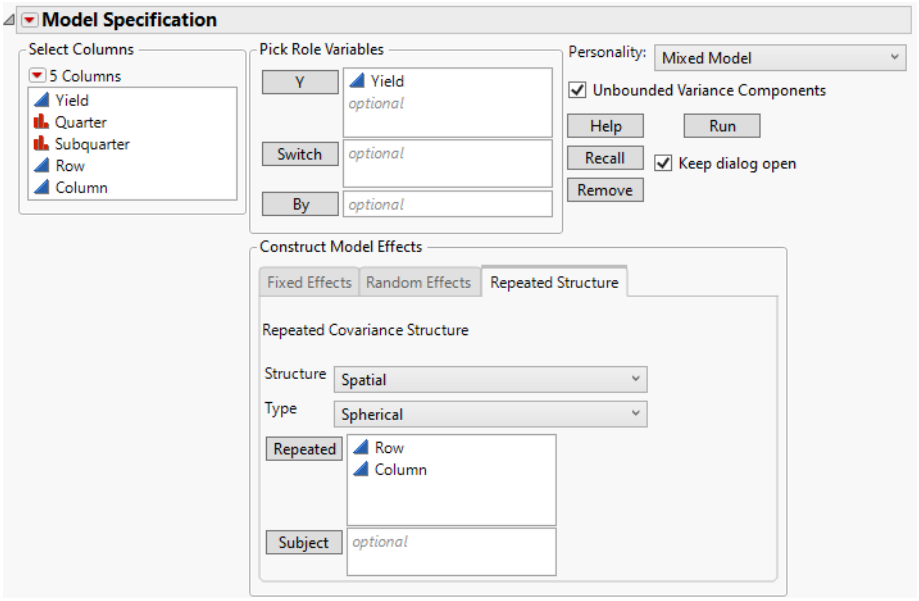
To determine whether there is significant spatial variability, you can fit a model that accounts for spatial variability. Then you can compare the likelihood for this spatial model to the likelihood for a model that does not account for spatial variability. You can do this because the independent errors model is nested within the spatial model family: The independent errors model is a spatial model with spatial correlation, ρ, equal to 0. This means that you can perform a formal likelihood ratio test of the two models.

First, fit the model that accounts for spatial structure.

1. Select **Help > Sample Data Folder** and open Uniformity Trial.jmp.
2. Select **Analyze > Fit Model**.
3. Select **Keep dialog open** so that you can return to the launch window in the next example.
4. Select Yield and click **Y**.
5. Select **Mixed Model** from the Personality list.
6. Select the **Repeated Structure** tab.
7. Choose **Spatial** from the list next to Structure.
8. Choose **Spherical** from the list next to Type.

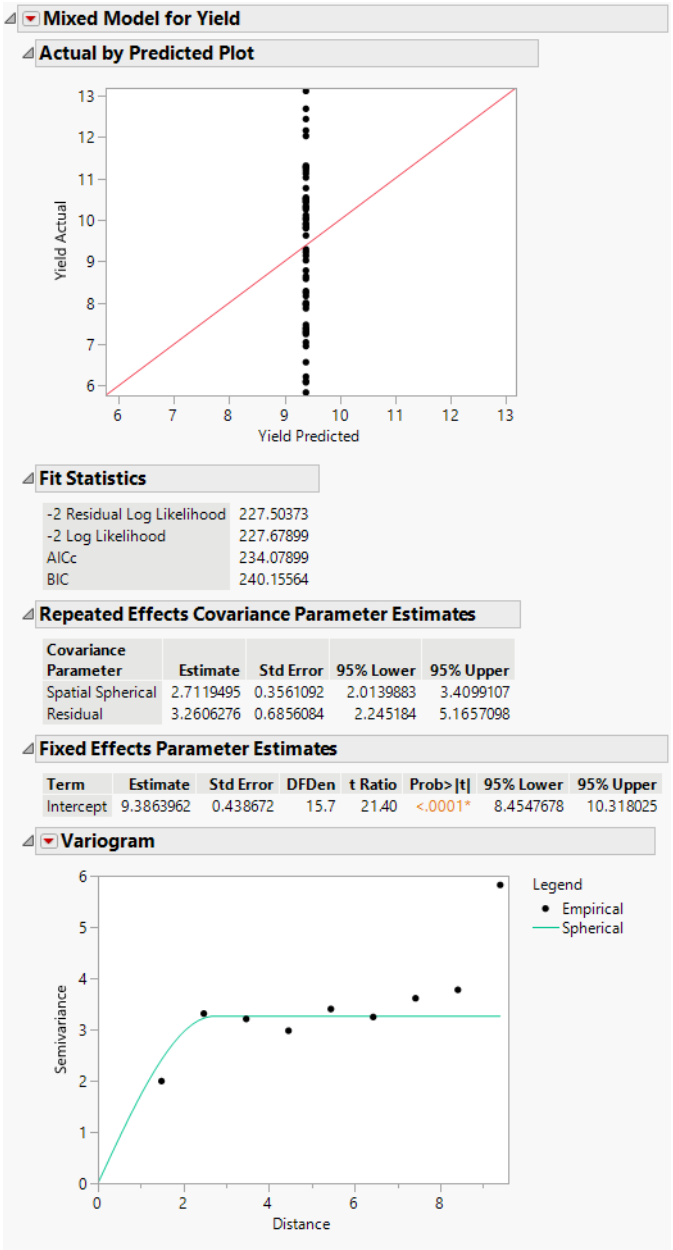
9. Select Row and Column and click **Repeated**.

Figure 8.32 Completed Fit Model Launch Window Showing Repeated Structure Tab



10. Click **Run**.

Figure 8.33 Mixed Model Report for Spatial Spherical Covariance Structure



In the Mixed Model report, the Actual by Predicted Plot shows that the predicted yield is a single value. This is because only spatial covariance was fit. The Fit Statistics report shows that -2 Log Likelihood is 227.68, and the AICc is 234.08.

Because an isotropic spatial structure was fit, a Variogram plot is shown. Because the trials are laid out in an 8 by 8 grid, there are more pairs of points at small distances than at very large distances. See Figure 8.37 for the layout. The Variogram shows that a spherical spatial structure is an excellent fit for distances up to about 8.4. The distance class for the final distance consists of only the two diagonal pairs of points.

The Repeated Effects Covariance Parameter Estimates report gives estimates of the range (Spatial Spherical = 2.71) and the sill (Residual = 3.26). See "Variogram".

JMP PRO Fit the Independent Errors Model

Next, fit the independent errors model.

1. Return to the Fit Model Launch Window.
2. Select **Repeated Structure** tab.
3. Select **Residual** from the Structure list.
4. Remove Row and Column from the Repeated effects list.

   Otherwise, a warning appears: "Repeated columns and subject columns are ignored when the Residual covariance structure is selected."
5. Click **Run**.

   The fit statistics for the independent errors model are: -2 Log Likelihood = 254.22, and AICc = 258.41. Each of these exceeds the corresponding value for the spatial correlation model, where -2 Log Likelihood is 227.68 and the AICc is 234.08. Because smaller values of these statistics indicate a better fit, the spatial model might provide a better fit.

JMP PRO Conduct a Likelihood Ratio Test (Optional)

A formal likelihood ratio test shows whether the spatial correlation model explains significant variation. One model must be nested in another model to create valid likelihood ratio tests.

Typically, spatial models are compared using AICc or BIC rather than through formal likelihood ratio testing. Evaluating the AICc or BIC is faster, and many spatial models are not nested.

You can conduct a likelihood ratio test in this example, because the independent errors model is nested within the spatial model family. The independent errors model is a spatial model with spatial correlation, ρ, equal to 0. This means that you can perform a formal likelihood ratio test of the two models.

In this example, the likelihood ratio test statistic is 254.22-227.68=26.54. Comparing this to a Chi-square distribution on one degree of freedom, the null hypothesis of no spatial correlation is rejected with a p-value < 0.0001. You can conclude that these data contain significant spatial variability.

JMP PRO Select the Type of Spatial Covariance

Next, you determine which spatial covariance structure best fits the data:

- with or without a nugget effect (variation over relatively small distances)
- isotropic (spatial correlation is equal in all directions) or anisotropic (spatial correlation differs in the two directions)
- type of structure, spherical, Gaussian, exponential, or power.

1. Return to the Fit Model launch window.
2. Select the **Repeated Structure** tab.
3. Select Row and Column and click **Repeated**.
4. Select **Spatial with Nugget** from the Structure list.
5. Select **Spherical** from the Type list.
6. Click **Run**.

   The Mixed Model report is shown in Figure 8.34. Notice that the log-likelihoods are essentially equal to those of the spherical with no nugget model, and the AICc is slightly higher (236.36 compared to 234.08). The Repeated Effects Covariance Parameter Estimates report shows that the Nugget covariance parameter has an estimate of zero. There does not appear to be any evidence for a nugget effect.

Figure 8.34 Mixed Model Report for Spatial Spherical with Nugget

Mixed Model for Yield

Fit Statistics

-2 Residual Log Likelihood	227.50373
-2 Log Likelihood	227.67899
AICc	234.07899
BIC	240.15564

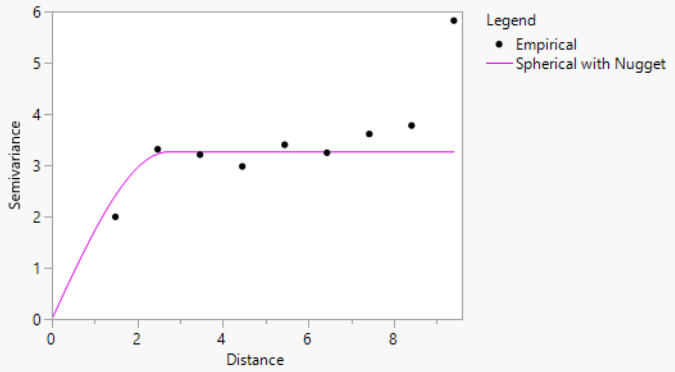
Repeated Effects Covariance Parameter Estimates

Covariance Parameter	Estimate	Std Error	95% Lower	95% Upper
Spatial Spherical	2.7119495	0.364264	1.9980051	3.4258939
Nugget	0	0.1198054	-0.234814	0.2348143
Residual	3.2606276	1.0268779	1.9121185	6.7753216

Fixed Effects Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|	95% Lower	95% Upper
Intercept	9.3863962	0.439083	13.3	21.38	<.0001*	8.4402696	10.332523

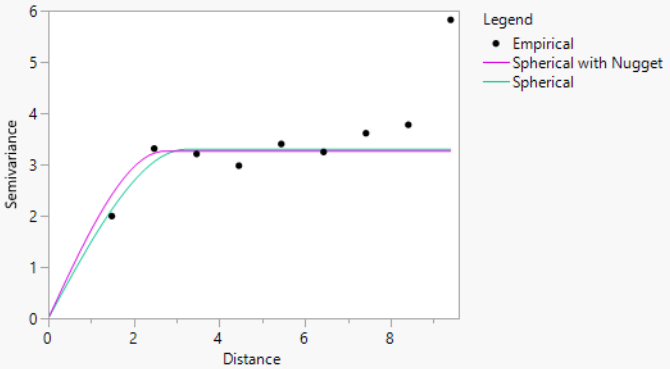
Variogram



7. Click the Variogram red triangle and select **Spatial > Spherical**.

Figure 8.35 Variogram in the Mixed Model Report

Variogram



The two variograms are virtually identical. This also suggests that there is no evidence of a nugget effect.

8. Return to the Fit Model launch window.
9. Select **Repeated Structure** tab.
10. To test anisotropicity, select **Spatial Anisotropic** from the Structure list.
11. Select **Spherical** from the Type list.
12. Click **Run**.

    The Mixed Model report is shown in Figure 8.36. The fit statistics indicate not as good a fit as the isotropic (spatial structure) spherical model (AICc 240.54 compared to 234.08). The Repeated Effects Covariance Parameter Estimates report shows that the estimates for the Row (Spatial Spherical Row) and Column (Spatial Spherical Column) covariances are very close. There is no evidence to suggest that spatial correlations within rows and columns of the grid differ.

Figure 8.36 Mixed Model Report for Spatial Anisotropic Spherical Covariance Structure

Mixed Model for Yield

Fit Statistics

-2 Residual Log Likelihood	232.09009
-2 Log Likelihood	231.86638
AICc	240.54434
BIC	248.50191

Repeated Effects Covariance Parameter Estimates

Covariance Parameter	Estimate	Std Error	95% Lower	95% Upper
Spatial Spherical Row	2.2453516	0.4114883	1.4388492	3.0518539
Spatial Spherical Column	2.2285839	0.3316821	1.5784989	2.8786689
Residual	3.0050408	0.6112702	2.0921756	4.6820483

Fixed Effects Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|	95% Lower	95% Upper
Intercept	9.4039776	0.3585807	21.3	26.23	<.0001*	8.658977	10.148978

JMP PRO Determine the Type of the Spatial Structure

An isotropic spatial structure with no nugget is appropriate. To determine the type of the spatial structure, you can compare the available types.

1. Return to the Fit Model launch window.
2. Select the **Repeated Structure** tab.
3. Select Row and Column and click **Repeated**.
4. Select **Spatial** from the Structure list.
5. Select **Power** from the Type list.
6. Click **Run**.

   Note the AICc and BIC values for the model fit.

7. Return to the Fit Model launch window.
8. Select **Exponential** from the Type list.
9. Click **Run**.

   Note the AICc and BIC values for the model fit.
10. Return to the Fit Model launch window.
11. Select **Gaussian** from the Type list.
12. Click **Run**.

    Note the AICc and BIC values for the model fit.

The observed AICc values for these types and the other fits that you performed are summarized in Table 8.2.

Table 8.2 Fit Statistics for Spatial Models Fit

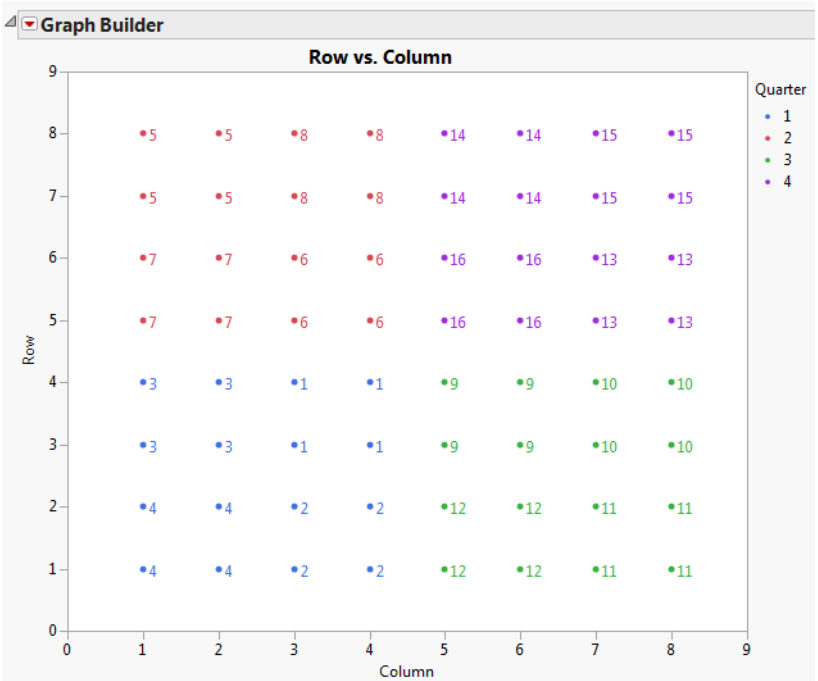
Structure	Type	AICc	BIC
Spatial	Spherical	234.08	240.16
Residual		258.41	262.53
Spatial with Nugget	Spherical	236.36	244.31
Spatial Anisotropic	Spherical	240.54	248.50
Spatial	Power	240.24	246.32
Spatial	Exponential	240.24	246.32
Spatial	Gaussian	238.37	244.44

The best fitting model (using the smallest AICc value) is the Spatial structure model with Spherical covariance structure. Now you will compare this model to the complete and incomplete block models to complete the objectives of the uniformity trial.

JMP PRO Compare the Model to Block Designs

1. Return to the Uniformity Trial data table.
2. In the Tables panel, run the Graph Builder script.

Figure 8.37 Graph Builder Plot of Proposed Complete and Incomplete Block Designs



This plot shows the proposed complete and incomplete block designs for the field. The color indicates the quarter fields that would serve as complete blocks. The numbered points represent the sub-quarter fields that would serve as incomplete blocks.

To fit the complete block model, follow these steps:

1. Return to the Fit Model launch window.
2. Select **Repeated Structure** tab.
3. Select **Residual** from the Structure list.
4. Remove Row and Column from the effect.

   Otherwise, a pop-up window appears, stating, “Repeated columns and subject columns are ignored when the Residual covariance structure is selected.”
5. Select the **Random Effects** tab.
6. Select Quarter and click **Add**.
7. Click **Run**.

To fit the incomplete block model, follow these steps:

1. Return to the Fit Model launch window.
2. Select the **Random Effects** tab.

3. Select Quarter and click **Remove**.
4. Select Subquarter and click **Add**.
5. Click **Run**.

The following list shows both AICc and BIC values for the competing models. The spherical covariance structure results in the best model fit. This indicates that, for future studies using this field, a completely randomized design with spatially correlated errors is preferred.

- Spherical model (see "Determine the Type of the Spatial Structure")
  - AICc: 234.08
  - BICc: 240.16
- Complete block (RCBD) model
  - AICc: 259.90
  - BICc: 265.97
- Incomplete block model
  - AICc: 248.77
  - BICc: 254.85

JMP PRO Example of a Correlated Response

Use the Mixed Model personality of the Fit Model platform to analyze the effect of two layouts that deal with wafer production. Each of 50 wafers is partitioned into four quadrants and a characteristic of interest is measured on each of these quadrants. Data of this type are usually presented in a format where each row contains all of the repeated measurements for one of the units of interest. Data of this type are often analyzed using separate models for each response. However, when repeated measurements are taken on a single unit, it is likely that there is within-unit correlation. Failure to account for this correlation can result in poor decisions and predictions. You can use the Mixed Model personality to account for and model the possible correlation.

For mixed model analysis of repeated measures data, each repeated measurement needs to be in its own row. If your data are in the typical format where all repeated measurements are in the same row, you can construct an appropriate data table for Mixed Model analysis by using Tables > Stack. See *Using JMP*.

In this example, you first fit univariate models using the usual data table format. Then you use the Mixed Model personality to fit models for the four responses while simultaneously accounting for possible correlation among the responses.

JMP PRO Fit Univariate Models

Use Standard Least Squares to fit a univariate model for each of the four quadrants.

1. Select **Help > Sample Data Folder** and open Wafer Quadrants.jmp.

   This data table is structured for Mixed Model analysis, with one row for each Y measurement on each Quadrant. To conduct univariate analyses, you split the table using a saved script.

2. In the Wafer Quadrants.jmp data table, click the green triangle next to the **Split Y by Quadrant** script.

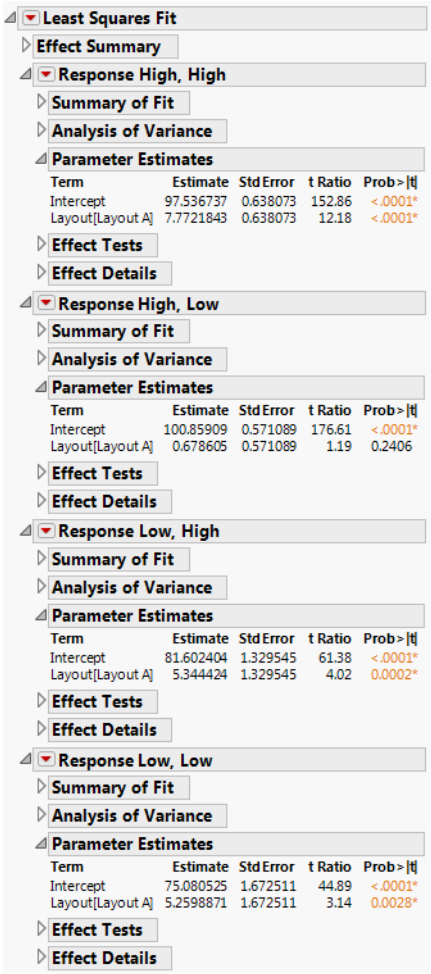
   The new data table is in the format often used to record repeated measures data. Each value of Wafer ID defines a row and the four measurements for that wafer are given in the single row.

3. Select **Analyze > Fit Model**.

   Since there is a Model script in the data table, the Model Specification window is already filled in. Note that the columns High, High through Low, Low are entered as Y and that Layout is the single model effect.

4. Click **Run**.

Figure 8.38 Four Univariate Models



The report indicates that Layout has a statistically significant effect for all quadrants except the High, Low quadrant.

JMP PRO Perform Mixed Model Analysis

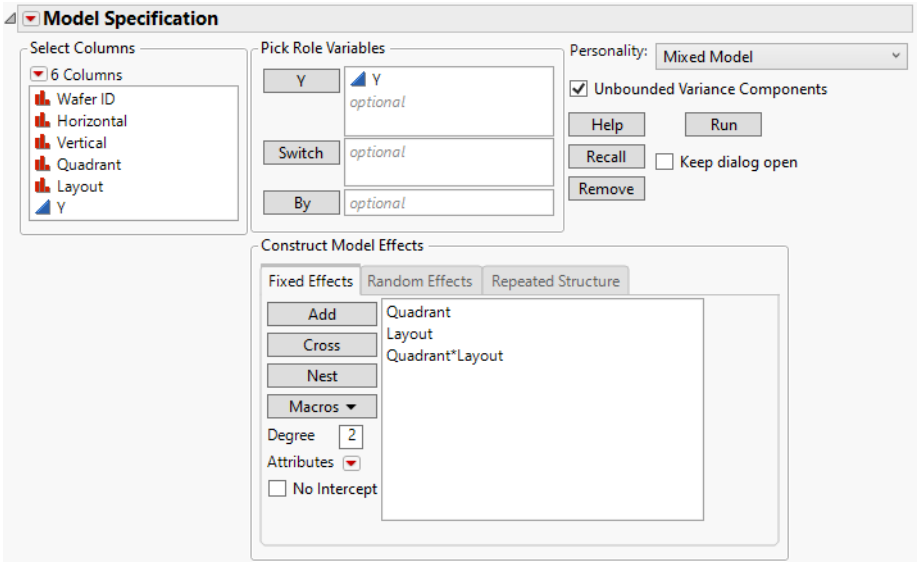
Using the Mixed Model analysis, you can obtain more information.

1. Return to Wafer Quadrants.jmp. Or, if you have closed it, select **Help > Sample Data Folder** and open Wafer Quadrants.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Select **Mixed Model** from the Personality list.

5. Select Quadrant and Layout from the Select Columns list and click **Macros > Full Factorial**.

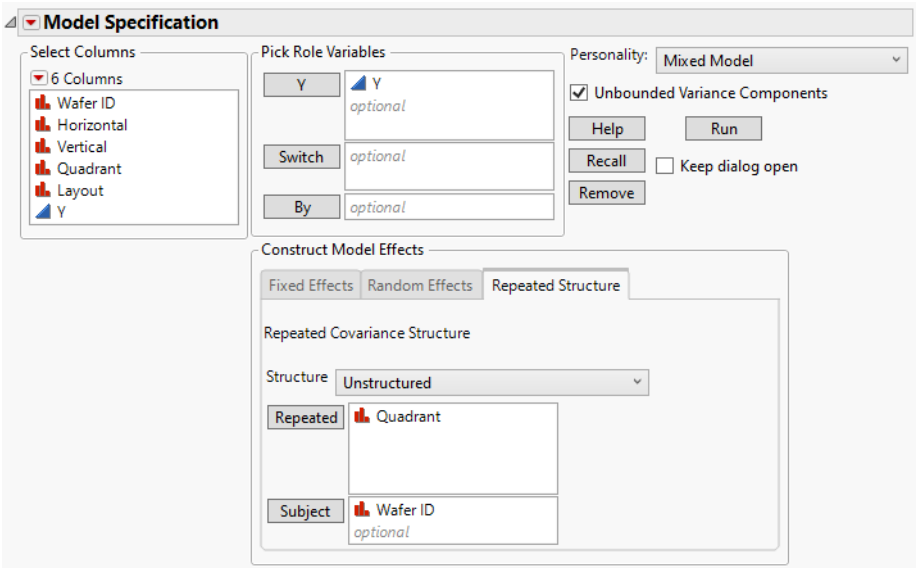
   This model specification enables you to explore the effect of Layout on the repeated measurements as well as the possible interaction of Layout with Quadrant.

Figure 8.39 Fit Model Launch Window Showing Completed Fixed Effects Tab



6. Select the **Repeated Structure** tab.
7. Select **Unstructured** from the Structure list.
8. Select Quadrant and click **Repeated**.
9. Select Wafer ID and click **Subject**.

Figure 8.40 Fit Model Launch Window Showing Repeated Structure Tab



10. Click **Run**.

Figure 8.41 Mixed Model Report with Fixed Effects Parameter Estimates Report Closed

Mixed Model for Y

Repeated Effects Covariance Parameter Estimates

Repeated Effect: Quadrant
Subject: Wafer ID

Covariance Parameter	Estimate	Std Error	95% Lower	95% Upper
Var(High, High)	19.835702	4.0489457	11.899914	27.77149
Cov(High, Low,High, High)	0.0111321	2.5624804	-5.011237	5.0335013
Var(High, Low)	15.889659	3.2434631	9.5325884	22.24673
Cov(Low, High,High, High)	15.893138	6.3915311	3.3659676	28.420309
Cov(Low, High,High, Low)	1.0055411	5.3413847	-9.463381	11.474463
Var(Low, High)	86.121899	17.579559	51.666597	120.5772
Cov(Low, Low,High, High)	14.723606	7.7996739	-0.563474	30.010686
Cov(Low, Low,High, Low)	8.0424897	6.8163252	-5.317262	21.402242
Cov(Low, Low,Low, High)	2.1512894	15.640276	-28.50309	32.805667
Var(Low, Low)	136.28412	27.81888	81.76012	190.80813

Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Quadrant	3	3	46.0	149.21352	<.0001*
Layout	1	1	48.0	51.757797	<.0001*
Quadrant*Layout	3	3	46.0	22.379307	<.0001*

The Repeated Effects Covariance Parameter Estimates report gives the estimated variances and covariances for the four responses. Note that the confidence interval for the covariance of Low, High with High, High does not include zero. This suggests that there is a positive covariance between measurements in these two quadrants. This is information that the Mixed Model analysis uses and that is not available when the responses are modeled independently.

The Fixed Effects Tests report indicates that there is a significant Quandrant by Layout interaction.

JMP PRO Explore the Layout by Characteristic Interaction with the Profiler

1. Click the Mixed Model red triangle and select **Marginal Model Inference > Profiler**.
2. In the Profiler plot, compare the predicted values for Y across the quadrants by first setting the vertical red dotted line at Layout A and then at Layout B.

Figure 8.42 Profile for Quadrant for Layout A

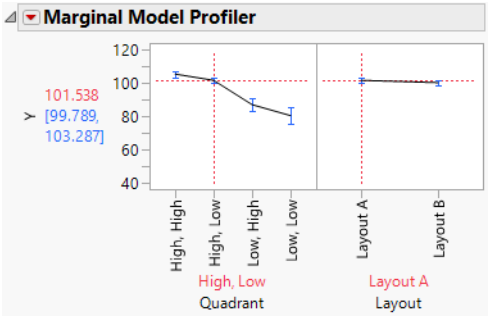
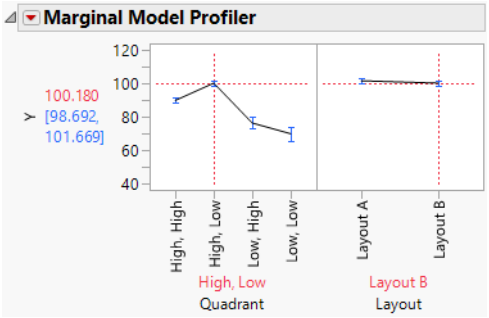


Figure 8.43 Profile for Quadrant for Layout B



The differences in the profiles at each setting of Layout give you insight into the significant interaction. It appears that the interaction is partially driven by the differences for the High, High quadrant.

JMP PRO Plot of Y by Layout and by Quadrant

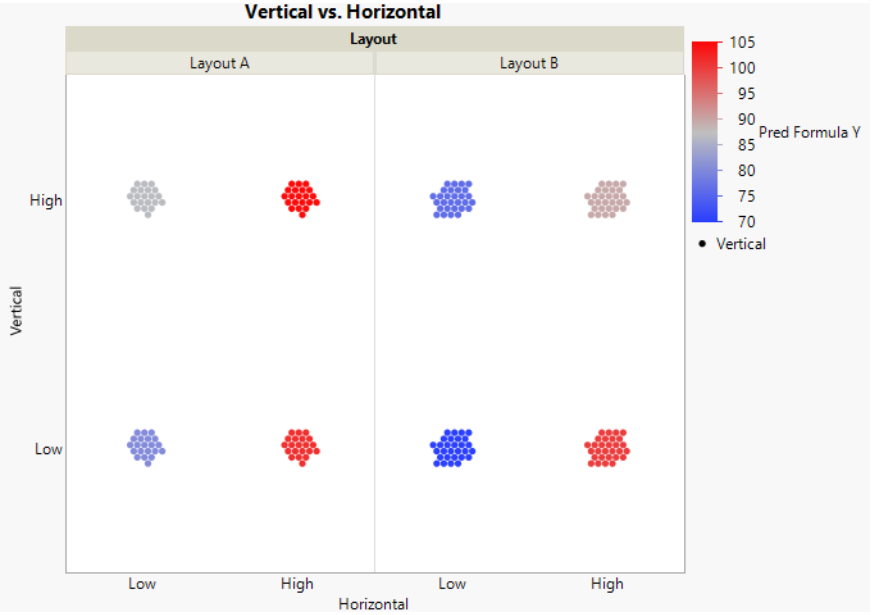
Use Graph Builder to explore the nature of the interaction.

1. Click the Mixed Model red triangle and select **Save Columns > Prediction Formula**.

   The prediction formula is saved to the data table in the column Pred Formula Y.

2. Select **Graph > Graph Builder**.
3. Drag Horizontal to the **X** zone.
4. Drag Vertical to the **Y** zone.
5. Drag Pred Formula Y to the **Color** zone.
6. Drag Layout to the **Wrap** zone.
7. Click **Done** to hide the control panel.

Figure 8.44 Completed Graph Builder Plot



The plot shows the predicted differences for the eight Layout and Quadrant combinations using a color intensity scale. The predicted values for the High, Low quadrant are in the lower right. The color gradient shows relatively little difference for these predicted values. Other differences are clearly indicated.

JMP PRO Statistical Details for the Mixed Model Personality

This section contains statistical details for the Mixed Model personality of the Fit Model platform.

- "Statistical Details for the Convergence Score Test"
- "Statistical Details for the Random Coefficient Model"
- "Statistical Details for Repeated Measures"
- "Statistical Details for Repeated Covariance Structures"
- "Statistical Details for Spatial and Temporal Variability"
- "Statistical Details for the Kackar-Harville Correction"

JMP PRO Statistical Details for the Convergence Score Test

In the Mixed Model report, the convergence failure warning shows the score test for the following hypothesis: that the unknown maximum likelihood estimate (MLE) is consistent with the parameter given in the final iteration of the model-fitting algorithm. This hypothesis test is possible because the relative gradient criterion is algebraically equivalent to the score test statistic. Remarkably, the score test does not require knowledge of the true MLE.

JMP PRO Score Test

Consider first the case of a single parameter, θ. Let l be the log-likelihood function for θ and let $\mathbf{x}$ be the data. The score is the derivative of the log-likelihood function with respect to θ:

$$U(\theta) = \frac{\partial l(\theta|\mathbf{x})}{\partial \theta}$$

The observed information is:

$$I(\theta) = -\frac{\partial^2}{\partial \theta^2} l(\theta|\mathbf{x})$$

The statistic for the score test of H_0: $\theta = \theta_0$ is:

$$S(\theta_0) = \frac{U(\theta_0)^2}{I(\theta_0)}$$

This statistic has an asymptotic Chi-square distribution with 1 degree of freedom under the null hypothesis.

The score test can be generalized to multiple parameters. Consider the vector of parameters θ. Then the test statistic for the score test of H_0: $\theta = \theta_0$ is:

$$S(\theta_0) = \mathbf{U}'(\theta_0)\mathbf{I}^{-1}(\theta_0)\mathbf{U}(\theta_0)$$

where

$$\mathbf{U}(\theta) = \frac{\partial l(\theta|\mathbf{x})}{\partial \theta}$$

and

$$\mathbf{I}(\theta) = -\frac{\partial^2 l(\theta|\mathbf{x})}{\partial \theta(\partial \theta')}$$

and $\mathbf{U}'$ denotes the transpose of the matrix $\mathbf{U}$.

The test statistic is asymptotically Chi-square distribution with k degrees of freedom. Here k is the number of unbounded parameters.

JMP PRO Relative Gradient

The convergence criterion for the Mixed Model fitting procedure is based on the relative gradient $g'\mathbf{H}^{-1}g$. Here, $g(\theta) = \mathbf{U}(\theta)$ is the gradient of the log-likelihood function and $\mathbf{H}(\theta) = -\mathbf{I}(\theta)$ is its Hessian.

Let θ_0 be the value of θ where the algorithm terminates. Note that the relative gradient evaluated at θ_0 is the score test statistic. A p-value is calculated using a Chi-square distribution with k degrees of freedom. This p-value gives an indication of whether the value of the unknown MLE is consistent with θ_0. The number of unbounded parameters listed in the Random Effects Covariance Parameter Estimates report equals k.

JMP PRO Statistical Details for the Random Coefficient Model

In the Mixed Model personality of the Fit Model platform, the standard random coefficient model specifies a random intercept and slope for each subject. Let y_{ij} denote the measurement of the j^{th} observation on the i^{th} subject. Then the random coefficient model can be specified as follows:

$$y_{ij} = a_i + b_i x_{ij} + e_{ij}$$

where

$$i = 1, 2, \ldots, t$$

$$j = 1, 2, \ldots, n_i$$

$$\begin{bmatrix} a_i \\ b_i \end{bmatrix} \sim iid\ N\left(\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \mathbf{G}\right)$$

$$\mathbf{G} = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix}$$

and

$$e_{ij} \sim iid\ N(0, \sigma^2)$$

You can reformulate the model to reflect the fixed and random components that are estimated:

$$y_{ij} = (\alpha + a_i^*) + (\beta + b_i^*)x_{ij} + e_{ij}$$

where

$$a_i^* = \alpha_i - \alpha$$

$$b_i^* = \beta_i - \beta$$

and

$$\begin{bmatrix} a_i^* \\ b_i^* \end{bmatrix} \sim iid\ N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{G}\right)$$

with **G** and e_{ij} defined as above.

JMP PRO Statistical Details for Repeated Measures

In the Mixed Model personality of the Fit Model platform, the form of the repeated measures model is $y_{ijk} = \alpha_{ij} + s_{ik} + e_{ijk}$, where

α_{ij} can be written as a treatment and time factorial

s_{ik} is the random effect of the k^{th} subject assigned to the i^{th} treatment

$j = 1,\ldots,m$ denotes the repeated measurements over time.

Assume that the s_{ik} are independent and identically distributed N(0, σ_s^2) variables. Denote the number of treatment factors by t and the number of subjects by s. Then the distribution of e_{ijk} is N(0, Σ), where

$$\Sigma = \mathbf{I}_{ts} \otimes Var(\mathbf{y}_{ik} | s_{ik})$$

and

$$\mathbf{y}'_{ik} | s_{ik} = \left(\left[y_{i1k} \; y_{i2k} \; \ldots \; y_{imk} \right] \middle| s_{ik} \right)$$

Denote the block diagonal component of the covariance matrix Σ corresponding to the ik^{th} subject within treatment by Σ_{ik}. In other words, $\Sigma_{ik} = Var(\mathbf{y}_{ik} | s_{ik})$. Because observations over time within a subject are not typically independent, it is necessary to estimate the variance of $y_{ijk} | s_{ik}$. Failure to account for the correlation leads to distorted inference.

See "Statistical Details for Repeated Covariance Structures" and "Statistical Details for Spatial and Temporal Variability" for more information about the covariance structures available for Σ_{ik}.

JMP PRO Statistical Details for Repeated Covariance Structures

This section gives the parameterizations for the following covariance structures in the Mixed Model personality of the Fit Model platform:

- "Unequal Variances Covariance Structure"
- "Unstructured Covariance Structure"
- "Compound Symmetry Covariance Structure"
- "AR(1) Covariance Structure"
- "Toeplitz Covariance Structure"
- "Antedependent Covariance Structure"

JMP PRO Unequal Variances Covariance Structure

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 & 0 & \dots & 0 \\ & \sigma_2^2 & 0 & \dots & 0 \\ & & \sigma_3^2 & \dots & 0 \\ & & & \ddots & \vdots \\ & & & & \sigma_m^2 \end{bmatrix}$$

Here, the variance among observations taken at time j is:

$$\sigma_j^2 = Var(y_{ijk} | s_{ik})$$

The variances are allowed to differ across the levels of the repeated column. The covariances between observations at different levels are zero.

JMP PRO Unstructured Covariance Structure

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \dots & \sigma_{1[m-1]} & \sigma_{1m} \\ & \sigma_2^2 & \sigma_{23} & \dots & \sigma_{2[m-1]} & \sigma_{2m} \\ & & \sigma_3^2 & \dots & \sigma_{3[m-1]} & \sigma_{3m} \\ & & & \ddots & \vdots & \vdots \\ & & & & \sigma_{m-1}^2 & \sigma_{[m-1]m} \\ & & & & & \sigma_m^2 \end{bmatrix}$$

Here, the variance among observations taken at time j is:

$$\sigma_j^2 = Var(y_{ijk} | s_{ik})$$

The covariance between observations taken at times j and j' is:

$$\sigma_{jj'} = Cov(y_{ijk}, y_{ij'k} | s_{ik})$$

The variances are allowed to differ across the levels of the repeated column. The covariances between observations at different levels is unique.

JMP PRO Compound Symmetry Covariance Structure

In JMP, a compound symmetry covariance structure is implemented using a mixed model with independent errors approach. Random effects are classified into two categories: G-side or R-side. See Searle et al. (1992).

The G-side random effects are associated with the design matrix for random effects. The R-side random effects are associated with residual error. Within-subject variance is part of the design structure and is modeled on the G-side. Between-subject variance falls into the residual structure and is modeled R-side. The variances in the independent structure are modeled in the following manner:

- The random effects G-side variance is modeled by $s_{ik} \sim$ iid N(0, σ_s^2).
- The R-side variance is modeled by $e_{ijk} \sim$ iid N(0, σ^2).

Then the covariance matrix is defined as follows:

$$\Sigma_{ik} = \sigma_s^2\mathbf{J} + \sigma^2\mathbf{I} = \begin{bmatrix} \sigma_s^2+\sigma^2 & \sigma_s^2 & \cdots & \sigma_s^2 \\ & \sigma_s^2+\sigma^2 & \cdots & \sigma_s^2 \\ & & \ddots & \vdots \\ & & & \sigma_s^2+\sigma^2 \end{bmatrix}$$

where **J** is a matrix consisting of *1*s and **I** is an identity matrix.

Alternatively, all variance could be modeled on the R-side. Under the Gaussian assumption, this compound-symmetry covariance structure is equivalent to the independence model (Type=CS in SAS). This structure is available in JMP by using the Compound Symmetry structure in the repeated structure tab. Here, the correlation between pairs of repeated observations is the same regardless of the time difference between the observations. Thus, the correlation matrix can be specified as follows:

$$C = \begin{bmatrix} 1 & \rho & \cdots & \rho \\ & 1 & \cdots & \rho \\ & & \ddots & \vdots \\ & & & 1 \end{bmatrix}$$

Using the Compound Symmetry structure in JMP also assumes a common variance, σ_e^2, among observations taken at any time point. The covariance structure is then $\Sigma = \sigma_e^2 C$ where

$$\sigma_e^2 = \sigma_s^2 + \sigma^2$$

and

$$\rho = \frac{\sigma_s^2}{\sigma_s^2 + \sigma^2} \ .$$

Here, ρ is the intra-class correlation coefficient and σ_e^2 is the residual variance. Another option is to use the Compound Symmetry Unequal Variances structure in JMP, which allows the variance to vary across time points. This leads to the following covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho\sigma_1\sigma_3 & \cdots & \rho\sigma_1\sigma_{m-1} & \rho\sigma_1\sigma_m \\ & \sigma_2^2 & \rho\sigma_2\sigma_3 & \cdots & \rho\sigma_2\sigma_{m-1} & \rho\sigma_2\sigma_m \\ & & \sigma_3^2 & \cdots & \rho\sigma_3\sigma_{m-1} & \rho\sigma_3\sigma_m \\ & & & \ddots & \vdots & \vdots \\ & & & & \sigma_{m-1}^2 & \rho\sigma_{m-1}\sigma_m \\ & & & & & \sigma_m^2 \end{bmatrix}$$

JMP PRO AR(1) Covariance Structure

$$\Sigma = \begin{bmatrix} \sigma^2 & \rho^{|t_1-t_2|}\sigma^2 & \rho^{|t_1-t_3|}\sigma^2 & \cdots & \rho^{|t_1-t_{m-1}|}\sigma^2 & \rho^{|t_1-t_m|}\sigma^2 \\ & \sigma^2 & \rho^{|t_2-t_3|}\sigma^2 & \cdots & \rho^{|t_2-t_{m-1}|}\sigma^2 & \rho^{|t_2-t_m|}\sigma^2 \\ & & \sigma^2 & \cdots & \rho^{|t_3-t_{m-1}|}\sigma^2 & \rho^{|t_3-t_m|}\sigma^2 \\ & & & \ddots & \vdots & \vdots \\ & & & & \sigma^2 & \rho^{|t_{m-1}-t_m|}\sigma^2 \\ & & & & & \sigma^2 \end{bmatrix}$$

Here t_j is the time of observation j. In this structure, observations taken at any given time have the same variance, σ^2. The parameter ρ, where $-1 < \rho < 1$, is the correlation between two observations that are one unit of time apart. As the time difference between observations increases, their covariance decreases because ρ is raised to a higher power. In many applications, AR(1) provides an adequate model of the within subject correlation, providing more power without sacrificing Type I error control.

JMP PRO Toeplitz Covariance Structure

In the Toeplitz structure, observations that are separated by a fixed number of time units have the same correlation. In contrast to the AR(1) correlation structure, the Toeplitz correlations at a fixed time difference are arbitrary. Denote the correlation for observations d units apart by ρ_d. The correlation matrix is defined as follows:

$$C = \begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{m-1} & \rho_m \\ & 1 & \rho_1 & \cdots & \rho_{m-2} & \rho_{m-1} \\ & & 1 & \cdots & \rho_{m-3} & \rho_{m-2} \\ & & & \ddots & \vdots & \vdots \\ & & & & 1 & \rho_1 \\ & & & & & 1 \end{bmatrix}$$

Two options in JMP use this correlation matrix:

- The Toeplitz structure assumes a common variance, σ^2, for observations from any time point. The covariance structure is $\Sigma = \sigma^2 C$.
- Alternatively, the Toeplitz Unequal Variances structure allows the variance to vary across time points:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho_1\sigma_1\sigma_2 & \rho_2\sigma_1\sigma_3 & \cdots & \rho_{m-1}\sigma_1\sigma_{m-1} & \rho_m\sigma_1\sigma_m \\ & \sigma_2^2 & \rho_1\sigma_2\sigma_3 & \cdots & \rho_{m-2}\sigma_2\sigma_{m-1} & \rho_{m-1}\sigma_2\sigma_m \\ & & \sigma_3^2 & \cdots & \rho_{m-3}\sigma_3\sigma_{m-1} & \rho_{m-2}\sigma_3\sigma_m \\ & & & \ddots & \vdots & \vdots \\ & & & & \sigma_{m-1}^2 & \rho_1\sigma_{m-1}\sigma_m \\ & & & & & \sigma_m^2 \end{bmatrix}$$

JMP PRO Antedependent Covariance Structure

The antedependence model is a general model that is flexible and allows the correlation structure to change over time. In this model, the correlation between two observations at adjacent time points j - 1 and j is unique and is denoted $\rho_{j[j-1]}$.

The correlation between pairs of observations at non-adjacent time points j and j' is the product of all the adjacent correlations in between.

$$Corr(y_{ijk}, y_{ij'k} | s_{ik}) = \prod_{k=j}^{j'-1} \rho_{k[k-1]}$$

For example, the correlation between the pair of observations at time points j=2 and j'=6 would be $\rho_{21}\rho_{32}\rho_{43}\rho_{54}$.

The correlation matrix is defined as follows:

$$C = \begin{bmatrix} 1 & \rho_{10} & \rho_{10}\rho_{21} & \cdots & \rho_{10}\cdots\rho_{[m-1][m-2]} & \rho_{10}\cdots\rho_{m[m-1]} \\ & 1 & \rho_{21} & \cdots & \rho_{21}\cdots\rho_{[m-1][m-2]} & \rho_{21}\cdots\rho_{m[m-1]} \\ & & 1 & \cdots & \rho_{32}\cdots\rho_{[m-1][m-2]} & \rho_{32}\cdots\rho_{m[m-1]} \\ & & & \ddots & \vdots & \vdots \\ & & & & 1 & \rho_{m[m-1]} \\ & & & & & 1 \end{bmatrix}$$

Two options in JMP use this correlation matrix:

- The Antedependent Equal Variance structure assumes equal variances across observation times while still allowing for the correlations to change. The variance among observations at any time is σ^2 and the covariance matrix is $\Sigma = \sigma^2 C$.
- The Antedependent structure allows the variance among observations at any given time to vary. Denote the variance among observations taken at time j is σ_j^2. Then the covariance matrix is defined as follows:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho_{10}\sigma_1\sigma_2 & \rho_{10}\rho_{21}\sigma_1\sigma_3 & \cdots & \rho_{10}\cdots\rho_{[m-1][m-2]}\sigma_1\sigma_{m-1} & \rho_{10}\cdots\rho_{m[m-1]}\sigma_1\sigma_m \\ & \sigma_2^2 & \rho_{21}\sigma_2\sigma_3 & \cdots & \rho_{21}\cdots\rho_{[m-1][m-2]}\sigma_2\sigma_{m-1} & \rho_{21}\cdots\rho_{m[m-1]}\sigma_2\sigma_m \\ & & \sigma_3^2 & \cdots & \rho_{32}\cdots\rho_{[m-1][m-2]}\sigma_3\sigma_{m-1} & \rho_{32}\cdots\rho_{m[m-1]}\sigma_3\sigma_m \\ & & & \ddots & \vdots & \vdots \\ & & & & \sigma_{m-1}^2 & \rho_{m[m-1]}\sigma_{m-1}\sigma_m \\ & & & & & \sigma_m^2 \end{bmatrix}$$

JMP PRO Statistical Details for Spatial and Temporal Variability

Consider the simple model $y_i = \mu + e_i$. The spatial or temporal structure in the Mixed Model personality of the Fit Model platform is modeled through the error term, e_i. In general, the spatial correlation model can be defined as $Var(e_i) = \sigma_i^2$ and $Cov(e_i, e_j) = \sigma_{ij}$.

Let $\mathbf{s}_i$ denote the location of y_i, where $\mathbf{s}_i$ is specified by coordinates reflecting space or time. The spatial or temporal structure is typically restricted by assuming that the covariance is a function of the Euclidean distance, d_{ij}, between $\mathbf{s}_i$ and $\mathbf{s}_j$. The covariance can be written as $Cov(e_i, e_j) = \sigma^2[f(d_{ij})]$, where $f(d_{ij})$ represents the correlation between observations y_i and y_j.

In the case of two or more location coordinates, if $f(d_{ij})$ does not depend on direction, then the covariance structure is *isotropic*. If it does, then the structure is *anisotropic*.

JMP PRO Spatial Correlation Structure

The correlation structures for spatial models available in JMP are shown below. These are parametrized by ρ, which is positive unless it is otherwise constrained.

- Spherical

$$f(d_{ij}) = [1 - 1.5(d_{ij}/\rho) + 0.5(d_{ij}/\rho)^3] \times 1_{\{d_{ij} < \rho\}}$$

$$\text{where } 1_{\{d_{ij} < \rho\}} = \begin{cases} 1, \text{ if } d_{ij} < \rho \\ 0, \text{ if } d_{ij} \geq \rho \end{cases}$$

- Exponential

$$f(d_{ij}) = \exp(-d_{ij}/\rho)$$

- Gaussian

$$f(d_{ij}) = \exp(-d_{ij}^2/\rho^2)$$

- Power

$$f(d_{ij}) = \rho^{d_{ij}}$$

For an anisotropic model, the correlation function contains a parameter, ρ_κ, for each direction.

JMP PRO Variogram

When the spatial process is second-order stationary, the structures listed in "Spatial Correlation Structure" define *variograms*. Borrowed from geostatistics, the variogram is the standard tool for describing and estimating spatial variability. It measures spatial variability as a function of the distance, d_{ij}, between observations using the semivariance.

Let $Z(\mathbf{s})$ denote the value of the response at a location $\mathbf{s}$. The *semivariance* between observations at $\mathbf{s}_i$ and $\mathbf{s}_j$ is defined as follows:

$$\gamma(\mathbf{s}_i, \mathbf{s}_j) = (Var(Z(\mathbf{s}_i) - Z(\mathbf{s}_j)))/2$$

If the response has a constant mean, then the expression can be simplified to the following:

$$\gamma(\mathbf{s}_i, \mathbf{s}_j) = E[(Z(\mathbf{s}_i) - Z(\mathbf{s}_j))^2]/2$$

If the process is isotropic, the semivariance depends only on the distance h between points and the function can be specified as follows:

$$\gamma(h) = E[(Z(\mathbf{s}_i) - Z((\mathbf{s}_i + h)))^2]/2$$

The following terms are associated with variograms:

Nugget Defined as the intercept. This represents a jump discontinuity at $h = 0$.

Sill Defined as the value of the semivariogram at the plateau reached for larger distances. It corresponds to the variance of an observation. In models with no nugget effect, the sill is σ^2. In models with a nugget effect, the sill is $\sigma^2 + c_1$, where c_1 represents the nugget. The *partial sill* is defined as σ^2.

Range Defined as the distance at which the semivariogram reaches the sill. At distances less than the range, observations are spatially correlated. For distances greater than or equal to the range, spatial correlation is effectively zero. In spherical models, ρ is the range. In exponential models, 3ρ is the practical range. In Gaussian models, $\rho\sqrt{3}$ is the practical range. The practical range is defined as the distance where covariance is reduced to 95% of the sill.

In Figure 8.34, the repeated effects covariance parameter estimates represent the various semivariogram features:

Spatial Spherical An estimate of the range, ρ.

Nugget A scaled estimate of c_1. The Residual times the Nugget is c_1.

Residual The partial sill or the sill in no nugget models.

JMP PRO Variogram Estimate

For a given isotropic spatial structure, the estimated variogram is obtained using a nonlinear least squares fit of the observed data to the appropriate function in "Spatial Correlation Structure".

JMP PRO Empirical Semivariance

To compute the *empirical semivariance*, the distances between all pairs of points for the variables selected for the variogram covariance are computed. The range of the distances is divided into 10 equal intervals. If the data do not allow for 10 intervals, then as many intervals as possible are constructed.

Distance classes consisting of pairs of points are constructed. The h^{th} distance class consists of all pairs of points whose distances fall in the h^{th} interval.

Consider the following notation:

n total number of pairs of points

C_h distance class consisting of points whose distance falls into the h^{th} largest interval

$Z(\mathbf{x})$ value of the response at $\mathbf{x}$, where $\mathbf{x}$ is a vector of temporal or spatial coordinates

$\gamma(h)$ semivariance for distance class C_h

The semivariance function, γ, is defined as follows:

$$\gamma(h) = \begin{cases} \frac{1}{2n}\left\{ \sum_{(x,y) \text{ in } C_h} [Z(\mathbf{x}) - Z(\mathbf{y})]^2 \right\} & \text{for } h > 0 \\ \hat{c}_1 & \text{for } h = 0 \end{cases}$$

Here $\hat{c}_1$ is an estimate of the nugget effect.

JMP PRO Statistical Details for the Kackar-Harville Correction

In the Mixed Model personality of the Fit Model platform, the variance matrix of the fixed effects is always modified to include a Kackar-Harville correction. The variance matrix of the BLUPs, and the covariances between the BLUPs and the fixed effects, are not Kackar-Harville corrected. The rationale for this approach is that corrections for BLUPs can be computationally and memory intensive when the random effects have many levels. In SAS, the Kackar-Harville correction is done for both fixed effects and BLUPs only when the `DDFM=KENWARDROGER` is set.

For covariance structures that have nonzero second derivatives with respect to the covariance parameters, the Kenward-Roger covariance matrix adjustment includes a second-order term. This term can result in standard error shrinkage. Also, the resulting adjusted covariance matrix can then be indefinite and is not invariant under reparameterization. The first-order Kenward-Roger covariance matrix adjustment eliminates the second derivatives from the calculation. All spatial structures and the AR(1) structure are covariance structures that generally lead to nonzero second derivatives.

Because JMP implements the Kenward-Roger first-order adjustment, note the following:

- Standard errors for linear combinations that involve only fixed effects parameters match `PROC MIXED DDFM=KENWARDROGER(FIRSTORDER)`. This presumes that one has taken care to transform between the different parameterizations used by `PROC MIXED` and JMP.
- Standard errors for linear combinations that involve BLUP parameters match `PROC MIXED DDFM=SATTERTHWAITE`.

JMP PRO Degrees of Freedom

The degrees of freedom for tests involving only linear combinations of fixed effect parameters are calculated using the first-order Kenward-Roger correction. Therefore, the JMP results for these tests match `PROC MIXED` using the `DDFM=KENWARDROGER(FIRSTORDER)` option. If there are BLUPs in the linear combination, JMP uses a Satterthwaite approximation to get the degrees of freedom. The results then follow a pattern similar to what is described for standard errors in the preceding paragraph.

For more information about the Kackar-Harville correction and the Kenward-Roger DF approach, see Kenward and Roger (1997). The Satterthwaite method is described in detail in the MIXED Procedure chapter in SAS Institute Inc. (2022d).

Chapter 9

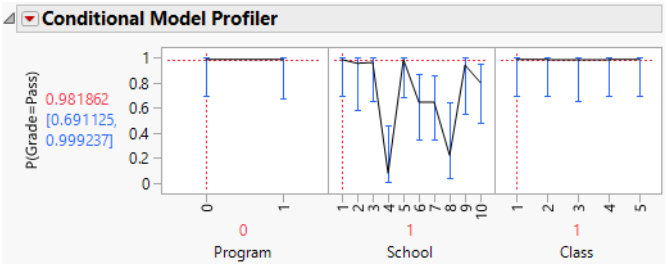
JMP PRO Generalized Linear Mixed Models

Fit a Variety of Mixed Models to Non-Normal Response Data

The Generalized Linear Mixed Model personality of the Fit Model platform is available only in JMP Pro.

The Generalized Linear Mixed Model personality fits models that have a non-Gaussian response variable and random design effects such as blocking. Using the generalized linear model framework enables you to accurately estimate standard errors for the parameters. Using the mixed model framework enables you to accurately represent random effects. The GLMM framework combines these two approaches and gives you the power to test hypotheses and have accurate estimation. The Generalized Linear Mixed Model personality can fit models for count and binomial response variables.

Figure 9.1 Conditional Model Profiler for a Generalized Linear Mixed Model



Contents

Overview of the Generalized Linear Mixed Models Personality 447
Example of a Generalized Linear Mixed Model 447
Launch the Generalized Linear Mixed Model Personality 450
Fit Model Launch Window 450
Data Format 454
Generalized Linear Mixed Model Options 454
Model Fit Reports 455
Fit Statistics and Model Summary 455
Random Effects Covariance Parameter Estimates 457
Fixed Effects Parameter Estimates 457
Random Coefficients 458
Fixed Effects Tests 458
Model Fit Options 459
Additional Example of the Generalized Linear Mixed Model Personality 463

JMP PRO Overview of the Generalized Linear Mixed Models Personality

The Generalized Linear Mixed Model (GLMM) personality of the Fit Model platform enables you to analyze models that have complex covariance structures and non-Gaussian response distributions. There are two non-Gaussian response distributions available: the Poisson distribution for discrete count data and the binomial distribution for binary responses. The GLMM framework is useful for the following types of model structures:

- Randomized complete and incomplete block designs
- Split-plot experiments
- Random coefficient models

The GLMM personality is a combination of two existing approaches: the linear mixed model framework and the generalized linear model framework.

Linear mixed models are fit in JMP using the Standard Least Squares or Mixed personalities of the Fit Model platform. Fitting a linear mixed model enables you to accurately represent random effects in the model. However, these models assume that the response variable is Gaussian, which means it is continuous with an infinite range. This assumption is problematic if you want to fit random effects, but have a non-Gaussian response.

Generalized linear models are fit in JMP using the Generalized Linear Model or Generalized Regression personalities of the Fit Model platform. Fitting a generalized linear model enables you to model non-Gaussian responses, such as discrete count data or binary data. However, you cannot include random effects.

Combining the linear mixed model and the generalized linear model frameworks enables you to test hypotheses and have accurate estimation for non-Gaussian response distributions when you also have random effects. For example, you can fit a logistic regression with random effects using the GLMM personality.

JMP PRO Example of a Generalized Linear Mixed Model

This example illustrates using the Generalized Linear Mixed Model personality of the Fit Model platform to evaluate two teaching programs. In an educational study, classes of students from multiple schools were assigned to one of two programs. At the end of the program, students took an evaluation test for which they receive a grade of Pass or Fail.

1. Select **Help > Sample Data Folder** and open Student Testing.jmp.
2. Select **Analyze > Fit Model**.

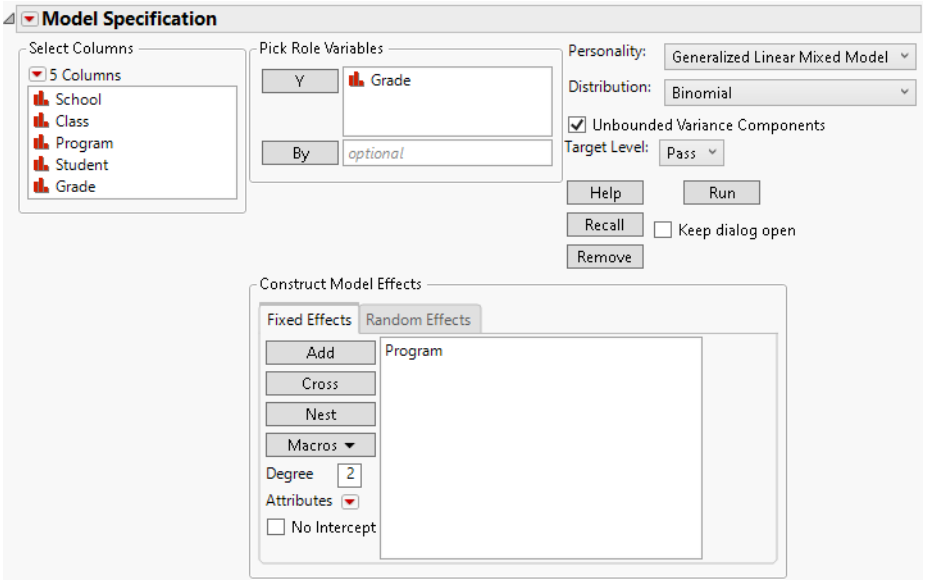
3. Select Grade and click **Y**.

   When you add this column as Y, the fitting Personality becomes Nominal Logistic.

4. Select **Generalized Linear Mixed Model** from the Personality list. Alternatively, you can select the Generalized Linear Mixed Model personality first, and then click **Y** to add Grade.

5. Select Program and click **Add** on the Fixed Effects tab.

Figure 9.2 Completed Fit Model Launch Window Showing Fixed Effects

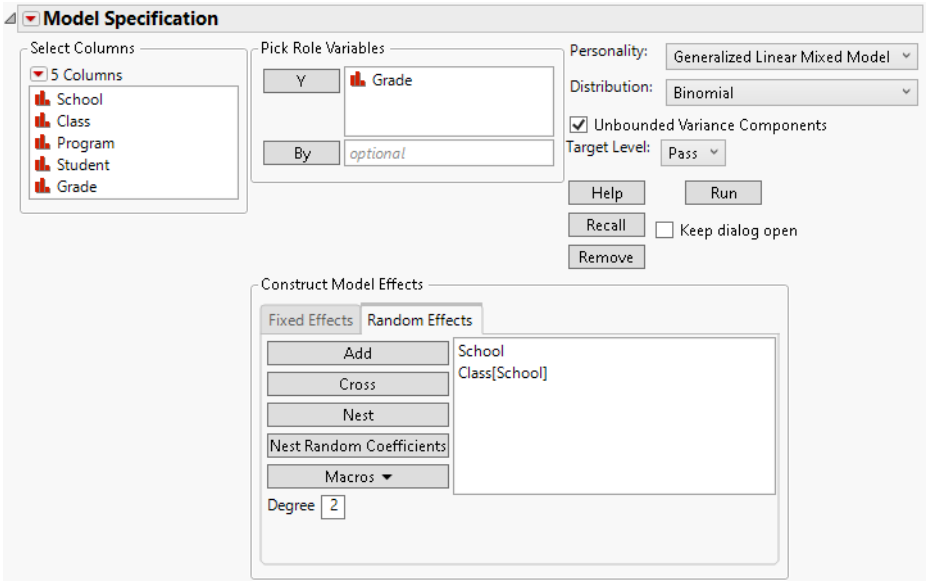


6. Select the **Random Effects** tab.

7. Select School and Class and click **Add**.

8. Select School from the Select Columns list, select Class from the Random Effects tab, and then click **Nest**.

Figure 9.3 Completed Fit Model Launch Window Showing Random Effects Tab



9. Click **Run**.

   The Generalized Linear Mixed Model report is shown in Figure 9.4. The effect of the teaching program is not statistically significant. However, the class effect nested within school is statistically significant.

Figure 9.4 Generalized Linear Mixed Model Report Window

Generalized Linear Mixed Model for Grade = Pass

Binomial

Fit Statistics

-2 Residual Log Pseudo-Likelihood	2598.7822
Generalized Chi-Square	272.85331
Generalized Chi-Square / DF	0.5478982

Model Summary

Response	Grade
Distribution	Binomial
Probability Model Link	Logit

Random Effects Covariance Parameter Estimates

Variance Component	Estimate	Std Error	95% Lower	95% Upper	Wald p-Value
School	1.8060322	1.5427052	-1.217614	4.8296789	0.2417
Class[School]	3.5936449	1.1550594	1.32977	5.8575197	0.0019*

Fixed Effects Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|	95% Lower	95% Upper
Intercept	1.7753397	0.5409684	7.1	3.28	0.0132*	0.5000999	3.0505794
Program[0]	-0.053192	0.3404391	38.8	-0.16	0.8766	-0.741929	0.6355445

Random Coefficients

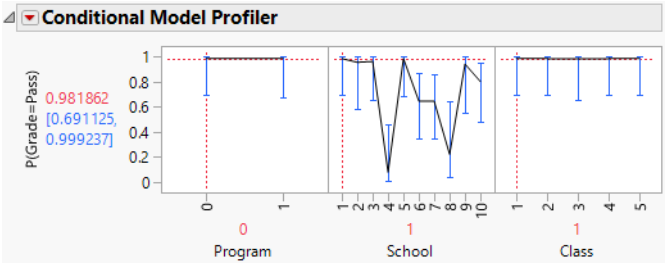
Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Program	1	1	38.8	0.024413	0.8766

10. Click the red triangle next to Binomial and select **Conditional Model Inference > Conditional Profiler**.

    You can use the Conditional Model Profiler to explore the differences in pass rates for various classes and schools.

Figure 9.5 Conditional Model Profiler



JMP PRO Launch the Generalized Linear Mixed Model Personality

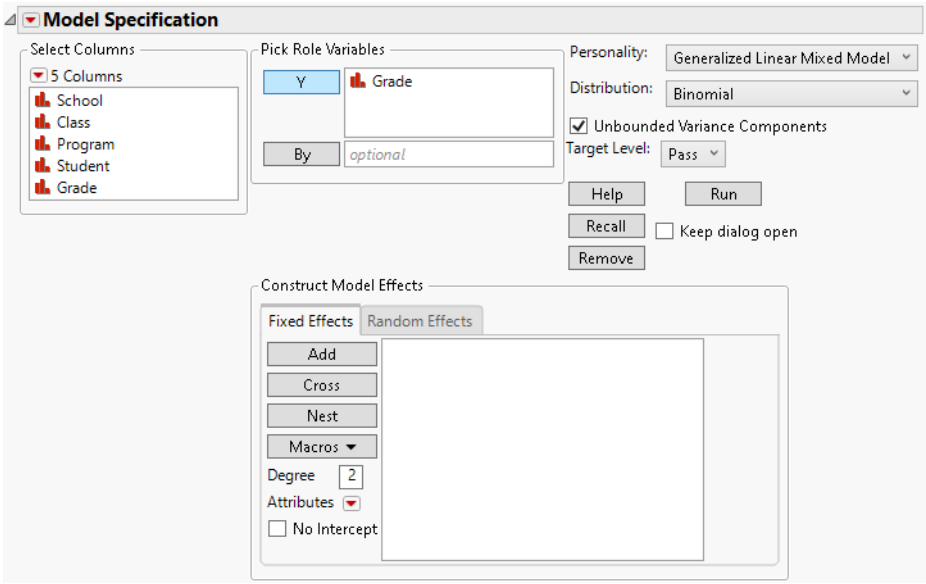
Launch the Generalized Linear Mixed Model personality by selecting **Analyze > Fit Model**, entering one or more columns for **Y**, and selecting **Generalized Linear Mixed Model** from the **Personality** menu.

JMP PRO Fit Model Launch Window

You can specify models with fixed effects, random effects, or a combination of fixed and random effects. You can also specify a distribution for the response variable. The options in the launch window differ based on the nature of the model that you specify. For more information about the options in the Select Columns red triangle menu, see *Using JMP*.

When fitting models using the Generalized Linear Mixed Model personality, you can allow unbounded variance components. This means that variance components that have negative estimates are not reported as zero. This option is selected by default. It should remain selected if you are interested in fixed effects, because bounding the variance estimates at zero leads to bias in the tests for fixed effects. See "Negative Variances" for more information about the Unbounded Variance Components option.

Figure 9.6 Fit Model Launch Window with Generalized Linear Mixed Model Selected



For more information about aspects of the Fit Model window that are common to all personalities, see "Model Specification". For more information about the options in the Select Columns red triangle menu, see *Using JMP*. Information specific to the Generalized Linear Mixed Model personality is presented here.

JMP PRO Specify a Distribution

In the Fit Model launch window, when you select Generalized Linear Mixed Model as the Personality, the Distribution option appears. Here you can specify a distribution for Y. The available distributions are described below.

Binomial Specifies that Y has a binomial distribution with parameters p and n. The response, Y, indicates the total number of successes in n independent trials with a fixed probability, p, for all trials. This distribution allows for the use of a sample size column. If no column is listed, it is assumed that the sample size is one. The link function for p is the logit. When you select a binary response variable that has a Nominal modeling type, Binomial is the only available response distribution.

When you select Binomial as the Distribution, the response variable must be specified in one of the following ways.

– Unsummarized: If your data are not summarized as frequencies of events, specify a single binary column as the response. If this column has a modeling type of Nominal, you can designate one of the levels to be the Target Level. The default Target Level value is the higher of the two levels based on the order of the levels.

– Summarized with sample size column entered as second Y: If your data are summarized as frequencies of events (successes) and trials, specify two continuous columns as Y in this order: the count of the number of successes, and the count of the number of trials.

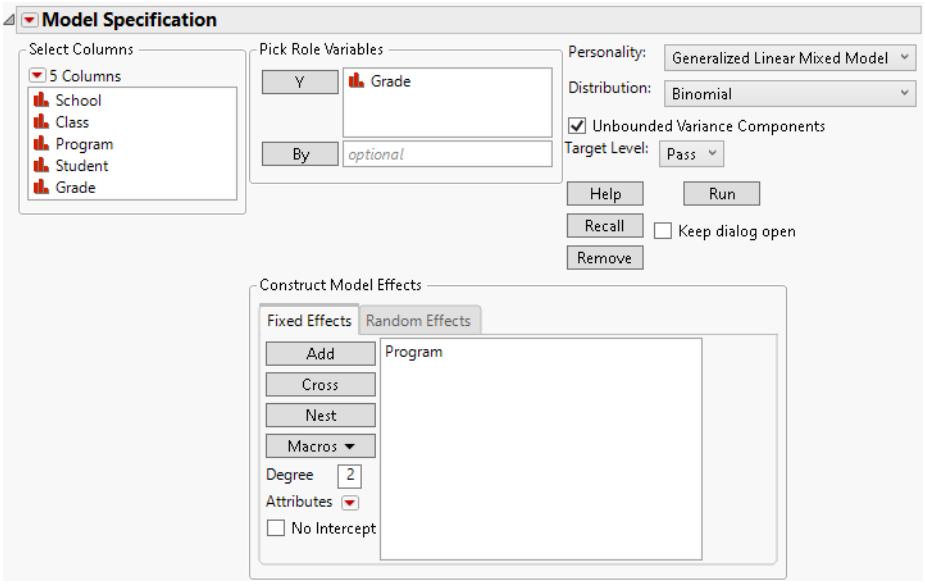
Poisson Specifies that Y has a Poisson distribution with mean λ. The Poisson distribution typically models the number of events in a given interval and is often expressed as count data. The link function for λ is the logarithm. Poisson regression is permitted even if Y assumes non-integer values.

JMP PRO Fixed Effects Tab

Add all fixed effects on the Fixed Effects tab. Use the Add, Cross, Nest, Macros, and Attributes options as needed. For more information about these options, see "Model Specification". Note that it is possible to have no fixed effects in the model.

Note: If a continuous column is involved in a random effect, that column is not centered, even if the Center Polynomials option in the Model Specifications red triangle menu is selected.

Figure 9.7 Completed Fit Model Launch Window Showing Fixed Effects



JMP PRO Random Effects Tab

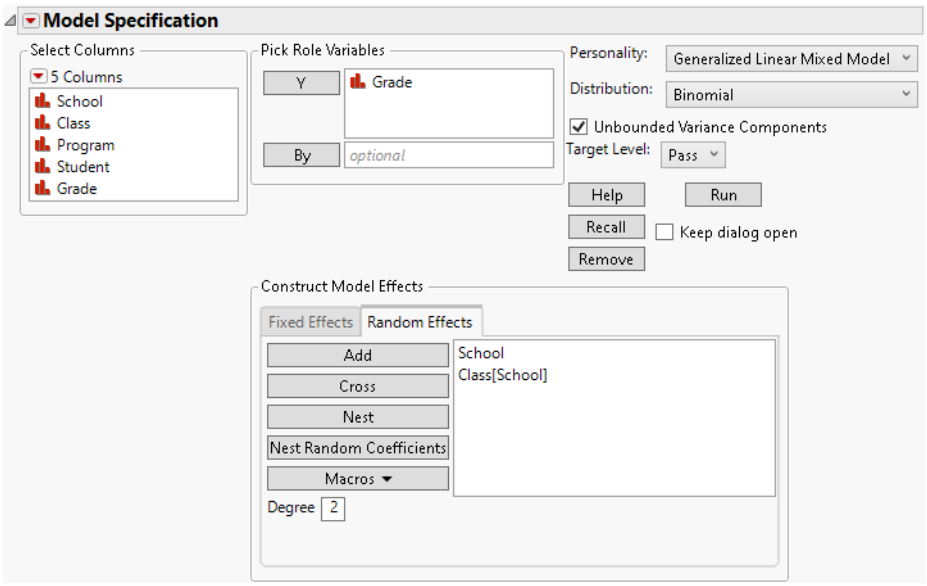
Specify traditional variance component models and random coefficients models using the Random Effects tab.

Note: If a continuous column is involved in a random effect, that column is not centered, even if the Center Polynomials option in the Model Specifications red triangle menu is selected.

Variance Components

For a traditional variance component model, specify terms such as random blocks, whole plot error terms, and subplot error terms using the Add, Cross, or Nest options. For more information about these options, see "Model Specification".

Figure 9.8 Completed Fit Model Launch Window Showing Random Effects



Random Coefficients

To construct random coefficients models, use the Nest Random Coefficients button to create groups of random coefficients.

1. Select the continuous columns from the Select Columns list that are predictors.
2. Select the **Random Effects** tab and then **Add**.
3. Select these effects in the Random Effects tab. Also select the column that contains the random effect whose levels define the individual regression models.
4. Click the **Nest Random Coefficients** button.

This last step creates random intercept and random slope effects that are correlated within the levels of the random effect. The subject is nested within the other effects due to the variability among subjects. If you believed that the intercept might be fixed for all groups, you would select Intercept[<*group*>]&Random Coefficients(1) and then click **Remove**.

You can define multiple groups of random coefficients in this fashion, as in hierarchical linear models. This might be necessary when you have both a random batch effect and a random batch by treatment effect on the slope and intercept coefficients. This might also be necessary in a hierarchical linear model: when you have a random student effect and random school effect on achievement scores and students are nested within school.

JMP PRO Data Format

The Generalized Linear Mixed Model personality of the Fit Model platform requires that all response measurements be contained in one response column. Repeated measures data are sometimes recorded in multiple columns, where each row is a subject and the repeated measurements are recorded in separate response columns. Data that are in this format must be stacked before running the Generalized Linear Mixed Model personality. The Cholesterol.jmp and Cholesterol Stacked.jmp sample data tables illustrate the wide format and the stacked format, respectively. Notice that each row in the wide table corresponds to one level of Patient in the stacked table.

JMP PRO Generalized Linear Mixed Model Options

The Generalized Linear Mixed Model red triangle menu contains the following options:

Model Dialog Shows the completed Fit Model launch window for the current analysis.

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Save By-Group Script Contains options that enable you to save a script that reproduces the platform report for all levels of a By variable to several destinations. Available only when a By variable is specified in the launch window.

JMP PRO Model Fit Reports

The Generalized Linear Mixed Model report contains a model fit report that is named for the distribution that was specified in the Fit Model launch window. The following reports appear by default:

- "Fit Statistics and Model Summary"
- "Random Effects Covariance Parameter Estimates"
- "Fixed Effects Parameter Estimates"
- "Random Coefficients"
- "Fixed Effects Tests"

Fit Statistics and Model Summary

In the Generalized Linear Mixed Model report, the Fit Statistics and Model Summary sections contain information that describes the model you have fit.

Fit Statistics

The columns in the Fit Statistics table depend on if there are random effects in the model.

No Random Effects

If there are no random effects, the table contains the following statistics for the model fit:

-2 Log Likelihood Twice the negative of the natural logarithm of the likelihood function for the model. See "Likelihood, AICc, and BIC". Use the log-likelihood for model comparisons in which the fixed, random, and repeated effects differ in any of the models.

Pearson Chi-Square The Pearson chi-square statistic for the model.

Pearson Chi-Square / DF The Pearson chi-square statistic for the model divided by the degrees of freedom for the model.

Tip: The Pearson Chi-Square / DF value can be used to evaluate if overdispersion is present in the data versus the model. If this value is much greater than 1, that is evidence of overdispersion. Specifically, if you fit a model with no random effects and the Pearson Chi-Square / DF is much greater than 1, the overdispersion could be due to a missing random effect. See "Additional Example of the Generalized Linear Mixed Model Personality".

Random Effects

If there are random effects, the table contains the following statistics for the model fit:

-2 Residual Pseudo-Likelihood Twice the negative of the natural logarithm of the residual pseudo-likelihood function for the model.

Generalized Chi-Square The generalized chi-square statistic for the model.

Generalized Chi-Square / DF The generalized chi-square statistic for the model divided by the degrees of freedom for the model.

Tip: The Generalized Chi-Square / DF value can be used to evaluate if overdispersion is present in the data versus the model. If this value is much greater than 1, that is evidence of overdispersion. See "Additional Example of the Generalized Linear Mixed Model Personality".

Model Summary

The Model Summary table contains the following information about the model:

Response The column assigned to the Y role in the Fit Model launch window.

Distribution The Distribution selected in the Fit Model launch window.

Probability Model Link (Appears only when the Distribution is Binomial.) The link function for the model for the probability.

Mean Model Link (Appears only when the Distribution is Poisson.) The link function for the mean.

Random Effects Covariance Parameter Estimates

In the Generalized Linear Mixed Model report, the Random Effects Covariance Parameter Estimates section describes the covariance parameters for the random effects that you specified in the model.

Variance Component The variance components for the random effects that you specified in the model.

Estimate The estimated variance component for the effect.

Std Error The standard error for the covariance component estimate.

95% Lower, 95% Upper The lower and upper 95% confidence limits for the variance component. You can change the α level in the Fit Model launch window by selecting Set Alpha Level from the Model Specification red triangle menu. See "Confidence Intervals for Variance Components".

Wald p-Value (Appears only when the Unbounded Variance Components option is selected in the Fit Model launch window.) The *p*-value for the test that the variance parameter is equal to zero.

Fixed Effects Parameter Estimates

In the Generalized Linear Mixed Model report, the Fixed Effects Parameter Estimates section describes the parameters for the fixed effects that you specified in the model.

Term The model term that corresponds to the estimated parameter of the fixed effect. The first term is always the intercept, unless you selected the No Intercept option in the Fit Model launch window. Continuous columns that are part of higher order terms are centered by default. Nominal or ordinal effects appear with values of levels in brackets. See "The Factor Models" for information about the coding of nominal and ordinal terms.

Note: If a continuous column is involved in a random effect, that column is not centered, even if the Center Polynomials option in the Model Specifications red triangle menu was selected.

Estimate The parameter estimate for each term. This is the estimate of the term's coefficient in the model.

Std Error The estimate of the standard error for the parameter estimate.

DFDen The denominator degrees of freedom, or the degrees of freedom for error, for the effect test. DFDen is calculated using the Kenward-Roger first order approximation.

t Ratio The test statistic for the test of whether the true value of the parameter is zero. The *t* Ratio is the ratio of the estimate to its standard error. Given the usual assumptions about the model, the *t* Ratio has a Student's *t* distribution under the null hypothesis.

Prob>|t| The *p*-value for a two-sided test of the *t* Ratio.

95% Lower, 95% Upper The lower and upper 95% confidence limits for the parameter estimate. You can change the α level in the Fit Model launch window by selecting Set Alpha Level from the Model Specification red triangle menu.

Note: When there are no random effects in the model, the value of DFDen is Infinity. This indicates that the *t* Ratio column contains a *z* Ratio, and the Prob>|t| and confidence interval columns are based on *z* intervals.

Random Coefficients

For each random effect in the model, the Generalized Linear Mixed Model report contains a section that shows estimated coefficients and a section that shows the matrix of covariance estimates. Each row of the coefficients table corresponds to one level of the random effect. The row shows all coefficient estimates that are associated with that level of the random effect. The random coefficient estimates are used in conjunction with fixed effect estimates to create predictions for any specific level of the random effect.

Fixed Effects Tests

In the Generalized Linear Mixed Model report, the Fixed Effect Tests section contains a table of significance tests for each fixed effect in the model. The test for a given effect tests that the null hypothesis that all parameters associated with that effect are zero. An effect, such as a single continuous explanatory variable, might have only one parameter. In this case, the test is equivalent to the *t* test for that term in the Fixed Effects Parameter Estimates report.

Source The fixed effects in the model.

Nparm The number of parameters that are associated with the effect. A continuous effect has one parameter. The number of parameters for a nominal or ordinal effect is one less than its number of levels. The number of parameters for a crossed effect is the product of the number of parameters for each individual effect.

DFNum The numerator degrees of freedom for the effect test.

DFDen The denominator degrees of freedom for the effect test, or the degrees of freedom for error. DFDen is calculated using the Kenward-Roger first order approximation.

F Ratio or ChiSquare The computed *F* ratio or chi-square statistic for testing that the effect is zero. If no random effects are present in the model, the test is a chi-square test. Otherwise, the test is an *F* test.

Prob > F or Prob>ChiSq The *p*-value for the effect test.

JMP PRO Model Fit Options

In the Generalized Linear Mixed Model report, each model fit report has a red triangle menu that contains the following options:

Model Reports Enables you to customize the reports that are shown for the specified model fit. The reports that are available are determined by the type of analysis that you conduct. Several of these reports are shown by default.

Fit Statistics Shows or hides a report for model fit statistics. See "Fit Statistics and Model Summary".

Random Effects Covariance Parameter Estimates (Available only when there are random effects specified in the launch window.) Shows or hides a report of random effects covariance parameter estimates. See "Random Effects Covariance Parameter Estimates".

Fixed Effects Parameter Estimates Shows or hides a report of fixed effects parameter estimates. See "Fixed Effects Parameter Estimates".

Random Coefficients (Available only when there are random effects specified in the launch window.) Shows or hides a report of random coefficients. See "Random Coefficients".

Random Effects Predictions (Available only when there are random effects specified in the launch window.) Shows or hides a report of random effect predictions. For each random effect in the model, the report provides an estimate known as the *best linear unbiased predictor* (BLUP), its standard error, degrees of freedom, and a Satterthwaite-based confidence interval. Estimation of the standard errors requires calculation of the BLUP covariance matrix, which can be time-intensive. If the calculation time is noticeable, a progress bar appears.

Indicator Parameterization Estimates (Available only when there are nominal columns among the fixed effects and the model contains an intercept.) Shows or hides the Indicator Function Parameterization report. This report gives parameter estimates for the fixed effects based on a model where nominal fixed effect columns are coded using indicator (SAS GLM) parameterization and are treated as continuous. Ordinal columns remain coded using the usual JMP coding scheme. The SAS GLM and JMP coding schemes are described in "The Factor Models".

Caution: Standard errors, t-ratios, and other results given in the Indicator Function Parameterization report differ from those in the Parameter Estimates report. This is because the estimates are estimating different parameters.

Fixed Effects Tests (Available only for models that contain at least one fixed effect.) Shows or hides the tests of fixed effects. See "Fixed Effects Tests".

Multiple Comparisons (Available only for models that contain at least one categorical fixed effect.) Opens the Multiple Comparisons launch window which provides various methods for comparing least squares means of main effects and interaction effects. For more information about the multiple comparisons options, see "Multiple Comparisons".

Once you click OK in the Multiple Comparisons launch window, a Multiple Comparisons report is added to the GLMM report window. A new Multiple Comparisons report is added each time you use the Multiple Comparisons option. Each Multiple Comparisons report contains estimates of the least squares means, standard error, and a 95% confidence interval on the original data scale. The report also contains estimates of the means or probabilities, standard errors, and a confidence interval on the inverse link scale. You can change the α level in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu. This report is followed by the multiple comparisons test that you select. The All Pairwise Comparisons report provides equivalence tests.

Diagnostic Bundle (Available only if Poisson is selected as the Distribution and there are no random effects in the model.) Shows or hides a set of four graphs including a plot of residuals by predicted values, residuals by row number, a histogram of the residuals, and a histogram of the probability of observing a response larger than the observed response.

The Fitted Probability of Observing a Larger Response histogram helps you assess goodness of fit of the model. The "correct" model should display an approximately uniform distribution of values.

Conditional Diagnostic Bundle (Available only if Poisson is selected as the Distribution and there are random effects in the model.) Shows or hides a set of four graphs including a plot of residuals by predicted values, residuals by row number, a histogram of the residuals,

and a histogram of the probability of observing a response larger than the observed response.

The Fitted Probability of Observing a Larger Response histogram helps you assess goodness of fit of the model. The "correct" model should display an approximately uniform distribution of values.

Marginal Model Inference Produces profilers that are based on marginal predicted values. The predicted values are shown in terms of counts for Poisson models and percents for binomial models.

Profiler Shows or hides a prediction profiler to examine the relationship between the response and model terms, without accounting for random effects.

Contour Profiler Shows or hides a contour profiler to examine the relationship between the response and model terms, without accounting for random effects.

Surface Profiler Shows or hides a surface profiler to examine the relationship between the response and model terms, without accounting for random effects.

Conditional Model Inference (Available only when there are random effects specified in the launch window.) Produces profilers that are based on conditional predicted values. Conditional predicted values reflect both fixed and random effects. The predicted values are shown in terms of counts for Poisson models and percents for binomial models.

Conditional Profiler Shows or hides a prediction profiler to examine the relationship between the response and the model terms, accounting for random effects.

Conditional Contour Profiler Shows or hides a contour profiler to examine the relationship between the response and the model terms, accounting for random effects.

Conditional Surface Profiler Shows or hides a surface profiler to examine the relationship between the response and the model terms, accounting for random effects.

Covariance and Correlation Matrices Contains options to view the covariance and correlation matrices that are associated with the model.

Covariance of Fixed Effects Shows or hides the covariance matrix for the fixed effects in the model.

Covariance of Covariance Parameters Shows or hides the covariance matrix for the random effects in the model.

Covariance of All Parameters Shows or hides the covariance matrix for all effects in the model.

Correlation of Fixed Effects Shows or hides the correlation matrix for the fixed effects in the model.

Save Columns Contains options to save various model results as one or more new columns in the data table. The predicted values are saved in terms of counts for Poisson models and percents for binomial models.

Prediction Formula Creates a new column called Pred Formula <colname> that contains both the formula and the marginal mean predicted values. A Predicting column property is added, noting the source of the prediction.

Standard Error of Predicted Creates a new column called StdErr Pred <colname> that contains standard errors for the predicted marginal mean responses.

Mean Confidence Interval Creates two new columns called Lower 95% Mean <colname> and Upper 95% Mean <colname>. These columns contain the lower and upper 95% confidence limits for the mean response. These intervals include the variation in the estimation, but not in the response. You can change the α level in the Fit Model launch window by selecting Set Alpha Level from the Model Specification red triangle menu.

Save Residual Formula (Not available if Binomial is selected as the Distribution.) Creates a new column called Residual <colname> that contains a formula for the residuals, given in the form Y minus the prediction formula.

Conditional Prediction Formula (Available only when there are random effects specified in the launch window.) Creates a new column called Cond Pred Formula <colname> that contains both the formula and the conditional mean predicted values. A Predicting column property is added, noting the source of the prediction.

Standard Error of Conditional Predicted (Available only when there are random effects specified in the launch window.) Creates a new column called StdErr Cond Pred <colname> that contains standard errors for the predicted conditional mean responses.

Conditional Mean CI (Available only when there are random effects specified in the launch window.)Creates two new columns called Lower 95% Cond Mean <colname> and Upper 95% Cond Mean <colname>. These columns contain the lower and upper 95% confidence limits for the expected value from conditional prediction. The confidence intervals include random effect estimates for models with random effects. You can change the α level in the Fit Model launch window by selecting Set Alpha Level from the Model Specification red triangle menu.

Save Conditional Residual Formula (Not available if Binomial is selected as the Distribution or if there are no random effects in the model.) Creates a new column called Cond Residual <colname> that contains the observed response values minus their conditional mean predicted values.

Save Simulation Formula (Not available when a By variable is specified in the Fit Model launch window.) Saves a column to the data table that contains a formula that generates simulated values using the estimated parameters for the model that you fit. This column can be used in the Simulate utility as a Column to Switch In. See *Basic Analysis*.

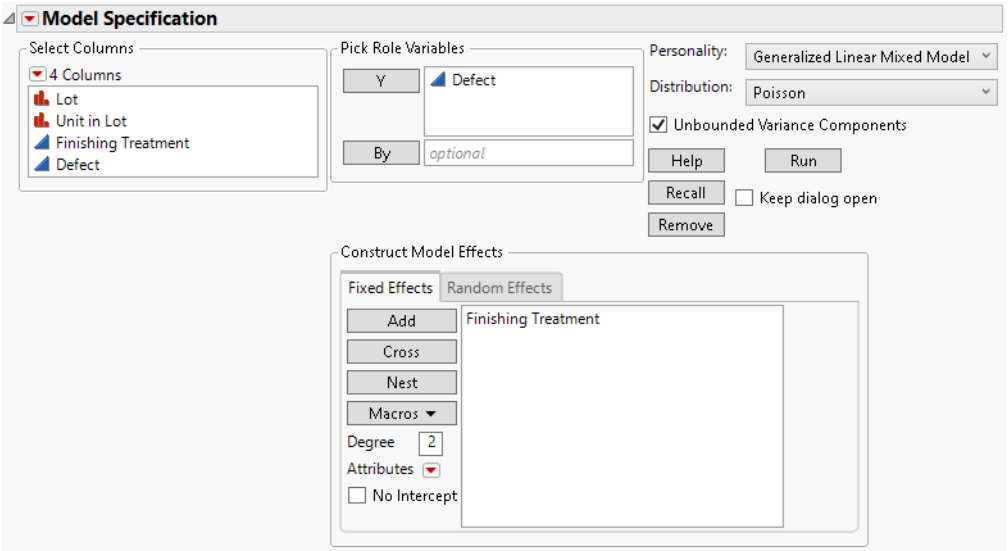
JMP PRO Additional Example of the Generalized Linear Mixed Model Personality

This example illustrates using the Generalized Linear Mixed Model personality of the Fit Model platform to analyze a manufacturing process where there are randomly selected lots, units within lots, and a finishing treatment that is applied to the units. The number of defects on each unit are counted, and the goal is to identify the level of finishing treatment that keeps the defect count less than 10. The design is a randomized complete block design with a non-Gaussian response variable (counts).

1. Select **Help > Sample Data Folder** and open Manufacturing Defect Counts.jmp.
2. Select **Analyze > Fit Model**.
3. Select Defect and click **Y**.

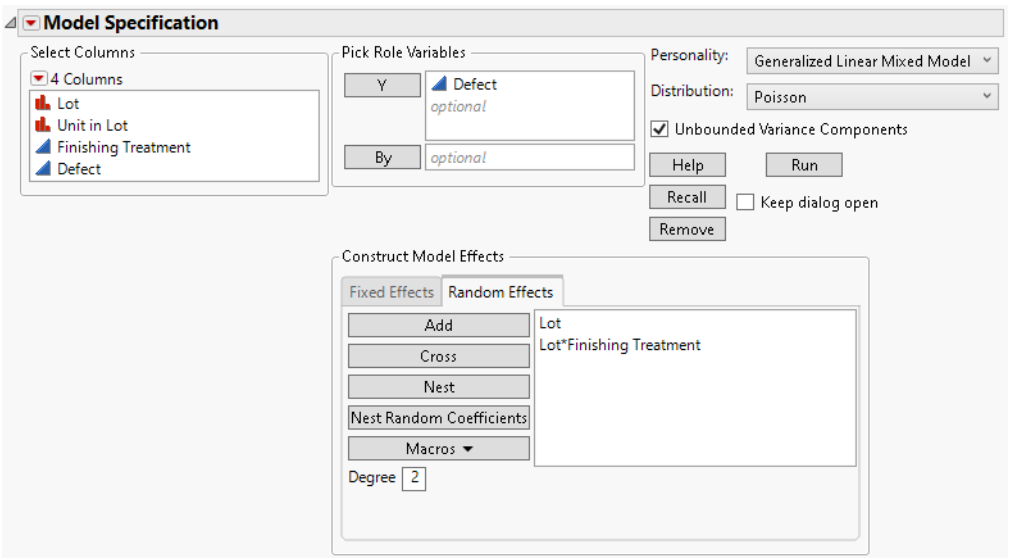
   When you add this column as Y, the fitting Personality becomes Standard Least Squares.
4. Select **Generalized Linear Mixed Model** from the Personality list. Alternatively, you can select the Generalized Linear Mixed Model personality first, and then click **Y** to add Defect.
5. From the Distribution list, select **Poisson**.
6. Select Finishing Treatment and click **Add** on the Fixed Effects tab.

Figure 9.9 Completed Fit Model Launch Window Showing Fixed Effects



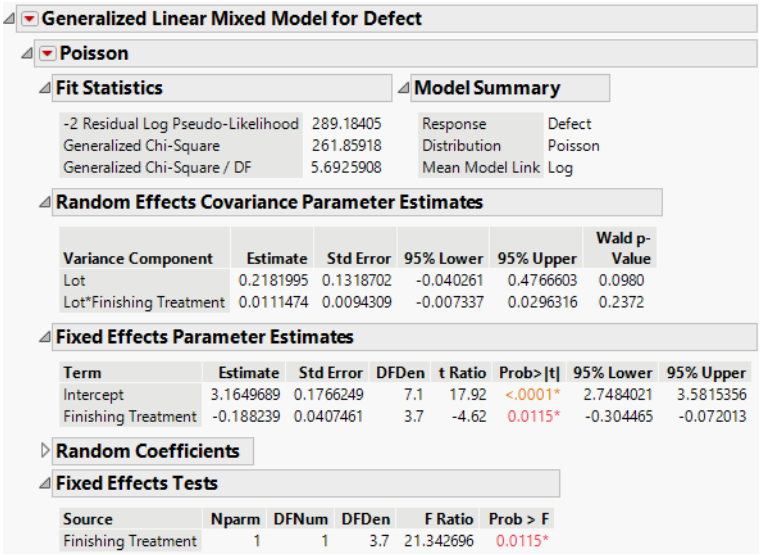
7. Select the **Random Effects** tab.
8. Select Lot and click **Add**.
9. Select Finishing Treatment from the Select Columns list, select Lot from the Random Effects tab, and then click **Cross**.

Figure 9.10 Completed Fit Model Launch Window Showing Random Effects Tab



10. Click **Run**.

Figure 9.11 Generalized Linear Mixed Model Report Window



Generalized Linear Mixed Model for Defect

Poisson

Fit Statistics

-2 Residual Log Pseudo-Likelihood	289.18405
Generalized Chi-Square	261.85918
Generalized Chi-Square / DF	5.6925908

Model Summary

Response	Defect
Distribution	Poisson
Mean Model Link	Log

Random Effects Covariance Parameter Estimates

Variance Component	Estimate	Std Error	95% Lower	95% Upper	Wald p-Value
Lot	0.2181995	0.1318702	-0.040261	0.4766603	0.0980
Lot*Finishing Treatment	0.0111474	0.0094309	-0.007337	0.0296316	0.2372

Fixed Effects Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|	95% Lower	95% Upper
Intercept	3.1649689	0.1766249	7.1	17.92	<.0001*	2.7484021	3.5815356
Finishing Treatment	-0.188239	0.0407461	3.7	-4.62	0.0115*	-0.304465	-0.072013

Random Coefficients

Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Finishing Treatment	1	1	3.7	21.342696	0.0115*

In the Fit Statistics section of the Generalized Linear Mixed Model report, note that the Generalized Chi-Square / DF statistic is 5.69. Since this value is much greater than 1, it is an indication that there is overdispersion in the data compared to the model. Next, you can fit a model with an additional term to account for the overdispersion.

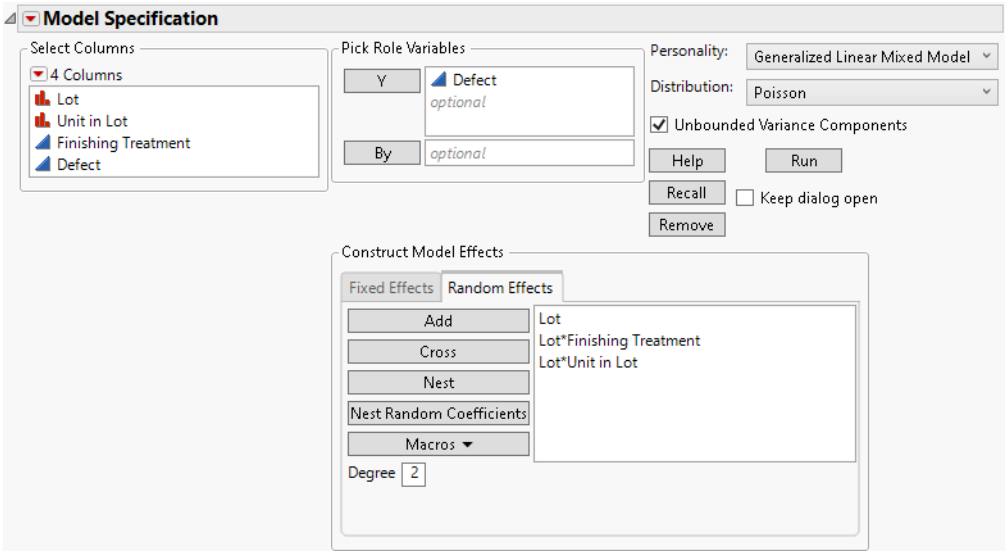
11. From the red triangle menu next to Generalized Linear Mixed Model for Defect, select **Model Dialog**.

    This option recalls the Fit Model launch window with the previous selections applied.

12. Select the **Random Effects** tab.

13. Select Unit in Lot from the Select Columns list, select Lot from the Random Effects tab, and then click **Cross**.

Figure 9.12 Completed Fit Model Launch Window Showing Random Effects Tab



14. Click **Run**.

Figure 9.13 Generalized Linear Mixed Model Report Window

Generalized Linear Mixed Model for Defect

Poisson

Fit Statistics

-2 Residual Log Pseudo-Likelihood	146.17242
Generalized Chi-Square	48.660739
Generalized Chi-Square / DF	1.0578421

Model Summary

Response	Defect
Distribution	Poisson
Mean Model Link	Log

Random Effects Covariance Parameter Estimates

Variance Component	Estimate	Std Error	95% Lower	95% Upper	Wald p-Value
Lot	0.1013655	0.2926026	-0.472125	0.674856	0.7290
Lot*Finishing Treatment	0.007238	0.0125458	-0.017351	0.0318273	0.5640
Lot*Unit in Lot	0.6708635	0.2020077	0.2749356	1.0667913	0.0009*

Fixed Effects Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|	95% Lower	95% Upper
Intercept	2.9144811	0.2523644	5.6	11.55	<.0001*	2.2861864	3.5427758
Finishing Treatment	-0.167797	0.0499182	4.5	-3.36	0.0233*	-0.300187	-0.035407

Random Coefficients

Fixed Effects Tests

Source	Nparm	DFNum	DFDen	F Ratio	Prob > F
Finishing Treatment	1	1	4.5	11.299297	0.0233*

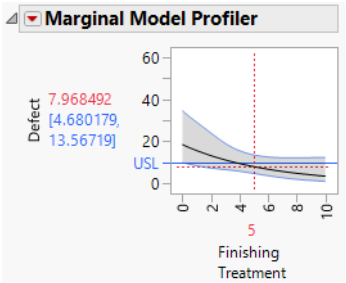
In the Fit Statistics section of the Generalized Linear Mixed Model report, note that the Generalized Chi-Square / DF statistic is now 1.06. Since this value is close to 1, it is an indication that the overdispersion in the data has been accounted for in the model. Next,

you can use this model to determine the level of the finishing treatment that keeps the defect count less than 10.

15. From the Poisson red triangle menu, select **Marginal Model Inference > Profiler**.

    The Marginal Model Profiler provides inference for the case where the lot effect is zero, so this profiler is useful for making inference about future lots.

Figure 9.14 Marginal Model Profiler for Defect Count



The profiler shows that for a finishing treatment level of 5, the predicted number of defects is approximately 8 and the 95% confidence interval around that prediction includes 10. Even at a finishing treatment level of 10, the confidence interval around the predicted defect count includes 10. Since 10 was the maximum finishing treatment level in the experiment, you recommend a new experiment that includes larger levels of finishing treatment to avoid extrapolation.

Chapter 10

Multivariate Response Models

Fit Relationships Using MANOVA

The Manova personality of the Fit Model platform enables you to fit multivariate models. Multivariate models fit several responses (Y variables) to a set of effects. The functions across the Y variables can be tested with appropriate response designs.

In addition to creating standard MANOVA (Multivariate Analysis of Variance) models, you can use the following techniques:

- Repeated measures analysis when repeated measurements are taken on each subject and you want to analyze effects both between subjects and within subjects across the measurements. This multivariate approach is especially important when the correlation structure across the measurements is arbitrary.
- Canonical correlation to find the linear combination of the X and Y variables that has the highest correlation.
- Discriminant analysis to find distance formulas between points and the multivariate means of various groups so that points can be classified into the groups that they are most likely to be in. A more complete implementation of discriminant analysis is in the Discriminant platform.

The multivariate fit begins with a rudimentary preliminary analysis that shows parameter estimates and least squares means. You can then specify a response design across the Y variables and multivariate tests are performed.

Contents

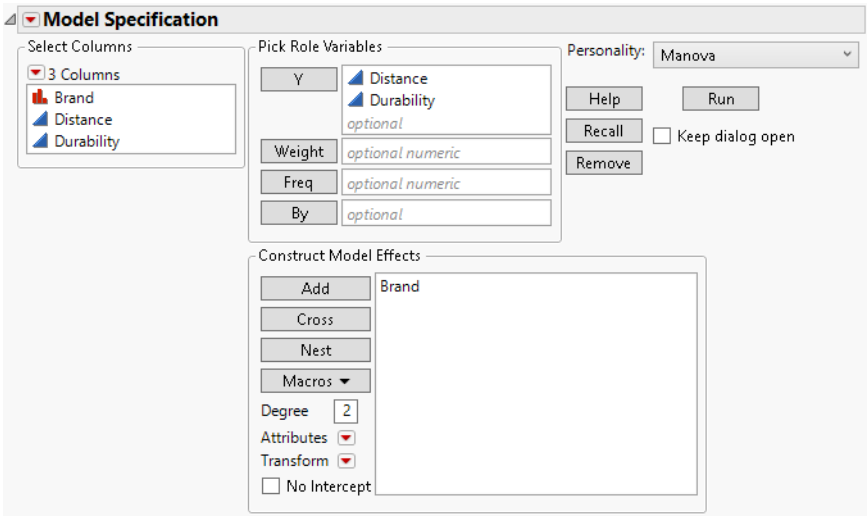
Example of a Multivariate Response Model 471
Launch the Manova Personality 473
The Manova Fit Report 473
The Manova Fit Options 474
Response Specification Panel 475
Multivariate Response Reports 476
Multivariate Tests in Multivariate Response Models 478
The Extended Multivariate Report 478
Comparison of Multivariate Tests 479
Univariate Tests and the Test for Sphericity 480
Multivariate Response Models with Repeated Measures 481
Discriminant Analysis in Multivariate Response Models 482
Additional Examples of the Manova Personality 482
Example of a Compound Multivariate Model 482
Example of a Repeated Measures Multivariate Model 483
Example of the Save Discrim Option 486
Example of Univariate and Sphericity Test 487
Example of Test Details 488
Example of Canonical Correlation Analysis 489
Statistical Details for the Manova Personality 490
Statistical Details for Multivariate Tests 490
Statistical Details for Approximate F-Tests 491
Statistical Details for Canonical Calculations 492

Example of a Multivariate Response Model

In this example, you are interested in testing the hypothesis that the distances traveled and durability are the same for three brands of golf balls. A robotic golfer hit a random sample of ten balls for each brand in a random sequence.

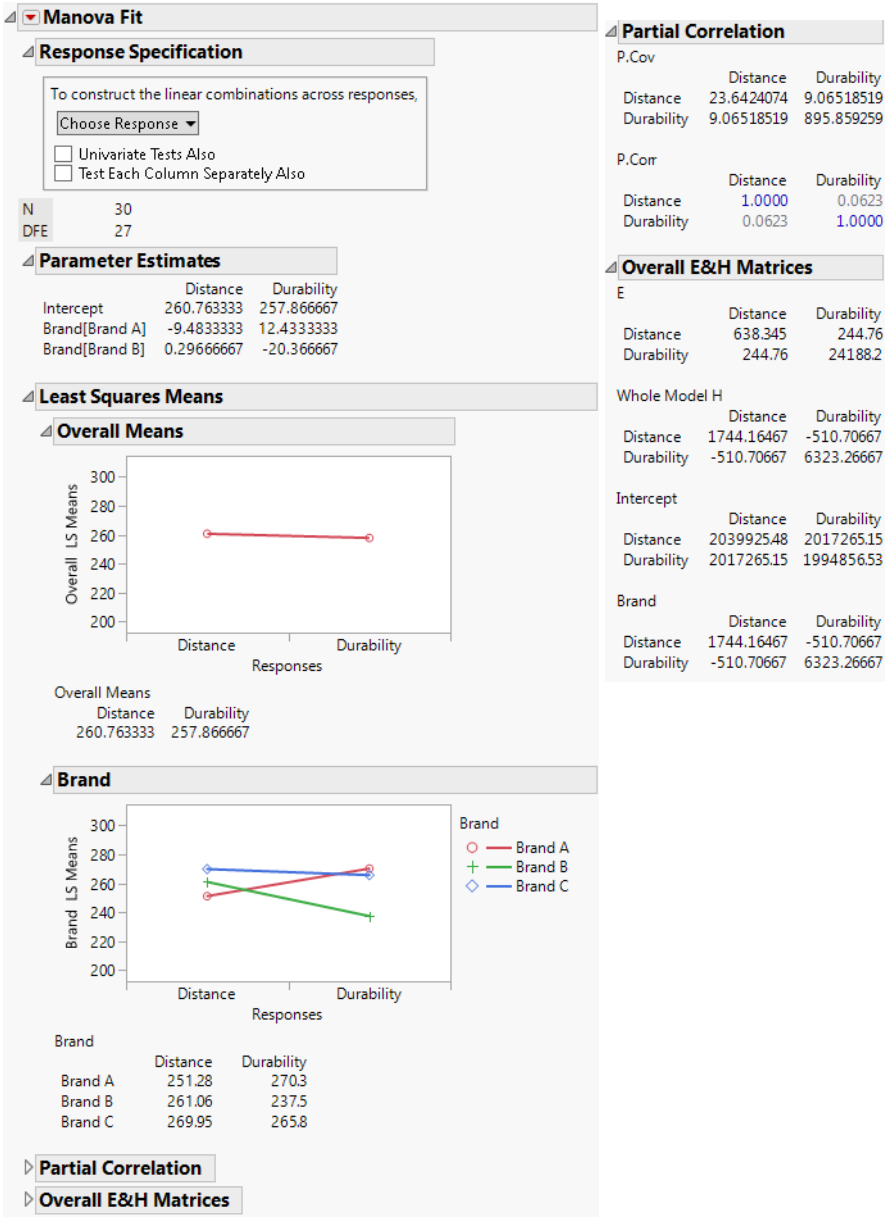
1. Select **Help > Sample Data Folder** and open Golf Balls.jmp.
2. Select **Analyze > Fit Model**.
3. Select Distance and Durability and click **Y**.
4. Select Brand and click **Add**.
5. For **Personality**, select **Manova**.

Figure 10.1 Manova Setup



6. Click **Run**.

Figure 10.2 Manova Report Window

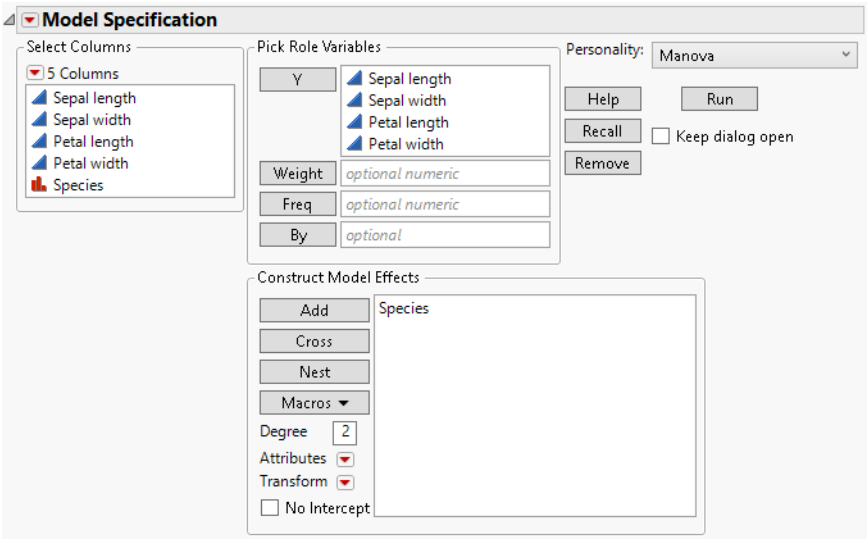


The initial results might not be very interesting in themselves, because no response design has been specified yet. After you specify a response design, the multivariate platform displays tables of multivariate estimates and tests. For more information about specifying a response design, see “Response Specification Panel”.

Launch the Manova Personality

Launch the Manova personality by selecting **Analyze > Fit Model**, entering one or more columns for **Y**, and selecting **Manova** from the **Personality** menu.

Figure 10.3 Fit Model Launch Window with Manova Selected



For more information about aspects of the Fit Model window that are common to all personalities, see "Model Specification". For more information about the options in the Select Columns red triangle menu, see *Using JMP*.

The Manova Fit Report

The Manova Fit report contains the following sections:

Response Specification Enables you to specify the response designs for various tests. See "Response Specification Panel".

Parameter Estimates Contains the parameter estimates for each response variable (without details like standard errors or *t tests*). There is a column for each response variable.

Least Squares Means Reports the overall least squares means of all of the response columns, least squares means of each nominal level, and least squares means plots of the means.

Partial Correlation Shows the covariance matrix and the partial correlation matrix of residuals from the initial fit, adjusted for the *X* effects.

Overall E&H Matrices Shows the E and H matrices:

- The elements of the **E** matrix are the cross products of the residuals.
- The **H** matrices correspond to hypothesis sums of squares and cross products.

There is an **H** matrix for the whole model and for each effect in the model. Diagonal elements of the **E** and **H** matrices correspond to the hypothesis (numerator) and error (denominator) sum of squares for the univariate *F* tests. New **E** and **H** matrices for any given response design are formed from these initial matrices, and the multivariate test statistics are computed from them.

The Manova Fit Options

The Manova Fit red triangle menu contains the following options:

Save Discrim Performs a discriminant analysis and saves the results to the data table. See "Discriminant Analysis in Multivariate Response Models".

Save Predicted Saves the predicted responses to the data table.

Save Residuals Saves the residuals to the data table.

Model Dialog Shows the completed launch window for the current analysis.

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

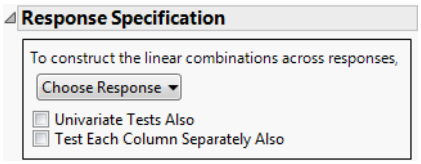
Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Save By-Group Script Contains options that enable you to save a script that reproduces the platform report for all levels of a By variable to several destinations. Available only when a By variable is specified in the launch window.

Response Specification Panel

In the Manova Fit report, specify the response designs for various tests using the Response Specification panel.

Figure 10.4 Response Specification Panel



Choose Response Provides choices for response design, which forms the M matrix. The columns of an M matrix define a set of transformation variables for the multivariate analysis. You can choose among the following options for the M matrix:

Repeated Measures Constructs and runs both Sum and Contrast responses.

Sum Sum of the responses that gives a single value.

Identity Uses each separate response, the identity matrix.

Contrast Compares each response and the first response.

Polynomial Constructs a matrix of orthogonal polynomials.

Helmert Compares each response with the combined responses listed below it.

Profile Compares each response with the following response.

Mean Compares each response with the mean of the others.

Compound Creates and runs several response functions that are appropriate if the responses are compounded from two effects.

Custom Uses any custom M matrix that you enter.

Note: The most typical response designs are Repeated Measures and Identity for multivariate regression. There is little difference in the tests given by the Contrast, Helmert, Profile, and Mean options, since they span the same space. However, the tests and details in the Least Squares means and Parameter Estimates tables for them show correspondingly different highlights. The Repeated Measures and the Compound options show dialogs to specify response effect names. They then fit several response functions without waiting for further user input. Otherwise, selections expand the control panel and give you more opportunities to refine the specification.

Univariate Tests Also Obtains adjusted and unadjusted univariate repeated measures tests and multivariate tests. Use in repeated measures models.

Test Each Column Separately Also Obtains univariate ANOVA tests and multivariate tests on each response.

The following buttons are available only after you have chosen a response option:

Run Performs the analysis and shows the multivariate estimates and tests. See "Multivariate Tests in Multivariate Response Models".

Help Shows the Help for the Response Specification panel.

Orthogonalize Orthonormalizes the matrix. Orthonormalization is done after the column contrasts (sum to zero) for all response types except Sum.

Delete Last Column Reduces the dimensionality of the transformation.

Multivariate Response Reports

Each time you choose a response and click Run in the Response Specification panel, a multivariate response report is added to the Manova Fit report. The multivariate response reports are named using the response specification names.

The red triangle menu in each response specification report contains the following option:

Custom Test Shows the Custom Test launch control in the report. This launch control enables you to set up custom tests of effect levels. For more information about creating custom tests, see "Custom Test".

Report and Options for Model Effects

Each model effect report contains a table of multivariate tests. For more information about these tests, see "Comparison of Multivariate Tests". The red triangle menu for each effect name in the multivariate response report contains the following options to request additional information about the multivariate fit:

Test Details Shows or hides canonical details about the test for the whole model or the specified effect. This report contains the eigenvalues and eigenvectors of the $\mathbf{E}^{-1}\mathbf{H}$ matrix used to construct multivariate test statistics.

Eigenvalue The eigenvalues of the $\mathbf{E}^{-1}\mathbf{H}$ matrix used in computing the multivariate test statistics.

Canonical Corr The canonical correlations associated with each eigenvalue. This is the canonical correlation of the transformed responses with the effects, corrected for all other effects in the model.

Eigvec The eigenvectors of the $\mathbf{E}^{-1}\mathbf{H}$ matrix, or equivalently of $(\mathbf{E} + \mathbf{H})^{-1}\mathbf{H}$.

Centroid Plot Shows or hides a table of centroid values and a plot of the centroids (multivariate least squares means) on the first two canonical variables formed from the test space.

The table of centroid values appears above the Canonical Centroid Plot. Click the CentroidVal disclosure icon to show the table of centroid values for up to four centroids.

In the Canonical Centroid Plot, the first canonical axis is the vertical axis. If the test space is one dimensional, the centroids align on a vertical axis. Relationships among groups of variables can be verified with biplot rays and the associated eigenvectors. See "Details for Centroid Plot Option".

The Canonical Centroid Plot red triangle menu contains the following options:

Centroid Circles Shows or hides the centroid points and circles on the canonical centroid plot. The centroid points appear with a circle corresponding to the 95% confidence region (Mardia et al. 1979). When centroid plots are created under effect tests, circles corresponding to the effect being tested appear in red. Other circles appear blue. The coordinates for the centroid points appear in the CentroidVal matrix above the Canonical Centroid Plot.

Biplot Rays Shows or hides biplot rays on the canonical centroid plot. The biplot rays show the directions of the original response variables in the test space. The intersection of the biplot rays is labeled Grand. The coordinates for the biplot ray intersection and endpoints appear in the CentroidVal matrix above the Canonical Centroid Plot.

Show Points Shows or hides the individual points on the canonical centroid plot.

Save Canonical Scores Saves the canonical scores as columns in the current data table. These columns have both the values and their formulas. The columns are called Canon[i], where i refers to the i^{th} canonical score for the Y variables. The canonical scores are computed based on the $\mathbf{E}^{-1}\mathbf{H}$ matrix used to construct the multivariate test statistic. Canonical scores are saved for eigenvectors corresponding to nonzero eigenvalues. See "Statistical Details for Canonical Calculations".

Tip: Canonical correlation analysis is not a specific option, but it can be performed using a sequence of options in the multivariate fitting platform. First, click the Whole Model red triangle and select **Test Details**. Then click the Whole Model red triangle and select **Save Canonical Scores**. The details list the canonical correlations (Canonical Corr) next to the eigenvalues. The saved variables are called Canon[1], Canon[2], and so on. These columns contain both the values and their formulas. To obtain the canonical variables for the *X* side, repeat the same steps, but interchange the *X* and *Y* variables. If you already have the columns Canon[n] appended to the data table, the new columns are called Canon[n] 2 (or another number) that makes the name unique. For an example of canonical correlation analysis, see "Example of Canonical Correlation Analysis".

Contrast Performs the statistical contrasts of treatment levels that you specify in the contrasts dialog.

Note: The **Contrast** option is the same as for regression with a single response. See "LSMeans Contrast" for a description and examples of the LSMeans Contrast options.

Multivariate Tests in Multivariate Response Models

In the Manova Fit report, the M Matrix report gives the response design that you specified. The M-transformed Parameter Estimates report gives the original parameter estimates matrix multiplied by the transpose of the M matrix.

Note: Initially in this chapter, the matrix names **E** and **H** refer to the error and hypothesis cross products. After specification of a response design, **E** and **H** refer to those matrices transformed by the response design, which are actually **M′EM** and **M′HM**.

The Extended Multivariate Report

In multivariate response model fits, the sums of squares due to hypothesis and error are matrices of squares and cross products instead of single numbers. And there are lots of ways to measure how large a value the matrix for the hypothesis sums of squares and cross products (called **H** or **SSCP**) is compared to that matrix for the residual (called **E**). JMP reports the four multivariate tests that are commonly described in the literature. If you are looking for a test at an exact significance level, you might need to go hunting for tables in reference books. Fortunately, all four tests can be transformed into an approximate *F* test. If the response design yields a single value, or if the hypothesis is a single degree of freedom, the multivariate tests are equivalent and yield the same exact *F* test. JMP labels the test Exact F. Otherwise, JMP labels it Approx. F.

In the golf balls example, there is only one effect, so the Whole Model test and the test for Brand are the same, which show the four multivariate tests with approximate *F* tests. There is only a single intercept with two DF (one for each response), so the *F* test for it is exact and is labeled Exact F.

The red triangle menus on the Whole Model, Intercept, and Brand reports contain options to generate additional information, which includes eigenvalues, canonical correlations, a list of centroid values, a centroid plot, and a Save option that lets you save canonical variates.

The effect (Brand in this example) pop-up menu also includes the option to specify contrasts.

The custom test and contrast features are the same as those for regression with a single response. See "Standard Least Squares Models".

To see formulas for the MANOVA table tests, see "Statistical Details for Multivariate Tests".

The extended Multivariate Report contains the following columns:

Test Labels each statistical test in the table. If the number of response function values (columns specified in the **M** matrix) is 1 or if an effect has only one degree of freedom per response function, the exact *F* test is presented. Otherwise, the standard four multivariate test statistics are given with approximate *F* tests: Wilks' Lambda (Λ), Pillai's Trace, the Hotelling-Lawley Trace, and Roy's Maximum Root.

Value Value of each multivariate statistical test in the report.

Approx. F (or Exact F) *F*-values corresponding to the multivariate tests. If the response design yields a single value or if the test is one degree of freedom, this is an exact *F* test.

NumDF Numerator degrees of freedom.

DenDF Denominator degrees of freedom.

Prob>F Significance probability corresponding to the *F*-value.

Note: For more information about the Sphericity Test table, see "Univariate Tests and the Test for Sphericity".

Comparison of Multivariate Tests

Although the four standard tests for multivariate response models often give similar results, there are situations where they differ, and one might have advantages over another. Unfortunately, there is no clear winner. In general, here is the order of preference in terms of power:

1. Pillai's Trace
2. Wilks' Lambda

3. Hotelling-Lawley Trace
4. Roy's Maximum Root

When there is a large deviation from the null hypothesis and the eigenvalues differ widely, the order of preference is the reverse (Seber 1984).

Univariate Tests and the Test for Sphericity

There are cases in multivariate response models, such as a repeated measures model, that allow transformation of a multivariate problem into a univariate problem (Huynh and Feldt 1970). Using univariate tests in a multivariate context is valid in the following situations:

- If the response design matrix **M** is orthonormal (**M**′**M** = Identity).
- If **M** yields more than one response the coefficients of each transformation sum to zero.
- If the *sphericity* condition is met. The sphericity condition means that the M-transformed responses are uncorrelated and have the same variance. **M**′Σ**M** is proportional to an identity matrix, where Σ is the covariance of the *Y* variables.

If these conditions hold, the diagonal elements of the **E** and **H** test matrices sum to make a univariate sums of squares for the denominator and numerator of an *F* test. Note that if the above conditions do not hold, then an error message appears. In the case of Golf Balls.jmp, an identity matrix is specified as the **M**-matrix. Identity matrices cannot be transformed to a full rank matrix after centralization of column vectors and orthonormalization. So the univariate request is ignored.

For an example of univariate and sphericity tests, see "Example of Univariate and Sphericity Test".

Multivariate Response Models with Repeated Measures

One common use of multivariate fitting is to analyze data with repeated measures, also called *longitudinal data*. A subject is measured repeatedly across time, and the data are arranged so that each of the time measurements form a variable. Because of correlation between the measurements, data should not be stacked into a single column and analyzed as a univariate model unless the correlations form a pattern termed *sphericity*. See the previous section, "Univariate Tests and the Test for Sphericity", for more information about this topic.

With repeated measures, the analysis is divided into two layers:

- Between-subject (or across-subject) effects are modeled by fitting the sum of the repeated measures columns to the model effects. This corresponds to using the **Sum** response function. This response function is an **M**-matrix that is a single vector of 1s.
- Within-subjects effects (repeated effects, or time effects) are modeled with a response function that fits differences in the repeated measures columns. This analysis can be done using the **Contrast** response function or any of the other similar differencing functions: **Polynomial**, **Helmert**, **Profile**, or **Mean**. When you model differences across the repeated measures, think of the differences as being a new within-subjects effect, usually time. When you fit effects in the model, interpret them as the interaction with the within-subjects effect. For example, the effect for Intercept becomes the Time (within-subject) effect, showing overall differences across the repeated measures. If you have an effect A, the within-subjects tests are interpreted to be the tests for the A*Time interaction, which model how the differences across repeated measures vary across the A effect.

Table 10.1 shows the relationship between the response function and the model effects compared with what a univariate model specification would be. Using both the **Sum** (between-subjects) and **Contrast** (within-subjects) models, you should be able to reconstruct the tests that would have resulted from stacking the responses into a single column and obtaining a standard univariate fit.

There is a direct and an indirect way to perform the repeated measures analyses:

- The direct way is to use the pop-up menu item Repeated Measures. This prompts you to name the effect that represents the within-subject effect across the repeated measures. Then it fits both the **Contrast** and the **Sum** response functions. An advantage of this way is that the effects are labeled appropriately with the within-subjects effect name.
- The indirect way is to specify the two response functions individually. First, do the **Sum** response function and second, do either **Contrast** or one of the other functions that model differences. You need to remember to associate the within-subjects effect with the model effects in the contrast fit.

Discriminant Analysis in Multivariate Response Models

Discriminant analysis is a method of predicting some level of a one-way classification based on known values of the responses. The technique is based on how close the measurement variables are to the multivariate means of the levels being predicted. Discriminant analysis is more fully implemented using the Discriminant Platform. See *Multivariate Methods*.

In the Manova personality of the Fit Model platform, specify the measurement variables as *Y* effects and the classification variable as a single *X* effect. The multivariate fitting platform gives estimates of the means and the covariance matrix for the data, assuming that the covariances are the same for each group. You obtain discriminant information with the Save Discrim option in the Manova Fit red triangle menu. The Save Discrim option saves distances and probabilities as columns in the current data table using the initial **E** and **H** matrices. For an example of the Save Discrim option, see "Example of the Save Discrim Option".

For a classification variable with *k* levels, the Save Discrim option adds *k* distance columns, *k* classification probability columns, the predicted classification column, and two columns of other computational information to the current data table.

Additional Examples of the Manova Personality

This section contains examples using the Manova personality of the Fit Model platform.

- "Example of a Compound Multivariate Model"
- "Example of a Repeated Measures Multivariate Model"
- "Example of the Save Discrim Option"
- "Example of Univariate and Sphericity Test"
- "Example of Test Details"
- "Example of Canonical Correlation Analysis"

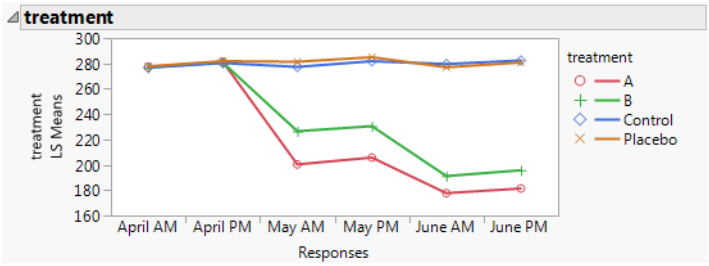
Example of a Compound Multivariate Model

In this example, you use the Manova personality of the Fit Model platform to model cholesterol treatment data with layers of repeated measures. Groups of five participants belong to one of four treatment groups called A, B, Control, and Placebo. Cholesterol was measured in the morning and again in the afternoon once a month for three months. In this example, the response columns are arranged chronologically with time of day within month.

1. Select **Help > Sample Data Folder** and open Cholesterol.jmp.

2. Select **Analyze > Fit Model**.
3. Select April AM, April PM, May AM, May PM, June AM, and June PM and click **Y**.
4. Select treatment and click **Add**.
5. Next to Personality, select **Manova**.
6. Click **Run**.

Figure 10.5 Treatment Graph



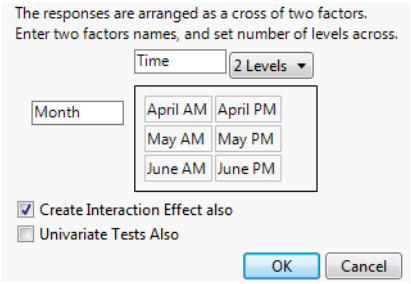
In the treatment graph, you can see that the four treatment groups began the study with very similar mean cholesterol values. The A and B treatment groups appear to have lower cholesterol values at the end of the trial period. The control and placebo groups remain unchanged.

7. Click the **Choose Response** menu and select **Compound**.

   Complete this window to tell JMP how the responses are arranged in the data table and the number of levels of each response. In the cholesterol example, the time of day columns are arranged within month. Therefore, you name time of day as one factor and the month effect as the other factor. Testing the interaction effect is optional.

8. Use the options in Figure 10.6 to complete the window.

Figure 10.6 Compound Window



9. Click **OK**.

   The tests for each effect appear. Parts of the report are shown in Figure 10.7. Note the following:

- The report for Time shows a *p*-value of 0.6038 for the interaction between Time and treatment, indicating that the interaction is not significant. This means that there is no evidence of a difference in treatment between AM and PM. Since Time has two levels (AM and PM), the exact *F* test appears.
- The report for Month shows *p*-values of <.0001 for the interaction between Month and treatment, indicating that the interaction is significant. This suggests that the differences between treatment groups change depending on the month. The treatment graph in Figure 10.5 indicates no difference among the groups in April, but the difference between treatment types (A, B, Control, and Placebo) becomes large in May and even larger in June.
- The report for Time*Month shows no significant *p*-values for treatment. This indicates that the three-way interaction effect involving Time, Month, and treatment is not statistically significant.

Figure 10.7 Cholesterol Study Results

Compound

Time

M Matrix

M-transformed Parameter Estimates

Whole Model

Intercept

treatment

Test	Value	Exact F	NumDF	DenDF	Prob>F
F Test	0.1188721	0.6340	3	16	0.6038

Compound

Month

M Matrix

M-transformed Parameter Estimates

Whole Model

Intercept

treatment

Test	Value	Approx. F	NumDF	DenDF	Prob>F
Wilks' Lambda	0.013025	38.8109	6	30	<.0001*
Pillai's Trace	1.3128917	10.1907	6	32	<.0001*
Hotelling-Lawley	50.753209	123.4368	6	18.326	<.0001*
Roy's Max Root	50.255302	268.0283	3	16	<.0001*

Compound

Time*Month

M Matrix

M-transformed Parameter Estimates

Whole Model

Intercept

treatment

Test	Value	Approx. F	NumDF	DenDF	Prob>F
Wilks' Lambda	0.6623742	1.1435	6	30	0.3619
Pillai's Trace	0.3582613	1.1638	6	32	0.3498
Hotelling-Lawley	0.4785668	1.1639	6	18.326	0.3671
Roy's Max Root	0.4008469	2.1378	3	16	0.1355

Example of a Repeated Measures Multivariate Model

In this example, you are interested in fitting a multivariate model to repeated measures data using the Manova personality of the Fit Model platform. Sixteen dogs are assigned to four groups defined by variables drug and dep1, each having two levels. The dependent variable is the blood concentration of histamine at 0, 1, 3, and 5 minutes after injection of the drug. The log of the concentration is used to minimize the correlation between the mean and variance of the data.

1. Select **Help > Sample Data Folder** and open Dogs.jmp.
2. Select **Analyze > Fit Model**.
3. Select LogHist0, LogHist1, LogHist3, and LogHist5 and click **Y**.
4. Select drug and dep1 and select **Full Factorial** from the **Macros** menu.
5. For Personality, select **Manova**.
6. Click **Run**.
7. In the **Choose Response** menu, select **Repeated Measures**.

   Time should be entered for YName. If you check the **Univariate Tests Also** check box, the report includes univariate tests, which are calculated as if the responses were stacked into a single column.
8. Click **OK**.

Figure 10.8 Repeated Measures Window

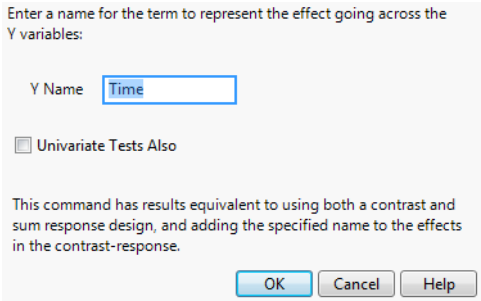


Table 10.1 shows how the multivariate tests for a **Sum** and **Contrast** response designs correspond to how univariate tests would be labeled if the data for columns LogHist0, LogHist1, LogHist3, and LogHist5 were stacked into a single *Y* column. The new rows are identified with a nominal grouping variable, Time.

Table 10.1 Corresponding Multivariate and Univariate Tests

Sum M-Matrix Between Subjects		**Contrast M-Matrix Within Subjects**	
Multivariate Test	**Univariate Test**	**Multivariate Test**	**Univariate Test**
intercept	intercept	intercept	time
drug	drug	drug	time*drug
depl	depl	depl	time*depl

The between-subjects analysis is produced first. This analysis is the same (except titling) as it would have been if **Sum** had been selected on the pop-up menu.

The within-subjects analysis is produced next. This analysis is the same (except titling) as it would have been if **Contrast** had been selected on the pop-up menu, though the within-subject effect name (Time) has been added to the effect names in the report. Note that the position formerly occupied by Intercept is Time, because the intercept term is estimating overall differences across the repeated measurements.

Example of the Save Discrim Option

The Save Discrim Option in the Manova personality of the Fit Model platform enables you to create columns that can be used in other JMP platforms to summarize the discriminant analysis with reports and graphs. In this example, there are $k = 3$ levels of the effect variable and four measures on each sample.

1. Select **Help > Sample Data Folder** and open Iris.jmp.
2. Select **Analyze > Fit Model**.
3. Select Sepal length, Sepal width, Petal length, and Petal width and click **Y**.
4. Select Species and click **Add**.
5. Next to Personality, select **Manova**.
6. Click **Run**.
7. Click the Manova Fit red triangle and select **Save Discrim**.

The following columns are added to the Iris.jmp sample data table:

SqDist[0] Quadratic form needed in the Mahalanobis distance calculations.

SqDist[setosa] Mahalanobis distance of the observation from the Setosa centroid.

SqDist[versicolor] Mahalanobis distance of the observation from the Versicolor centroid.

SqDist[virginica] Mahalanobis distance of the observation from the Virginica centroid.

Prob[0] Sum of the negative exponentials of the Mahalanobis distances, used below.

Prob[setosa] Probability of being in the Setosa category.

Prob[versicolor] Probability of being in the Versicolor category.

Prob[virginica] Probability of being in the Virginica category.

Pred Species Species that is most likely from the probabilities.

Now you can use the new columns in the data table with other JMP platforms to summarize the discriminant analysis with reports and graphs. For example:

1. From the updated Iris.jmp data table (that contains the new columns) select **Analyze > Fit Y by X**.
2. Select Species and click **Y, Response**.
3. Select Pred Species and click **X, Factor**.
4. Click **OK**.

The Contingency Table summarizes the discriminant classifications. Three misclassifications are identified.

Figure 10.9 Contingency Table of Predicted and Actual Species

Contingency Table

Pred Species	Count Total % Col % Row %	Species: setosa	Species: versicolor	Species: virginica	Total
setosa		50 33.33 100.00 100.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	50 33.33
versicolor		0 0.00 0.00 0.00	48 32.00 96.00 97.96	1 0.67 2.00 2.04	49 32.67
virginica		0 0.00 0.00 0.00	2 1.33 4.00 3.92	49 32.67 98.00 96.08	51 34.00
Total		50 33.33	50 33.33	50 33.33	150

Example of Univariate and Sphericity Test

In this example, you use the Manova personality of the Fit Model platform to perform univariate and sphericity tests on a multivariate response model.

1. Select **Help > Sample Data Folder** and open Dogs.jmp.
2. Select **Analyze > Fit Model**.

3. Select LogHist0, LogHist1, LogHist3, and LogHist5 and click **Y**.
4. Select drug and dep1 and click **Add**.
5. In the Construct Model Effects panel, select drug. In the Select Columns panel, select dep1. Click **Cross**.
6. For Personality, select **Manova**.
7. Click **Run**.
8. Select the check box next to **Univariate Tests Also**.
9. In the **Choose Response** menu, select **Repeated Measures**.

   Time should be entered for YName, and **Univariate Tests Also** should be selected.
10. Click **OK**.

Figure 10.10 Sphericity Test

Sphericity Test	
Mauchly Criterion	0.1752641
ChiSquare	16.930873
DF	5
Prob >Chisq	0.0046328

The sphericity test checks the appropriateness of an unadjusted univariate *F* test for the within-subject effects using the Mauchly criterion to test the sphericity assumption (Anderson 1958). The sphericity test and the univariate tests are always done using an orthonormalized **M** matrix. Use the following guidelines to interpret the sphericity test:

- If the true covariance structure is spherical, you can use the unadjusted univariate *F* tests.
- If the sphericity test is significant, the test suggests that the true covariance structure is not spherical. Therefore, you can use the multivariate or the adjusted univariate tests.

The univariate *F* statistic has an approximate *F* distribution even without sphericity, but the degrees of freedom for numerator and denominator are reduced by some fraction epsilon (ε). Box (1954), Greenhouse and Geisser (1959), and Huynh-Feldt (1976) offer techniques for estimating the epsilon degrees-of-freedom adjustment. Muller and Barton (1989) recommend the Greenhouse-Geisser version, based on a study of power.

The epsilon adjusted tests in the multivariate report are labeled G-G (Greenhouse-Geisser) or H-F (Huynh-Feldt). The epsilon adjustment is shown in the value column.

Example of Test Details

In this example, you use the Manova personality of the Fit Model platform to fit an identity multivariate response model and examine the details of the multivariate tests.

1. Select **Help > Sample Data Folder** and open Iris.jmp.

The Iris data (Mardia et al. 1979) have three levels of Species named Virginica, Setosa, and Versicolor. There are four measures (Petal length, Petal width, Sepal length, and Sepal width) taken on each sample.

2. Select **Analyze > Fit Model**.
3. Select Petal length, Petal width, Sepal length, and Sepal width and click **Y**.
4. Select Species and click **Add**.
5. For **Personality**, select **Manova**.
6. Click **Run**.
7. Click the **Choose Response** button and select **Identity**.
8. Click **Run**.
9. Click the Species red triangle and select **Test Details**.

   The eigenvalues, eigenvectors, and canonical correlations appear.

Figure 10.11 Test Details

Species

Test	Value	Approx. F	NumDF	DenDF	Prob>F
Wilks' Lambda	0.0234386	199.1453	8	288	<.0001*
Pillai's Trace	1.1918988	53.4665	8	290	<.0001*
Hotelling-Lawley	32.47732	582.1970	8	2034	<.0001*
Roy's Max Root	32.191929	1166.9574	4	145	<.0001*

Eigenvalue	Canonical Corr
32.1919292	0.98482089
0.28539104	0.47119702
1.235e-15	0
-6.174e-16	0

Eigvec

Sepal length	-0.0684059	0.00198791	-0.2350196	0.1176771
Sepal width	-0.1265612	0.1785267	0.21657608	0.04510419
Petal length	0.18155288	-0.0768636	0.23964446	0.06563465
Petal width	0.23180286	0.23417227	-0.2865277	-0.2438953

Example of Canonical Correlation Analysis

In this example, you use the Manova personality of the Fit Model platform to perform a canonical correlation analysis.

1. Select **Help > Sample Data Folder** and open Exercise.jmp.
2. Select **Analyze > Fit Model**.
3. Select chins, situps, and jumps and click Y.
4. Select weight, waist, and pulse and click **Add**.
5. For **Personality**, select **Manova**.
6. Click **Run**.
7. Click the **Choose Response** button and select **Identity**.

8. Click **Run**.
9. Click the Whole Model red triangle and select **Test Details**.
10. Click the Whole Model red triangle and select **Save Canonical Scores**.

Figure 10.12 Canonical Correlations

Whole Model

Test	Value	Approx. F	NumDF	DenDF	Prob>F
Wilks' Lambda	0.3503905	2.0482	9	34.223	0.0635
Pillai's Trace	0.6784815	1.5587	9	48	0.1551
Hotelling-Lawley	1.7719415	2.6397	9	19.053	0.0357*
Roy's Max Root	1.7247387	9.1986	3	16	0.0009*

Eigenvalue	Canonical Corr
1.72473874	0.79560815
0.0419084	0.20055604
0.00529433	0.07257029

Eigvec			
chins	0.02503681	-0.016636	0.05641878
situps	0.00637953	0.0004622	-0.004547
jumps	-0.0052909	0.0048507	0.0018787

The output canonical variables use the eigenvectors shown as the linear combination of the Y variables. For example, Canon[1] is calculated as follows:

```
0.02503681*chins + 0.00637953*situps + -0.0052909*jumps
```

This canonical analysis does not produce a standardized variable with mean 0 and standard deviation 1, but it is easy to define a new standardized variable with the calculator that has these features.

Statistical Details for the Manova Personality

This section provides statistical details for the Manova personality of the Fit Model platform.

- "Statistical Details for Multivariate Tests"
- "Statistical Details for Approximate F-Tests"
- "Statistical Details for Canonical Calculations"

Statistical Details for Multivariate Tests

In the following, **E** is the residual cross product matrix and **H** is the model cross product matrix. Diagonal elements of **E** are the residual sums of squares for each variable. Diagonal elements of **H** are the sums of squares for the model for each variable. In the discriminant analysis literature, **E** is often called **W**, where **W** stands for *within*.

Test statistics in the multivariate results tables are functions of the eigenvalues λ of $\mathbf{E}^{-1}\mathbf{H}$. The following list describes the computation of each test statistic.

Note: After specification of a response design, the initial **E** and **H** matrices are premultiplied by **M′** and postmultiplied by **M**.

- Wilks' Lambda

$$\Lambda = \frac{\det(\mathbf{E})}{\det(\mathbf{H}+\mathbf{E})} = \prod_{i=1}^{n}\left(\frac{1}{1+\lambda_i}\right)$$

- Pillai's Trace

$$V = \text{Trace}[\mathbf{H}(\mathbf{H}+\mathbf{E})^{-1}] = \sum_{i=1}^{n}\frac{\lambda_i}{1+\lambda_i}$$

- Hotelling-Lawley Trace

$$U = \text{Trace}(\mathbf{E}^{-1}\mathbf{H}) = \sum_{i=1}^{n}\lambda_i$$

- Roy's Max Root

$\Theta = \lambda_1$, the maximum eigenvalue of $\mathbf{E}^{-1}\mathbf{H}$.

E and **H** are defined as follows:

$$\mathbf{E} = \mathbf{Y}'\mathbf{Y} - \mathbf{b}'(\mathbf{X}'\mathbf{X})\mathbf{b}$$

$$\mathbf{H} = (\mathbf{L}\mathbf{b})'(\mathbf{L}(\mathbf{X}'\mathbf{X})^{-}\mathbf{L}')^{-1}(\mathbf{L}\mathbf{b})$$

where **b** is the estimated vector for the model coefficients and $\mathbf{A}^{-}$ denotes the generalized inverse of a matrix **A**.

The whole model **L** is a column of zeros (for the intercept) concatenated with an identity matrix having the number of rows and columns equal to the number of parameters in the model. **L** matrices for effects are subsets of rows from the whole model **L** matrix.

Statistical Details for Approximate F-Tests

To compute *F*-values and degrees of freedom, let p be the rank of $\mathbf{H}+\mathbf{E}$. Let q be the rank of $\mathbf{L}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{L}'$, where the **L** matrix identifies elements of $\mathbf{X}'\mathbf{X}$ associated with the effect being tested. Let v be the error degrees of freedom and s be the minimum of p and q. Also let $m = 0.5(|p-q|-1)$ and $n = 0.5(v-p-1)$.

Table 10.2, gives the computation of each approximate F from the corresponding test statistic.

Table 10.2 Approximate *F*-statistics

Test	Approximate F	Numerator DF	Denominator DF
Wilks' Lambda	$F = \left(\frac{1-\Lambda^{1/t}}{\Lambda^{1/t}}\right)\left(\frac{rt-2u}{pq}\right)$	pq	$rt-2u$
Pillai's Trace	$F = \left(\frac{V}{s-V}\right)\left(\frac{2n+s+1}{2m+s+1}\right)$	$s(2m+s+1)$	$s(2n+s+1)$
Hotelling-Lawley Trace	$F = \frac{2(sn+1)U}{s^2(2m+s+1)}$	$s(2m+s+1)$	$2(sn+1)$
Roy's Max Root	$F = \frac{\Theta(v-\max(p,q)+q)}{\max(p,q)}$	$\max(p,q)$	$v-\max(p,q)+q$

Statistical Details for Canonical Calculations

This section contains details for the canonical calculations used in the Manova personality of the Fit Model platform.

Details for the Test Details Option

When you select the Test Details option for a given test, eigenvalues, canonical correlations, and eigenvectors are shown in the report.

The canonical correlations produced by the Test Details option are computed as follows:

$$\rho_i = \sqrt{\frac{\lambda_i}{1+\lambda_i}}$$

where λ_i is the i^{th} eigenvalue of the $\mathbf{E}^{-1}\mathbf{H}$ matrix used in computing the multivariate test statistics

The matrix labeled Eigvec is the **V** matrix, which is the matrix of eigenvectors of $\mathbf{E}^{-1}\mathbf{H}$ for the given test.

Note: The **E** and **H** matrices for the given test refer to **M′EM** and **M′HM** in terms of the original **E** and **H** matrices. The **M** matrix is defined by the response design. The **E** and **H** used in this section are defined in "Statistical Details for Multivariate Tests".

Details for Centroid Plot Option

The total sample centroid and centroid values for effects are computed as follows:

$$\text{Grand} = (c'_1\bar{y}, c'_2\bar{y}, \ldots, c'_g\bar{y})$$

$$\text{Effect}_j = (c'_1\bar{x}_j, c'_2\bar{x}_j, \ldots, c'_g\bar{x}_j)$$

where

$$c_i = \left(v'_i\left(\frac{E}{N-r}\right)v_i\right)^{-1/2} v_i$$

N is the number of observations

v_i is the i^{th} column of **V**, the eigenvector matrix of $\mathbf{E}^{-1}\mathbf{H}$ for the given test

$\bar{x}_j$ is the multivariate least squares mean for the j^{th} effect

$\bar{y}$ is the overall mean of the responses

g is the number of eigenvalues of $\mathbf{E}^{-1}\mathbf{H}$ greater than 0

r is the rank of the **X** matrix

Note: The **E** and **H** matrices for the given test refer to **M′EM** and **M′HM** in terms of the original **E** and **H** matrices. The **M** matrix is defined by the response design. The **E** and **H** used in this section are defined in "Statistical Details for Multivariate Tests".

The centroid radii for effects are calculated as follows:

$$d = \sqrt{\frac{\chi^2_g(0.95)}{L(X'X)^{-1}L'}}$$

where g is the number of eigenvalues of $\mathbf{E}^{-1}\mathbf{H}$ greater than 0 and the **L** matrices in the denominator are from the multivariate least squares means calculations.

Details for the Save Canonical Scores Option

The canonical Y values are calculated as follows:

$$\tilde{Y} = \mathbf{YM'V}$$

where

Y is the matrix of response variables

M′ is the transpose of the response design matrix

V is the matrix of eigenvectors of $\mathbf{E}^{-1}\mathbf{H}$ for the given test

Note: The **E** and **H** matrices for the given test refer to **M′EM** and **M′HM** in terms of the original **E** and **H** matrices. The **M** matrix is defined by the response design. The **E** and **H** used in this section are defined in "Statistical Details for Multivariate Tests".

Canonical Y values are saved for eigenvectors corresponding to eigenvalues larger than zero.

Chapter 11

Loglinear Variance Models

Model the Variance and the Mean of the Response

The Loglinear Variance personality of the Fit Model platform enables you to model both the expected value and the variance of a response using regression models. The log of the variance is fit to one linear model and the expected response is fit to a different linear model simultaneously.

Note: The estimates are demanding in their need for a lot of well-designed, well-fitting data. You need more data to fit variances than you do means.

For many engineers, the goal of an experiment is not to maximize or minimize the response itself, but to aim at a target response and achieve minimum variability. The loglinear variance model provides a very general and effective way to model variances, and can be used for unreplicated data, as well as data with replications.

Contents

Overview of the Loglinear Variance Model 497
Example Using Loglinear Variance 498
Launch the Loglinear Variance Personality 500
The Loglinear Variance Fit Report 501
Loglinear Variance Fit Report Options 502
Additional Examples of Loglinear Variance Models 504
Example of Examining the Residuals in a Loglinear Variance Model 504
Example of Profiling a Fitted Loglinear Variance Model 505

Overview of the Loglinear Variance Model

The loglinear variance model provides a way to model variance through a linear model. In addition to having regressor terms to model the mean response, there are regressor terms in a linear model to model the log of the variance:

mean model: $E(y) = \mathbf{X}\beta$

variance model: $\log(Variance(y)) = \mathbf{Z}\,\lambda$,

or equivalently

$Variance(y) = \exp(\mathbf{Z}\,\lambda)$

where the columns of X are the regressors for the mean of the response, and the columns of Z are the regressors for the variance of the response. The regular linear model parameters are represented by β, and λ represents the parameters of the variance model. For more information about loglinear variance models, see Harvey (1976), Cook and Weisberg (1983), Aitken (1987), and Carroll and Ruppert (1988).

Loglinear variance models are estimated using REML.

A *dispersion* or *log-variance* effect can model changes in the variance of the response. This is implemented in the Fit Model platform by a fitting personality called the Loglinear Variance personality.

Dispersion Effects

Modeling dispersion effects is not very widely covered in textbooks, with the exception of the Taguchi framework. In a Taguchi-style experiment, this is handled by taking multiple measurements across settings of an outer array, constructing a new response that measures the variability off-target across this outer array, and then fitting the model to find out the factors that produce minimum variability. This type of modeling requires a specialized design that is a complete Cartesian product of two designs. The method of this chapter models variances in a more flexible, model-based approach. The particular performance statistic that Taguchi recommends for variability modeling is $STD = -\log(s)$. In the methodology used in JMP, the $\log(s^2)$ is modeled and combined with a model that has a mean. The two are basically equivalent, since $\log(s^2)=2\log(s)$.

Model Specification

Loglinear variance effects are specified in the Fit Model dialog by highlighting them and selecting **LogVariance Effect** from the **Attributes** drop-down menu. **&LogVariance** appears at the end of the effect. When you use this attribute, it also changes the fitting **Personality** at the top to **LogLinear Variance**. If you want an effect to be used for both the mean and variance of the response, then you must specify it twice, once with the **LogVariance** option.

The effects that you specify with the log-variance attribute become the effects that generate the Z variables in the model, and the other effects become the X variables in the model.

Notes

Every time another parameter is estimated for the mean model, at least one more observation is needed, and preferably more. But with variance parameters, several more observations for each variance parameter are needed to obtain reasonable estimates. It takes more data to estimate variances than it does means.

The loglinear variance model is a very flexible way to fit dispersion effects, and the method deserves much more attention than it has received so far in the literature.

Example Using Loglinear Variance

This example demonstrates fitting a loglinear variance model to data from a designed experiment. The data table contains the experimental results from a 7-factor 2^{7-3} fractional factorial design with four added centerpoints. Preliminary investigation determined that the mean response only seemed to vary with the first two factors, Mold Temperature, and Screw Speed, and the variance seemed to be affected by Holding Time.

Figure 11.1 Injection Molding Data

InjectionMolding
Locked File C:\Program
Model

Columns (9/0)
Pattern
MoldTemp *
Screw Speed *
Hold Time *
Gate Size *
Cycle Time + *
Moisture + *
Pressure + *
Shrinkage *

Rows
All rows 20
Selected 0
Excluded 0
Hidden 0
Labeled 0

	Pattern	MoldTemp	Screw Speed	Hold Time	Gate Size	Cycle Time	Moisture	Pressure	Shrinkage
1	-------	-1	-1	-1	-1	-1	-1	-1	6
2	+-----+	1	-1	-1	-1	1	-1	1	10
3	-+---+-	-1	1	-1	-1	1	1	-1	32
4	++--++-	1	1	-1	-1	-1	1	1	60
5	--+-+--	-1	-1	1	-1	1	1	1	4
6	+-+-+-+	1	-1	1	-1	-1	1	-1	15
7	-++--++	-1	1	1	-1	-1	-1	1	26
8	+++----	1	1	1	-1	1	-1	-1	60
9	---++++	-1	-1	-1	1	-1	1	1	8
10	+--+--+	1	-1	-1	1	1	1	-1	12
11	-+-+-+-	-1	1	-1	1	1	-1	1	34
12	++-+--+	1	1	-1	1	-1	-1	-1	60
13	--++--+	-1	-1	1	1	1	-1	-1	16
14	+-++-+-	1	-1	1	1	-1	-1	1	5
15	-++++--	-1	1	1	1	-1	1	-1	37
16	+++++++	1	1	1	1	1	1	1	52
17	0000000	0	0	0	0	0	0	0	25
18	0000000	0	0	0	0	0	0	0	29
19	0000000	0	0	0	0	0	0	0	24
20	0000000	0	0	0	0	0	0	0	27

1. Select **Help > Sample Data Folder** and open InjectionMolding.jmp.
2. Select **Analyze > Fit Model**.

Since the variables in the data table have been assigned preselected roles, the analysis runs automatically.

Figure 11.2 Loglinear Variance Report Window

Loglinear Variance Fit

Fit Statistics

-2 Residual Log Likelihood	90.712809
-2 Log Likelihood	87.777035
AICc	106.23857
BIC	105.75143

Mean Model for Shrinkage

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	27.582516	0.380697	72.45	<.0001*
MoldTemp	7.683713	0.392907	19.56	<.0001*
Screw Speed	18.673515	0.392907	47.53	<.0001*
MoldTemp*Screw Speed	5.765297	0.392907	14.67	<.0001*

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
MoldTemp	1	1	5.758	382.4387	<.0001*
Screw Speed	1	1	5.758	2258.768	<.0001*
MoldTemp*Screw Speed	1	1	5.758	215.3094	<.0001*

Variance Model for Shrinkage

Likelihood Ratio Test for Equal Variance

Source	-2 Residual Log Likelihood
Equal Variances Initially	101.76083063
After Fitted Variances	90.712808548
Difference: Chi-sq	11.048022086
Degrees of Freedom	1
Prob > ChiSquare	0.0008878186

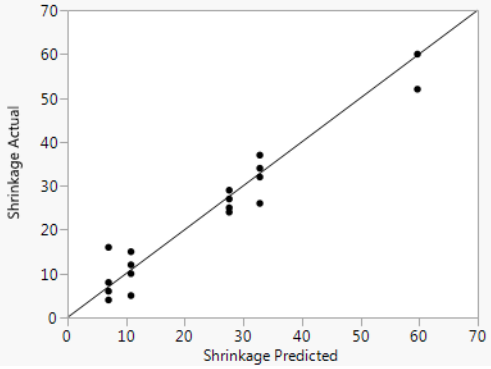
Variance Parameter Estimates

Term	Estimate	Std Error	ChiSquare	Prob>ChiSq	Lower 95% Limit	Upper 95% Limit
Hold Time	1.5788108	0.412603	11.0480	0.0009*	0.7057476	2.37325
Residual	6.6912718	2.445524	7.4864	0.0062*	3.2689762	13.696373

Variance Effect Likelihood Ratio Tests

Source	Test Type	DF	ChiSquare	Prob>ChiSq
Hold Time	Likelihood	1	11.0480	0.0009*

Actual by Predicted Plot

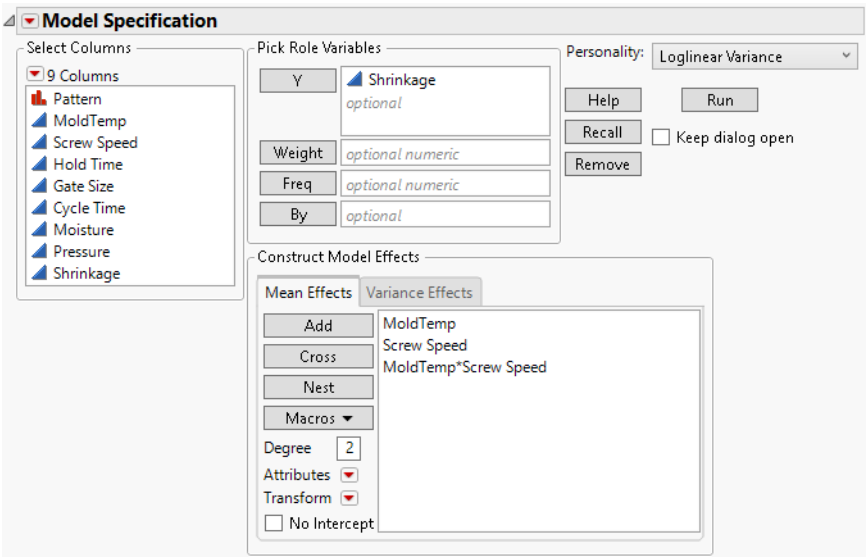


The Mean Model for Shrinkage report gives the parameters for the mean model, and the Variance Model for Shrinkage report gives the parameters for the variance model.

Launch the Loglinear Variance Personality

Launch the Loglinear Variance personality by selecting **Analyze > Fit Model**, entering one or more columns for **Y**, and selecting **Loglinear Variance** from the **Personality** menu.

Figure 11.3 Fit Model Launch Window with Loglinear Variance Selected



For more information about aspects of the Fit Model window that are common to all personalities, see "Model Specification". For more information about the options in the Select Columns red triangle menu, see *Using JMP*. Information specific to the Loglinear Variance personality is presented here.

If your model effects have missing values, you can treat these missing values as informative categories. Select the Informative Missing option from the Model Specification red triangle menu.

The Loglinear Variance Fit Report

The Loglinear Variance Fit report contains information about the overall model, the mean model, and the variance model. The report also contains a plot of actual versus predicted values for the loglinear model. The Parameter Estimates and Fixed Effect Tests sections of the mean model and variance model reports are similar to output found in the Standard Least Squares personality, though they are derived from restricted maximum likelihood (REML).

Figure 11.4 Mean Model Output

Loglinear Variance Fit

Fit Statistics

-2 Residual Log Likelihood	90.712809
-2 Log Likelihood	87.777035
AICc	106.23857
BIC	105.75143

Mean Model for Shrinkage

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	27.582516	0.380697	72.45	<.0001*
MoldTemp	7.683713	0.392907	19.56	<.0001*
Screw Speed	18.673515	0.392907	47.53	<.0001*
MoldTemp*Screw Speed	5.765297	0.392907	14.67	<.0001*

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
MoldTemp	1	1	5.758	382.4387	<.0001*
Screw Speed	1	1	5.758	2258.768	<.0001*
MoldTemp*Screw Speed	1	1	5.758	215.3094	<.0001*

Figure 11.5 Variance Model Output

Variance Model for Shrinkage

Likelihood Ratio Test for Equal Variance

Source	-2 Residual Log Likelihood
Equal Variances Initially	101.76083063
After Fitted Variances	90.712808548
Difference: Chi-sq	11.048022086
Degrees of Freedom	1
Prob > ChiSquare	0.0008878186

Variance Parameter Estimates

Term	Estimate	Std Error	ChiSquare	Prob>ChiSq	Lower 95% Limit	Upper 95% Limit
Hold Time	1.5788108	0.412603	11.0480	0.0009*	0.7057476	2.37325
Residual	6.6912718	2.445524	7.4864	0.0062*	3.2689762	13.696373

Variance Effect Likelihood Ratio Tests

Source	Test Type	DF	ChiSquare	Prob>ChiSq
Hold Time	Likelihood	1	11.0480	0.0009*

The second portion of the report shows the fit of the variance model. The Variance Parameter Estimates report shows the estimates and relevant statistics. Two hidden columns are provided:

- The hidden column **exp(Estimate)** is the exponential of the estimate. So, if the factors are coded to have +1 and -1 values, then the +1 level for a factor multiplies the variance by the

exp(Estimate) value. Likewise, the -1 level multiplies the variance by the reciprocal of this column. To see a hidden column, right-click the report and select the name of the column from the **Columns** menu that appears.

- The hidden column labeled **exp(2|Estimate|)**is the ratio of the higher to the lower variance if the regressor has the range -1 to +1.

The report also shows the standard error, chi-square, *p*-value, and profile likelihood confidence limits of each estimate. The residual parameter is the overall estimate of the variance, given all other regressors are zero.

Does the variance model fit significantly better than the original model? The likelihood ratio test for this question compares the fitted model with the model where all parameters are zero except the intercept, the model of equal-variance. In this case the *p*-value is highly significant. Changes in Hold Time change the variance.

The Variance Effect Likelihood Ratio Tests refit the model without each term in turn to create the likelihood ratio tests. These are generally more trusted than Wald tests.

Loglinear Variance Fit Report Options

The Loglinear Variance Fit red triangle menu contains the following options:

Save Columns Contains the following options to save columns to the data table.

Prediction Formula Creates a new column called Mean. The new column contains the predicted values for the mean, as computed by the specified model.

Variance Formula Creates a new column called Variance. The new column contains the predicted values for the variance, as computed by the specified model.

Std Dev Formula Creates a new column called Std Dev. The new column contains the predicted values for the standard deviation, as computed by the specified model.

Residuals Creates a new column called Residual that contains the residuals, which are the observed response values minus predicted values. See "Example of Examining the Residuals in a Loglinear Variance Model".

Studentized Residuals Creates a new column called Studentized Resid. The new column values are the residuals divided by their standard error.

Std Error of Predicted Creates a new column called Std Err Pred. The new column contains the standard errors of the predicted values.

Std Error of Individual Creates a new column called Std Err Indiv. The new column contains the standard errors of the individual predicted values.

Mean Confidence Interval Creates two new columns, Lower 95% Mean and Upper 95% Mean. The new columns contain the bounds for a confidence interval for the prediction mean.

Indiv Confidence Interval Creates two new columns, Lower 95% Indiv and Upper 95% Indiv. The new columns contain confidence limits for individual response values.

Row Diagnostics Contains the following options to plot row diagnostics.

Plot Actual by Predicted Plots the observed values by the predicted values of *Y*. This is the leverage plot for the whole model.

Plot Studentized Residual by Predicted Plots the Studentized residuals by the predicted values of *Y*.

Plot Studentized Residual by Row Plots the Studentized residuals by row.

Profilers Opens the Profiler, Contour Profiler, or Surface Profiler. Use the Profiler, Contour Profiler, or Surface Profiler to gain further insight into the fitted model. See "Factor Profiling".

Model Dialog Shows the completed launch window for the current analysis.

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Save By-Group Script Contains options that enable you to save a script that reproduces the platform report for all levels of a By variable to several destinations. Available only when a By variable is specified in the launch window.

Additional Examples of Loglinear Variance Models

This section contains examples of fitting loglinear variance models.

- "Example of Examining the Residuals in a Loglinear Variance Model"
- "Example of Profiling a Fitted Loglinear Variance Model"

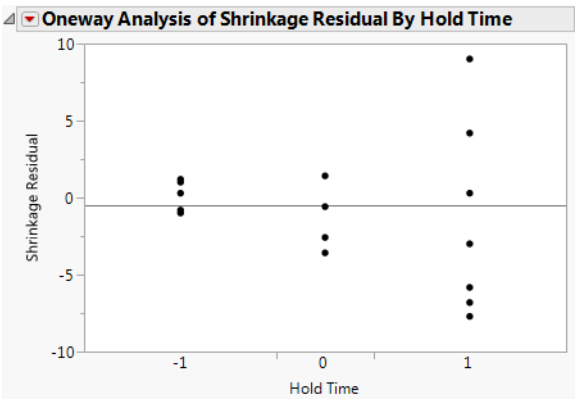
Example of Examining the Residuals in a Loglinear Variance Model

After fitting a loglinear variance model, you can analyze the residuals by saving the residuals to the data table.

1. Select **Help > Sample Data Folder** and open InjectionMolding.jmp.
2. Select **Analyze > Fit Model**.

   Since the variables in the data table have been assigned preselected roles, the analysis runs automatically.
3. Click the Loglinear Variance Fit red triangle and select **Save Columns > Residuals**.
4. In the InjectionMolding.jmp sample data table, right-click the continuous icon next to Hold Time in the Columns panel, and select **Nominal**.
5. Select **Analyze > Fit Y by X**.
6. Select Shrinkage Residual and click **Y, Response**.
7. Select Hold Time and click **X, Factor**.
8. Click **OK**.

Figure 11.6 Residual by Dispersion Effect



In this plot it is easy to see the variance go up as the Hold Time increases. This is done by treating Hold Time as a nominal factor.

Example of Profiling a Fitted Loglinear Variance Model

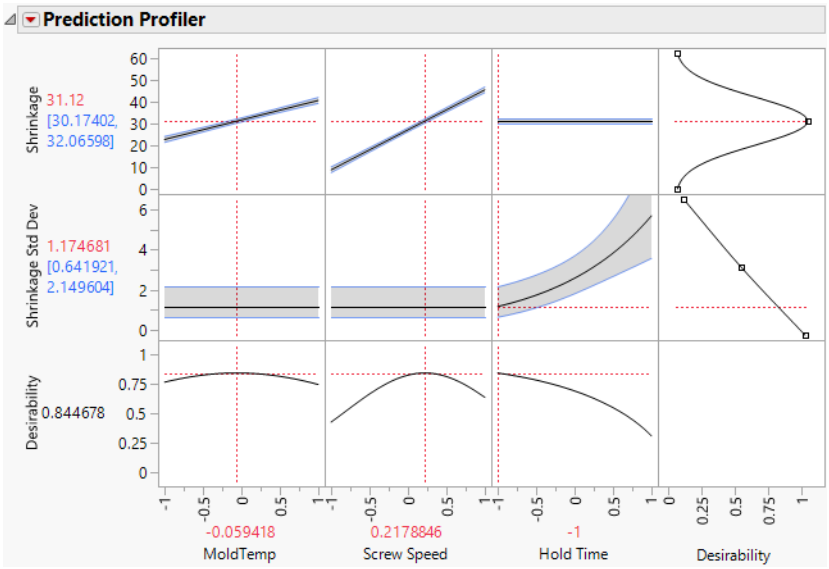
This example demonstrates the use of the Prediction Profiler to find factor settings that achieve a specific target for the response while minimizing variance. Fit the models and then use the Profiler to match a target value for a mean and to minimize variance.

1. Select **Help > Sample Data Folder** and open InjectionMolding.jmp.
2. Select **Analyze > Fit Model**.

   Since the variables in the data table have been assigned preselected roles, the analysis runs automatically.
3. Click the Loglinear Variance Fit red triangle and select **Profilers > Profiler**.
4. Click the Prediction Profiler red triangle and select **Optimization and Desirability > Set Desirabilities**.
5. In the Response Goal window that appears, change Maximize to Match Target.
6. Click **OK**.
7. In the second Response Goal window, click **OK**.
8. Click the Prediction Profiler red triangle and select **Optimization and Desirability > Maximize Desirability**.
9. Click the Prediction Profiler red triangle and select **Prediction Intervals**.

Note: Your results might vary due to random starting values in the optimization process.

Figure 11.7 Profiler to Match Target and Minimize Variance with Prediction Intervals



One of the best ways to see the relationship between the mean and the variance (both modeled with the LogVariance personality) is through looking at the individual prediction confidence intervals about the mean. Regular confidence intervals (those shown by default in the Profiler) do not show information about the variance model as well as individual prediction confidence intervals do. Prediction intervals show both the mean and variance model in one graph.

If Y is the modeled response, and you want a prediction interval for a new observation at x_n, then:

$$s^2\left|x_n = s^2_Y\right|x_n + s^2_{\hat{Y}}\left|x_n\right.$$

where:

$s^2\left|x_n\right.$ is the variance for the individual prediction at x_n

$s^2_Y\left|x_n\right.$ is the variance of the distribution of Y at x_n

$s^2_{\hat{Y}}\left|x_n\right.$ is the variance of the sampling distribution of $\hat{Y}$, and is also the variance for the mean.

Because the variance of the individual prediction contains the variance of the distribution of Y, the effects of the changing variance for Y can be seen. Not only are the individual prediction intervals wider, but they can change shape with a change in the variance effects.

Chapter 12

Logistic Regression Models

Fit Regression Models for Nominal or Ordinal Responses

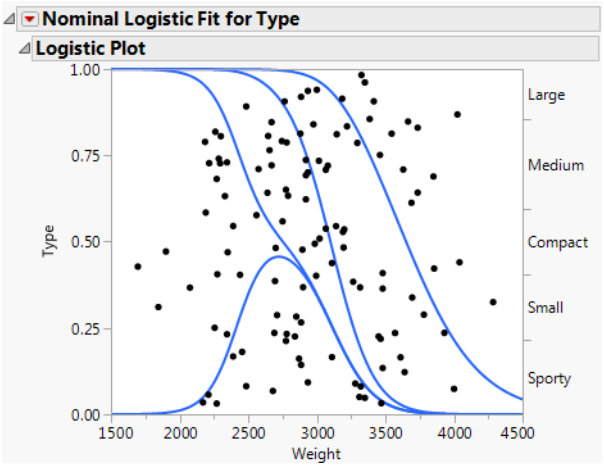
When your response variable has discrete values, you can use the Fit Model platform to fit a logistic regression model. The Fit Model platform provides two personalities for fitting logistic regression models. The personality that you use depends on the modeling type (Nominal or Ordinal) of your response column.

For nominal response variables, the Nominal Logistic personality fits a linear model to a multi-level logistic response function.

For ordinal response variables, the Ordinal Logistic personality fits the cumulative response probabilities to the logistic distribution function of a linear model.

Both personalities provide likelihood ratio tests for the model, a confusion matrix, and ROC and lift curves. When the response is binary, the Nominal Logistic personality also provides odds ratios (with corresponding confidence intervals).

Figure 12.1 Logistic Plot for a Nominal Logistic Regression Model



Contents

Overview of the Nominal and Ordinal Logistic Personalities 509
About Nominal Logistic Regression 509
About Ordinal Logistic Regression 509
Other JMP Platforms That Fit Logistic Regression Models 510
Examples of Logistic Regression 510
Example of Nominal Logistic Regression 510
Example of Ordinal Logistic Regression 512
Launch the Nominal and Ordinal Logistic Personalities 515
Validation in Logistic Regression Models 516
The Logistic Fit Report 516
Whole Model Test 518
Fit Details 519
Lack of Fit Test 520
Logistic Fit Platform Options 520
Options for Nominal and Ordinal Fits 520
Options for Nominal Fits 523
Options for Ordinal Fits 524
Additional Examples of Logistic Regression 525
Example of Inverse Prediction in Fit Model 525
Example of Using Effect Summary for a Nominal Logistic Model 527
Example of a Quadratic Ordinal Logistic Model 529
Example of Stacking Counts in Multiple Columns 532
Statistical Details for the Nominal and Ordinal Logistic Personalities 533
Statistical Details for the Logistic Regression Model 534
Statistical Details for Odds Ratios 534
Statistical Details for Logistic Regression Statistical Tests 535

Overview of the Nominal and Ordinal Logistic Personalities

Logistic regression models the probabilities of the levels of a categorical *Y* response variable as a function of one or more *X* effects. The Fit Model platform provides two personalities for fitting logistic regression models. The personality that you use depends on the modeling type (Nominal or Ordinal) of your response column.

For more information about fitting logistic regression models, see Walker and Duncan (1967), Nelson (1976), Harrell (1986), and McCullagh and Nelder (1989).

For more information about the parameterization of the logistic regression model, see "Statistical Details for the Logistic Regression Model".

About Nominal Logistic Regression

When the response variable has a nominal modeling type, the platform fits a linear model to a multi-level logistic response function using maximum likelihood. Therefore, all but one response level is modeled by a logistic curve that represents the probability of the response level given the value of the *X* effects. The probability of the final response level is 1 minus the sum of the other fitted probabilities. As a result, at all values of the *X* effects, the fitted probabilities for the response levels sum to 1.

If the response variable is binary, you can set the Target Level in the Fit Model window to specify the level whose probability you want to model. By default, the model estimates the probability of the higher level of the response variable.

For more information about fitting models for nominal response variables, see "Nominal Responses".

About Ordinal Logistic Regression

When the response variable has an ordinal modeling type, the platform fits the cumulative response probabilities to the logistic function of a linear model using maximum likelihood. Therefore, the cumulative probability of being at or below each response level is modeled by a curve. The curves are the same for each level except that they are shifted to the right or left.

Tip: If there are many response levels, the ordinal model is much faster to fit and uses less memory than the nominal model.

For more information about fitting models with ordinal response variables, see "Ordinal Responses".

Other JMP Platforms That Fit Logistic Regression Models

There are many other places in JMP where you can fit logistic regression models:

- To fit logistic regression models with a single continuous main effect, you can use the Fit Y by X platform to see a cumulative logistic probability plot for each effect. See *Basic Analysis*.
- To perform variable selection in logistic regression models, you can use the Stepwise personality of the Fit Model platform. See "Stepwise Regression Models".
- To fit logistic regression models that use a link function other than the Logit link, you can use the Generalized Linear Model personality of the Fit Model platform. See "Generalized Linear Models".
- JMP PRO To perform variable selection in logistic regression models and fit penalized logistic regression models, you can use the Generalized Regression personality of the Fit Model platform. See "Generalized Regression Models".

Examples of Logistic Regression

This section contains two examples, one for each of the logistic regression personalities in the Fit Model platform (Nominal and Ordinal):

- "Example of Nominal Logistic Regression"
- "Example of Ordinal Logistic Regression"

Example of Nominal Logistic Regression

Use the Nominal Logistic personality of the Fit Model platform to fit a nominal logistic regression model. An experiment was performed on metal ingots that were prepared with different heating and soaking times and then tested for readiness to roll. In this example, you fit the probability of readiness to roll using a logistic regression model with two regressors.

1. Select **Help > Sample Data Folder** and open Ingots.jmp.

   The values of the categorical variable ready, Ready and Not Ready, indicate whether an ingot is ready to roll.

2. Select **Analyze > Fit Model**.
3. Select ready and click **Y**.

   Because you selected a column with the Nominal modeling type, the Fit Model Personality updates to Nominal Logistic.

Because ready is a Nominal column with only two levels, the Target Level option appears. This option enables you to specify the response level whose probability you want to model.

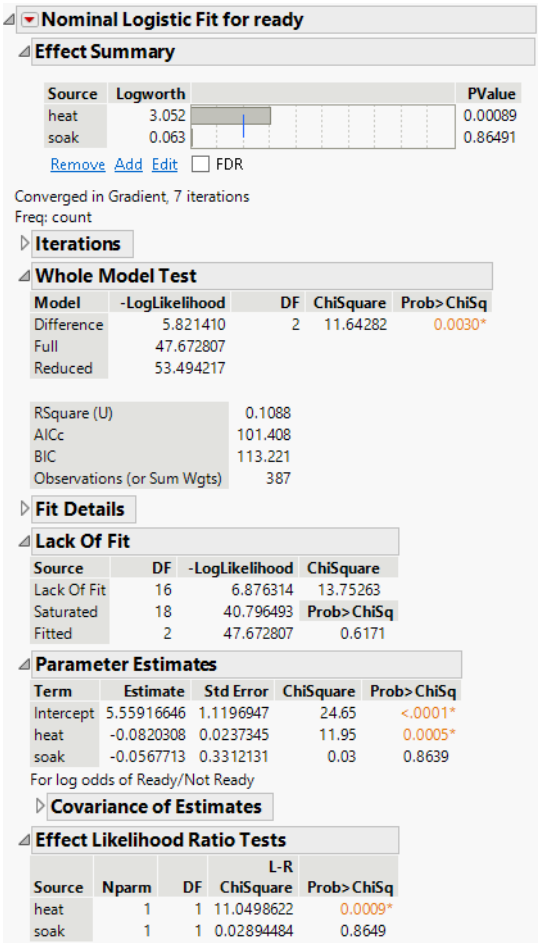
4. From the Target Level list, select Ready.

   In this model, the Target Level is Ready, so you are modeling the probability of the Ready response.

5. Select heat and soak and click **Add**.
6. Select count and click **Freq**.
7. Click **Run**.

   When the fitting process converges, the nominal regression report appears.

Figure 12.2 Nominal Logistic Fit Report



Nominal Logistic Fit for ready

Effect Summary

Source	Logworth	PValue
heat	3.052	0.00089
soak	0.063	0.86491

Remove Add Edit FDR

Converged in Gradient, 7 iterations
Freq: count

Iterations

Whole Model Test

Model	-LogLikelihood	DF	ChiSquare	Prob>ChiSq
Difference	5.821410	2	11.64282	0.0030*
Full	47.672807			
Reduced	53.494217			

RSquare (U)	0.1088
AICc	101.408
BIC	113.221
Observations (or Sum Wgts)	387

Fit Details

Lack Of Fit

Source	DF	-LogLikelihood	ChiSquare
Lack Of Fit	16	6.876314	13.75263
Saturated	18	40.796493	Prob>ChiSq
Fitted	2	47.672807	0.6171

Parameter Estimates

Term	Estimate	Std Error	ChiSquare	Prob>ChiSq
Intercept	5.55916646	1.1196947	24.65	<.0001*
heat	-0.0820308	0.0237345	11.95	0.0005*
soak	-0.0567713	0.3312131	0.03	0.8639

For log odds of Ready/Not Ready

Covariance of Estimates

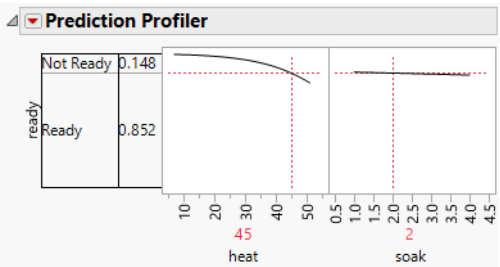
Effect Likelihood Ratio Tests

Source	Nparm	DF	L-R ChiSquare	Prob>ChiSq
heat	1	1	11.0498622	0.0009*
soak	1	1	0.02894484	0.8649

In the Whole Model Test report, the chi-square statistic (11.64) has a small *p*-value (0.0030), which indicates that the overall model is significant. However, the parameter estimate for soak has a *p*-value of 0.8639, which indicates that soaking time is not statistically significant.

8. Click the red triangle next to Nominal Logistic Fit for ready and select **Profiler**.

Figure 12.3 Prediction Profiler



When heat is set at 45 and soak is set at 2 the probability of ready is 0.85.

At this point, you might also be interested in using inverse prediction to find the heating time at a specific soaking time and given a particular probability of readiness to roll. See "Example of Inverse Prediction in Fit Model" for a continuation of this example.

Example of Ordinal Logistic Regression

Use the Ordinal Logistic personality of the Fit Model platform to fit an ordinal logistic regression model. An experiment was conducted to test whether various cheese additives (A to D) had an effect on cheese taste. Taste was measured by a tasting panel and recorded on an ordinal scale from 1 (strong dislike) to 9 (excellent taste). In this example, you fit the probability of each response using a logistic regression model with one regressor.

1. Select **Help > Sample Data Folder** and open Cheese.jmp.
2. Select **Analyze > Fit Model**.
3. Select Response and click **Y**.

   Because you selected a column with the Ordinal modeling type, the Fit Model Personality updates to Ordinal Logistic.
4. Select Cheese and click **Add**.
5. Select Count and click **Freq**.
6. Click **Run**.

Figure 12.4 Ordinal Logistic Fit Report

Ordinal Logistic Fit for Response

Freq: Count

Whole Model Test

Model	-LogLikelihood	DF	ChiSquare	Prob>ChiSq
Difference	74.22695	3	148.4539	<.0001*
Full	355.67395			
Reduced	429.90090			

RSquare (U)	0.1727
AICc	734.695
BIC	770.061
Observations (or Sum Wgts)	208

Fit Details

Lack Of Fit

Source	DF	-LogLikelihood	ChiSquare
Lack Of Fit	21	10.15410	20.30819
Saturated	24	345.51986	Prob>ChiSq
Fitted	3	355.67395	0.5018

Parameter Estimates

Term	Estimate	Std Error	ChiSquare	Prob>ChiSq
Intercept[1]	-4.6051335	0.4267239	116.46	<.0001*
Intercept[2]	-3.5499488	0.3073868	133.37	<.0001*
Intercept[3]	-2.4503882	0.2398702	104.36	<.0001*
Intercept[4]	-1.381774	0.2001699	47.65	<.0001*
Intercept[5]	-0.0455233	0.1803299	0.06	0.8007
Intercept[6]	0.90648953	0.1886609	23.09	<.0001*
Intercept[7]	2.408151	0.2380517	102.34	<.0001*
Intercept[8]	3.96800057	0.3466551	131.02	<.0001*
Cheese[A]	-0.8622328	0.2289236	14.19	0.0002*
Cheese[B]	2.48960592	0.2703151	84.82	<.0001*
Cheese[C]	0.8476542	0.2280359	13.82	0.0002*

Effect Likelihood Ratio Tests

Source	Nparm	DF	L-R ChiSquare	Prob>ChiSq
Cheese	3	3	148.453899	<.0001*

The model fit in this example reduces the –LogLikelihood of 429.9 for the intercept-only model to 355.67 for the full model. This reduction yields a likelihood ratio chi-square statistic for the whole model of 148.45 with 3 degrees of freedom. Therefore, the difference in perceived cheese taste is highly significant.

The most preferred cheese additive is the one with the most negative parameter estimate. Cheese[D] does not appear in the Parameter Estimates report, because it does not have its own column of the design matrix. However, Cheese D's effect can be computed as the negative sum of the others, and is shown in Table 12.1.

Table 12.1 Preferences for Cheese Additives in Cheese.jmp

Cheese	Estimate	Preference
A	–0.8622	2nd place
B	2.4896	least liked
C	0.8477	3rd place

Table 12.1 Preferences for Cheese Additives in Cheese.jmp *(Continued)*

D	–2.4750	most liked

Comparison to Nominal Logistic Model

The Lack of Fit report shows a test of whether the model fits the data well.

As an ordinal problem, each of the first eight response levels has an intercept, but there are only three parameters for the four levels of Cheese. As a result, there are 3 degrees of freedom in the ordinal model. The ordinal model is the Fitted model in the Lack of Fit test.

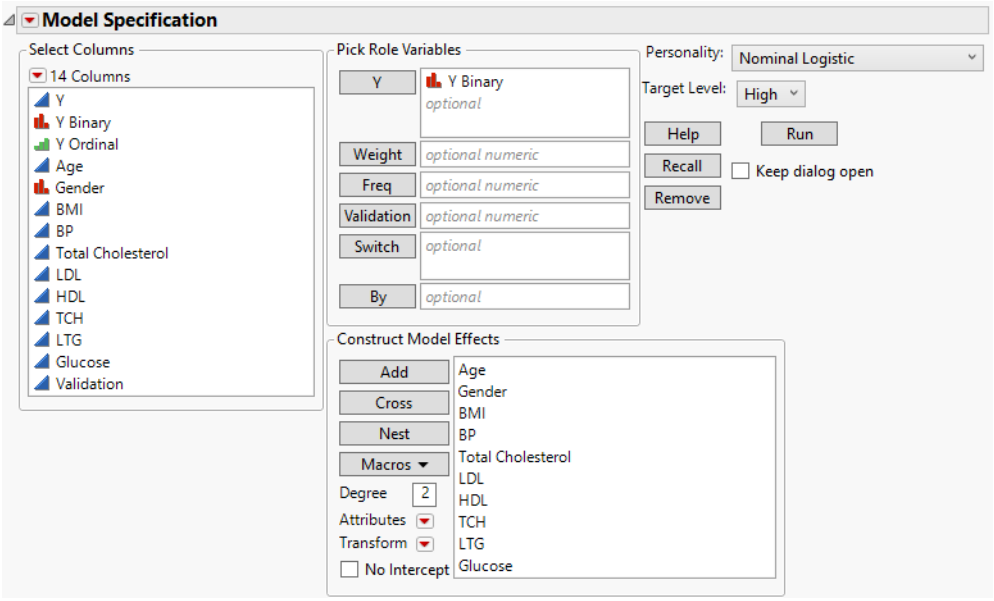
As a nominal problem, each of the first eight response levels has an intercept as well as three parameters for the four levels of Cheese. As a result, there are 8 x 3 = 24 degrees of freedom in the nominal model. Therefore, the nominal model is the Saturated model in the Lack of Fit test.

In this example, the Lack of Fit test for the ordinal model happens to be testing the ordinal response model against the nominal model. The nonsignificance of Lack of Fit leads one to believe that the ordinal model is reasonable.

Launch the Nominal and Ordinal Logistic Personalities

Launch the Nominal Logistic and Ordinal Logistic personalities by selecting **Analyze > Fit Model** and entering one or more non-continuous columns for **Y**. If multiple columns are entered for **Y**, a model for each response is fit.

Figure 12.5 Fit Model Launch Window with Nominal Logistic Selected



For more information about aspects of the Fit Model window that are common to all personalities, see "Model Specification". For more information about the options in the Select Columns red triangle menu, see *Using JMP*. Information specific to the Nominal Logistic and Ordinal Logistic personalities is presented here.

If your model effects have missing values, you can treat these missing values as informative categories. Select the Informative Missing option from the Model Specification red triangle menu.

To specify a model without an intercept term, select the No Intercept option in the Construct Model Effects panel of the Fit Model window. The No Intercept option is not available for the Ordinal Logistic personality.

The event of interest in the logistic regression model is defined by the Target Level option in the Fit Model window. This option is available only when you specify a binary response column in the Nominal Logistic personality.

Note: The Logistic personalities in the Fit Model platform require that your data be in a stacked format such that all of the responses are in one column. Sometimes, your data are formatted in multiple columns. See "Example of Stacking Counts in Multiple Columns" for an example of converting responses in multiple columns into a single column response.

JMP PRO Validation in Logistic Regression Models

Validation is the process of using part of a data set to estimate model parameters, and using the other part to assess the predictive ability of the model.

- The *training* set is used to estimate model parameters.
- The *validation* set is used in the model fitting to assess or validate the predictive ability of the model.
- The *test* set is a final, independent assessment of the model's predictive ability. The test set is available only when using a validation column.

The training, validation, and test sets are created as subsets of the original data. This is done through the use of a validation column in the Fit Model launch window.

The validation column's values determine how the data is split, and what method is used for validation:

- If the column has two distinct values, then training and validation sets are created.
- If the column has three distinct values, then training, validation, and test sets are created.
- If the column has more than three distinct values, or only one, then no validation is performed.

When a validation column is used, model fit statistics are given for the training, validation, and test sets in the Fit Details report. There is also a separate ROC curve, lift curve, and confusion matrix for each of the Training, Validation, and Test sets.

For more information about how a Validation column is used in JMP modeling platforms, see *Predictive and Specialized Modeling*.

The Logistic Fit Report

When you fit a model using the Nominal Logistic or Ordinal Logistic personality, you obtain a Nominal or Ordinal Logistic Fit report. By default, these reports contain the following sections:

Effect Summary An interactive report that enables you to add or remove effects from the model. See "Effect Summary Report".

Logistic Plot (Available only if the model consists of a single continuous effect.) The logistic probability plot illustrates what the logistic model is fitting. At each value on the horizontal axis, the probability scale in the vertical direction is partitioned into probabilities for each response category. The probabilities are measured as the vertical distance between the curves, with the total across all response category probabilities summing to 1.

The points in the logistic plot represent the observations from the data table. The horizontal position of each point is determined by its value of continuous factor. The vertical position of each point is randomly chosen to be between curves that correspond to the value of its response category. Because a fixed random seed is used, the vertical positions do not differ across multiple fits of the same model.

Iterations (Available only in the Nominal Logistic personality.) After launching Fit Model, an iterative estimation process begins and is reported iteration by iteration. After the fitting process completes, you can open the Iteration History report and see the iteration steps.

Whole Model Test Shows tests that compare the whole-model fit to the model that omits all the regressor effects except the intercept parameters. The test is analogous to the Analysis of Variance table for continuous responses. For more information about the Whole Model Test report, see "Whole Model Test".

Fit Details Shows various measures of fit for the model. See "Fit Details".

Lack of Fit (Available only when there are replicated points with respect to the *X* effects and the model is not saturated.) Shows a lack of fit test, also called a goodness of fit test, that addresses whether more terms are needed in the model. See "Lack of Fit Test".

Parameter Estimates Shows the parameter estimates, standard errors, and associated hypothesis tests. The Covariance of Estimates report gives the variances and covariances of the parameter estimates.

Note: The Covariance of Estimates report appears only for nominal response variables, and does not appear for ordinal response variables.

Singularity Details (Appears only when there are linear dependencies among the model terms.) Shows a report that contains the linear functions that the model terms satisfy.

Effect Likelihood Ratio Tests The likelihood ratio chi-square tests are calculated as twice the difference of the log-likelihoods between the full model and the model constrained by the hypothesis to be tested. The constrained model is the model that does not contain the effect. These tests can take time to do because each test requires a separate set of iterations.

Note: Likelihood ratio tests are the platform default if they are projected to take less than 20 seconds to complete. Otherwise, the default effect tests are Wald tests.

Whole Model Test

In the Logistic Fit report, the Whole Model Test table shows tests that compare the whole-model fit to the model that omits all the regression parameters except the intercept parameters. The test is analogous to the Analysis of Variance table for continuous responses. The negative log-likelihood corresponds to the sums of squares, and the chi-square test corresponds to the *F* test.

The Whole Model Test table shows these quantities:

Model Lists the model labels.

Difference The difference between the Full model and the Reduced model. This model is used to measure the significance of the regressors as a whole to the fit.

Full The complete model that includes the intercepts and all effects.

Reduced The model that includes only the intercept parameters.

–LogLikelihood The negative log-likelihood for the respective models. See "Likelihood, AICc, and BIC".

DF The degrees of freedom (DF) for the Difference between the Full and Reduced model.

Chi-Square The likelihood ratio chi-square test statistic for the hypothesis that all regression parameters are zero. The test statistic is computed by taking twice the difference in negative log-likelihoods between the fitted model and the reduced model that has only intercepts.

Prob>ChiSq The probability of obtaining a greater chi-square value if the specified model fits no better than the model that includes only intercepts.

RSquare (U) The proportion of the total uncertainty that is attributed to the model fit, defined as the ratio of the Difference to the Reduced negative log-likelihood values. RSquare ranges from zero for no improvement in fit to 1 for a perfect fit. An RSquare (U) value of 1 indicates that the predicted probabilities for events that occur are equal to one: There is no uncertainty in predicted probabilities. Because certainty in the predicted probabilities is rare for logistic models, RSquare (U) tends to be small.

RSquare (U) is sometimes referred to as *U*, the uncertainty coefficient, or as *McFadden's pseudo* R^2.

AICc The corrected Akaike Information Criterion. See "Likelihood, AICc, and BIC".

BIC The Bayesian Information Criterion. See "Likelihood, AICc, and BIC".

Observations (or Sum Weights) Total number of observations in the sample. If a Freq or Weight column is specified in the Fit Model window, this value is the sum of the values of a column assigned to the Freq or Weight role.

Fit Details

In the Logistic Fit report, the Fit Details section contains the following statistics:

Entropy RSquare Equivalent to RSquare (U). See "Whole Model Test".

Generalized RSquare A measure that can be applied to general regression models. It is based on the likelihood function L and is scaled to have a maximum value of 1. The Generalized RSquare measure simplifies to the traditional RSquare for continuous normal responses in the standard least squares setting. Generalized RSquare is also known as the Nagelkerke or Craig and Uhler R^2, which is a normalized version of Cox and Snell's pseudo R^2. See Nagelkerke (1991).

Mean -Log p The average of -log(p), where p is the fitted probability associated with the event that occurred.

RASE The root average square error, where the differences are between the response and p (the fitted probability for the event that actually occurred).

Mean Abs Dev The average of the absolute values of the differences between the response and p (the fitted probability for the event that actually occurred).

Misclassification Rate The rate for which the response category with the highest fitted probability is not the observed category.

N The number of observations.

For Entropy RSquare and Generalized RSquare, values closer to 1 indicate a better fit. For Mean -Log p, RASE, Mean Abs Dev, and Misclassification Rate, smaller values indicate a better fit.

To test that the effects as a whole are significant (the Whole Model test), a chi-square statistic is computed by taking twice the difference in negative log-likelihoods between the fitted model and the reduced model that has only intercepts.

If you specified a validation column, the Fit Details report contains columns for each of the Training, Validation, and Test sets.

Lack of Fit Test

In the Logistic Fit report, the Lack of Fit test addresses whether there is enough information in the current model or whether more complex terms are needed. This test is sometimes called a goodness-of-fit test. The lack of fit test calculates a pure-error negative log-likelihood by constructing categories for every combination of the model effect values in the data. The Saturated row in the Lack Of Fit table contains this log-likelihood. The Lack of Fit report also contains a test of whether the Saturated log-likelihood is significantly better than the Fitted model.

The Saturated degrees of freedom is m–1, where m is the number of unique populations. The Fitted degrees of freedom is the number of parameters not including the intercept.

The Lack of Fit table contains the negative log-likelihood for error due to Lack of Fit, error in a Saturated model (pure error), and the total error in the Fitted model. The chi-square statistic tests for lack of fit.

Logistic Fit Platform Options

The Nominal Logistic Fit and Ordinal Logistic Fit red triangle menus contain the following options:

- "Options for Nominal and Ordinal Fits"
- "Options for Nominal Fits"
- "Options for Ordinal Fits"

Options for Nominal and Ordinal Fits

The following options are available in both the Nominal Logistic Fit and Ordinal Logistic Fit red triangle menus:

Logistic Plot (Available only if the model consists of a single continuous effect.) Shows or hides the Logistic Plot report. See "The Logistic Fit Report".

Likelihood Ratio Tests Shows or hides the Effect Likelihood Ratio Tests report. The likelihood ratio chi-square tests are calculated as twice the difference of the log-likelihoods between the full model and the model constrained by the hypothesis to be tested. The constrained model is the model that does not contain the effect). These tests can take time to do because each test requires a separate set of iterations. Therefore, they could take a long time for large problems.

Note: Likelihood ratio tests are the platform default if they are projected to take less than 20 seconds to complete. This default option is highly recommended.

Wald Tests Shows or hides the Effect Wald Tests report. The Wald chi-square is a quadratic approximation to the likelihood ratio test, and it is a by-product of the calculations. Though Wald tests are considered less trustworthy, they do provide an adequate significance indicator for screening effects. Each parameter estimate and effect is shown with a Wald test. This is the default test if the likelihood ratio tests are projected to take more than 20 seconds to complete.

Confidence Intervals Shows or hides profile-likelihood confidence intervals for the model parameters. You can change the confidence level by selecting Set Alpha Level in the Model Specification red triangle menu in the Fit Model window. Each confidence limit requires a set of iterations in the model fit and can take a long time to compute. Furthermore, the effort does not always succeed in finding limits.

ROC Curve Shows or hides an ROC curve for the model. Receiver Operating Characteristic (ROC) curves measure the sorting efficiency of the model's fitted probabilities to sort the response levels. ROC curves can also aid in setting criterion points in diagnostic tests. The higher the curve from the diagonal, the better the fit. An introduction to ROC curves is found in *Basic Analysis*.

If the logistic fit has more than two response levels, it produces a generalized ROC curve (identical to the one in the Partition platform). In such a plot, there is a curve for each response level, which is the ROC curve of that level versus all other levels. See *Predictive and Specialized Modeling*.

If you specified a validation column, an ROC curve is shown for each of the Training, Validation, and Test sets.

Lift Curve Shows or hides a lift curve for the model. A lift curve shows the same information as an ROC curve, but in a way to dramatize the richness of the ordering at the beginning. The vertical axis shows the ratio of how rich that portion of the population is in the chosen response level compared to the rate of that response level as a whole. If you specified a validation column, a lift curve is shown for each of the Training, Validation, and Test sets. See *Predictive and Specialized Modeling* for more information about lift curves.

Confusion Matrix Shows or hides a report of confusion statistics, which contains a Confusion Matrix report and a Confusion Rates report. Both reports are two-way classifications of the actual response levels and the predicted response levels. The predicted response level is the Target Level specified in the launch window. The Confusion Rates report is equal to the Confusion Matrix report, with the numbers divided by the row totals.

For a good model, predicted response levels should be the same as the actual response levels. The Confusion Matrix report provides a way to assess how the predicted responses align with the actual responses. If you specified a validation column, a confusion matrix is shown for each of the Training, Validation, and Test sets.

If the response is nominal and has a Profit Matrix column property, a Decision Matrix report also appears when this option is selected. For more information about the Decision Matrix report, see *Predictive and Specialized Modeling*.

Decision Threshold (Available only for binary responses.) Shows or hides Decision Thresholds reports for the training, validation, and test sets, if specified. Each report contains a graph of the distribution of fitted probabilities for each model, confusion matrices for each model, and classification graphs to compare the model fits. See *Predictive and Specialized Modeling* for more information about the Decision Thresholds report.

Profiler Shows or hides the prediction profiler, showing the fitted values for a specified response probability as the values of the factors in the model are changed. This feature is available for both nominal and ordinal responses. For more information about the options in the red triangle menu, see *Profilers*.

Contour Profiler (Available only when the model contains more than one continuous factor.) Shows or hides the Contour Profiler. For more information about the options in the red triangle menu, see *Profilers*.

Model Dialog Shows the completed launch window for the current analysis.

Effect Summary Shows or hides the Effect Summary report, which enables you to interactively update the effects in the model. See "The Logistic Fit Report".

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Save By-Group Script Contains options that enable you to save a script that reproduces the platform report for all levels of a By variable to several destinations. Available only when a By variable is specified in the launch window.

Options for Nominal Fits

The following options are available in the Nominal Logistic Fit red triangle menu:

Plot Options (Available only if the model consists of a single continuous effect.) The Plot Options menu contains the following options:

Show Points Toggles the points in the Logistic Plot on or off.

Show Rate Curve Enables you to compare the rate at various values of the effect variable with the fitted logistic curve. This option is useful only if you have several points for each value of the effect. In these cases, you get reasonable estimates of the rate at each value, and compare this rate with the fitted logistic curve. To prevent too many degenerate points, usually at zero or one, JMP shows only the rate value if there are at least three points at the *x*-value.

Line Color Specifies the color of the plot curves.

Odds Ratios Shows or hides an Odds Ratios report that contains Unit Odds Ratios and Range Odds Ratios.

Figure 12.6 Odds Ratios

Odds Ratios

For ready odds of Ready versus Not Ready

Unit Odds Ratios

Per unit change in regressor

Term	Odds Ratio	Lower 95%	Upper 95%	Reciprocal
heat	0.921244	0.87937	0.965111	1.0854892
soak	0.94481	0.493646	1.808313	1.0584137

Range Odds Ratios

Per change in regressor over entire range

Term	Odds Ratio	Lower 95%	Upper 95%	Reciprocal
heat	0.027069	0.003496	0.209605	36.942229
soak	0.8434	0.120295	5.913179	1.185677

Tests and confidence intervals on odds ratios are Wald based.

Inverse Prediction (Available only for two-level nominal responses.) Finds the *x* value that results in a specified probability. See the appendix "Statistical Details" for more information about inverse prediction.

Save Probability Formula Creates columns in the current data table that contain formulas for linear combinations of the response levels, prediction formulas for the response levels, and a prediction formula giving the most likely response.

For a nominal response model with *r* levels, JMP creates the following columns:

- columns called Lin[j] that contain a linear combination of the regressors for response levels *j* = 1, 2, ... *r* - 1
- a column called Prob[r], with a formula for the fit to the last level, *r*
- columns called Prob[j] for *j* < *r* with a formula for the fit to level *j*
- a column called Most Likely response name that selects the most likely level of each row based on the computed probabilities.

JMP PRO **Publish Probability Formulas** Creates probability formulas and saves them as formula column scripts in the Formula Depot platform. If a Formula Depot report is not open, this option creates a Formula Depot report. See *Predictive and Specialized Modeling*.

Indicator Parameterization Estimates (Available only when there are nominal columns among the model effects.) Shows or hides the Indicator Function Parameterization report, which gives parameter estimates for the model where nominal columns are coded using indicator (SAS GLM) parameterization and are treated as continuous. Ordinal columns remain coded using the usual JMP coding scheme. The SAS GLM and JMP coding schemes are described in "The Factor Models".

Caution: Standard errors and chi-square values given in the Indicator Function Parameterization report differ from those in the Parameter Estimates report. This is because the estimates are estimating different model parameters.

Options for Ordinal Fits

The following options are available in the Ordinal Logistic Fit red triangle menu:

Save Contains the following Save options:

Save Probability Formula Creates columns in the current data table that contain formulas for linear combinations of the response levels, prediction formulas for the response levels, and a prediction formula giving the most likely response.

For an ordinal response model with *r* levels, JMP creates the following columns:

- a column called Linear that contains the formula for a linear combination of the regressors without an intercept term

- columns called Cum[j] that each contain a formula for the cumulative probability that the response is less than or equal to level *j*, for levels *j* = 1, 2, ... *r* - 1. There is no Cum[j = 1, 2, ... r - 1] that is 1 for all rows
- columns called Prob[j = 1, 2, ... r - 1], for 1 < *j* < *r* that each contain the formula for the probability that the response is level *j*. Prob[j] is the difference between Cum[j] and Cum[j – 1]. Prob[1] is Cum[1], and Prob[r] is 1 – Cum[r – 1].
- a column called Most Likely response name that selects the most likely level of each row based on the computed probabilities.

JMP PRO **Publish Probability Formulas** Creates probability formulas and saves them as formula column scripts in the Formula Depot platform. If a Formula Depot report is not open, this option creates a Formula Depot report. See *Predictive and Specialized Modeling*.

Save Quantiles (Available only when the response is numeric and has the ordinal modeling type.) Creates columns in the current data table named OrdQ.05, OrdQ.50, and OrdQ.95 that fit the quantiles for these three probabilities.

Save Expected Value (Available only when the response is numeric and has the ordinal modeling type.) Creates a column in the current data table called Ord Expected. This column contains the linear combination of the response values with the fitted response probabilities for each row and gives the expected value.

Additional Examples of Logistic Regression

This section contains examples using logistic regression in the Fit Model platform.

- "Example of Inverse Prediction in Fit Model"
- "Example of Using Effect Summary for a Nominal Logistic Model"
- "Example of a Quadratic Ordinal Logistic Model"
- "Example of Stacking Counts in Multiple Columns"

Example of Inverse Prediction in Fit Model

Inverse prediction enables you to predict the value for one independent variable at a specified probability of the response. If there is more than one independent variable, you must specify values for all of them except the one you are predicting.

An experiment was performed on metal ingots. The ingots were prepared with different heating and soaking times and then tested for readiness to roll.

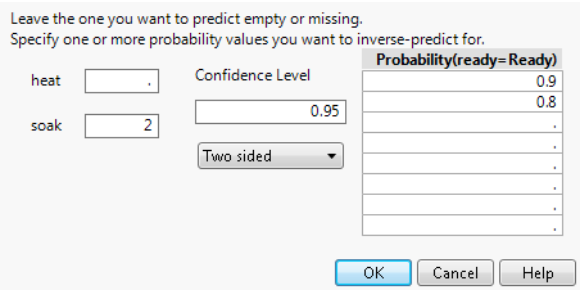
You are interested in predicting the heating time at a soaking time of 2 for probabilities of readiness to roll of 0.8 and 0.9.

1. For the analysis, follow the steps in "Example of Nominal Logistic Regression".
2. Click the red triangle next to Nominal Logistic Fit for ready and select **Inverse Prediction**.
3. Delete the value for heat.

   You want to find the predicted value of heat, so you leave the heat value empty.
4. Enter 2 for soak.

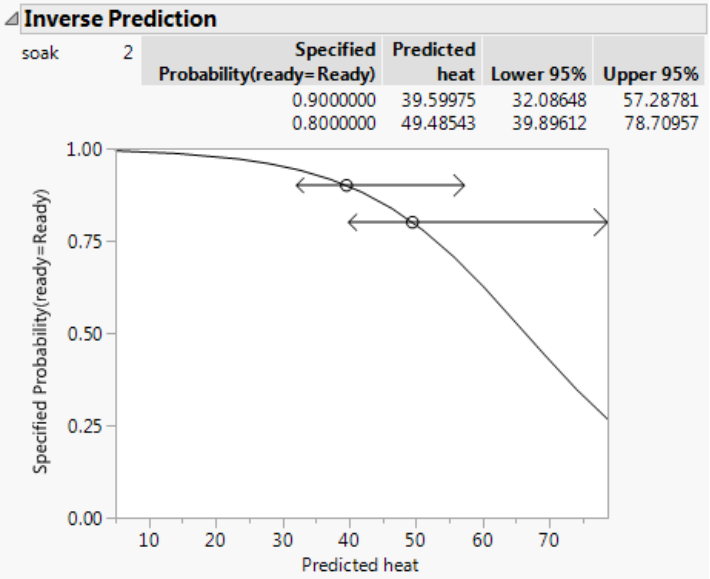
   You want to predict heating time when soak is 2.
5. Under Probability, enter 0.9 and 0.8 in the first two rows.

Figure 12.7 The Inverse Prediction Specification Window



6. Click **OK**.

Figure 12.8 Inverse Prediction Report



When soak is 2, the predicted value of heat for which there is a 90% chance of an ingot being ready to roll is 39.60 with a 95% confidence interval from 32.09 to 57.29. When soak is 2, the predicted value for heat for which there is an 80% chance of an ingot being ready to roll is 49.49 with a 95% confidence interval from 39.90 to 78.71.

Example of Using Effect Summary for a Nominal Logistic Model

A market research study was undertaken to evaluate preference for a brand of detergent based on a set of variables. You are interested in finding the variables that most contribute to brand preference. The model is defined by the following variables:

- the response variable, brand, with values m and x
- an effect called softness (water softness) with values soft, medium, and hard
- an effect called previous use with values yes and no
- an effect called temperature with values high and low
- a count variable, count, which gives the frequency counts for each combination of effect categories

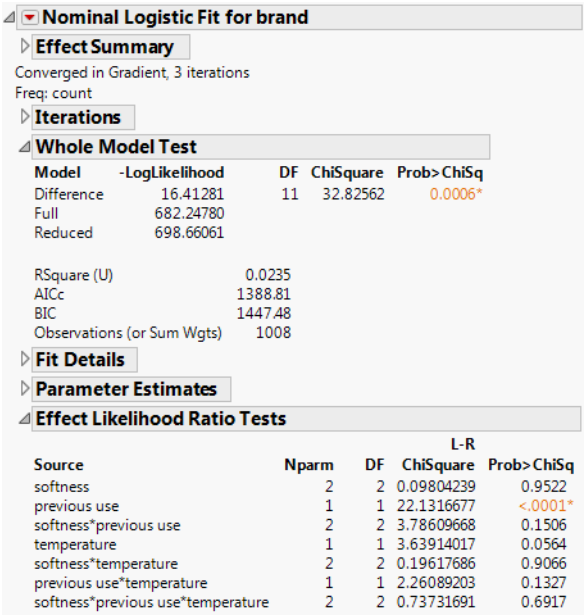
The study begins by specifying the full three-factor factorial model.

1. Select **Help > Sample Data Folder** and open Detergent.jmp.
2. Select **Analyze > Fit Model**.
3. Select brand from the Select Columns list and click **Y**.

   Because you specified a nominal response variable, the Personality changes to Nominal Logistic.

   Because brand is a Nominal column with only two levels, the Target Level option appears. This option enables you to specify the response level whose probability you want to model.
4. From the Target Level list, select m.
5. Select count and click **Freq**.
6. Select softness through temperature and click **Macros > Full Factorial**.
7. Click **Run**.

Figure 12.9 Nominal Logistic Fit for Three-Factor Factorial Model



Nominal Logistic Fit for brand

Effect Summary

Converged in Gradient, 3 iterations
Freq: count

Iterations

Whole Model Test

Model	-LogLikelihood	DF	ChiSquare	Prob>ChiSq
Difference	16.41281	11	32.82562	0.0006*
Full	682.24780			
Reduced	698.66061			

RSquare (U)	0.0235
AICc	1388.81
BIC	1447.48
Observations (or Sum Wgts)	1008

Fit Details

Parameter Estimates

Effect Likelihood Ratio Tests

Source	Nparm	DF	L-R ChiSquare	Prob>ChiSq
softness	2	2	0.09804239	0.9522
previous use	1	1	22.1316677	<.0001*
softness*previous use	2	2	3.78609668	0.1506
temperature	1	1	3.63914017	0.0564
softness*temperature	2	2	0.19617686	0.9066
previous use*temperature	1	1	2.26089203	0.1327
softness*previous use*temperature	2	2	0.73731691	0.6917

The Whole Model Test report shows that the three-factor full factorial model as a whole is significant (Prob>ChiSq = 0.0006).

The Effect Likelihood Ratio Tests report shows that the effects that include softness do not contribute significantly to the model fit. This leads you to consider removing softness from the model. You can do this from the Effect Summary report.

8. In the Effect Summary report, select softness*previous use through softness under Source and click **Remove**.

   The report updates to show the two-factor factorial model (Figure 12.10). The Whole Model Test report shows that the two-factor model is also significant as a whole.

Figure 12.10 Nominal Logistic Fit for Two-Factor Factorial Model

Nominal Logistic Fit for brand

Effect Summary

Converged in Gradient, 3 iterations
Freq: count

Iterations

Whole Model Test

Model	-LogLikelihood	DF	ChiSquare	Prob>ChiSq
Difference	13.58479	3	27.16958	<.0001*
Full	685.07582			
Reduced	698.66061			

RSquare (U)	0.0194
AICc	1378.19
BIC	1397.81
Observations (or Sum Wgts)	1008

Fit Details

Parameter Estimates

Term	Estimate	Std Error	ChiSquare	Prob>ChiSq
Intercept	0.03043265	0.0666827	0.21	0.6481
previous use[no]	-0.3148498	0.0666827	22.29	<.0001*
temperature[low]	-0.1331196	0.0666827	3.99	0.0459*
previous use[no]*temperature[low]	0.11106673	0.0666827	2.77	0.0958

For log odds of m/x

Covariance of Estimates

Effect Likelihood Ratio Tests

Source	Nparm	DF	L-R ChiSquare	Prob>ChiSq
previous use	1	1	22.6511053	<.0001*
temperature	1	1	3.99625478	0.0456*
previous use*temperature	1	1	2.78794934	0.0950

You conclude that previous use of a detergent brand and water temperature have an effect on detergent preference. You also note that the interaction between temperature and previous use is not statistically significant, so there is no evidence that temperature depends on previous use.

Example of a Quadratic Ordinal Logistic Model

Use the Ordinal Logistic personality of the Fit Model platform to fit a quadratic surface to optimize the probabilities of higher or lower levels of an ordinal response.

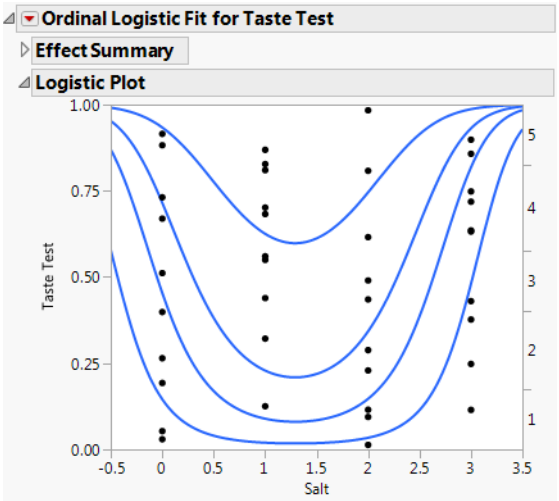
In this example, a microwave popcorn manufacturer wants to find out how much salt consumers like in their popcorn. To do this, the manufacturer looks for the maximum probability of a favorable response as a function of how much salt is added to the popcorn package. In this experiment, the salt amounts are controlled at 0, 1, 2, and 3 teaspoons. Respondents rate the taste on a scale of 1 (low) to 5 (high). The optimal amount of salt is the amount that maximizes the probability of more favorable responses. The ten observations for each of the salt levels are shown in Table 12.2.

Table 12.2 Salt in Popcorn

Salt Amount	Salt Rating Responses
no salt	1, 3, 2, 4, 2, 2, 1, 4, 3, 4
1 tsp.	4, 5, 3, 4, 5, 4, 5, 5, 4, 5
2 tsp.	4, 3, 5, 1, 4, 2, 5, 4, 3, 2
3 tsp.	3, 1, 2, 3, 1, 2, 1, 2, 1, 2

1. Select **Help > Sample Data Folder** and open Salt in Popcorn.jmp.
2. Select **Analyze > Fit Model**.
3. Select Taste Test from the Select Columns list and click **Y**.
4. Select Salt from the Select Columns list and click **Macros > Response Surface**.
5. Click **Run**.
6. Click the disclosure icon next to Response Surface to open the report.

Figure 12.11 Ordinal Logistic Fit for Salt in Popcorn.jmp



The report shows how the quadratic model fits the response probabilities. The curves, instead of being shifted logistic curves, become stacked U-shaped curves where each curve achieves its minimum at the same point. The critical value is at Mean(X) – 0.5 * (b1/b2) where b1 is the linear coefficient and b2 is the quadratic coefficient. This formula is for centered X. From the Parameter Estimates table, you can compute that the optimal amount of salt is 1.5 - 0.5 * (0.5637/1.3499) = 1.29 teaspoons.

The vertical distance at a specific amount of salt between each curve measures the probability of each of the five response levels for the specific amount of salt. The probability for the highest response level is the distance from the top curve to the top of the plot rectangle. This distance reaches a maximum when the amount of salt is about 1.3 teaspoons. All curves share the same critical point.

The parameter estimates for Salt and Salt*Salt are the coefficients used to find the critical value. Although the critical value appears on the logistic plot as a minimum, it is a maximum in the sense of maximizing the probability of the highest response. The Solution portion of the report is shown under Response Surface in Figure 12.11, where 1.29 is shown under Critical Value.

Example of Stacking Counts in Multiple Columns

When data that are frequencies (counts) are listed in several columns of your data table, you must transform the data into the form that you need for logistic regression. For example, the Ingots2.jmp data table (Figure 12.12) has columns Nready and Nnotready. These columns give the number of ingots that are ready to roll and ingots that are not ready to roll for each combination of Heat and Soak values.

Figure 12.12 Ingots2.jmp Sample Data Table

	Heat	Soak	Nready	Nnotready	Ntotal	P	Loss
1	7	1	10	0	10	0.5000	6.9315
2	7	1.7	17	0	17	0.5000	11.7835
3	7	2.2	7	0	7	0.5000	4.8520
4	7	2.8	12	0	12	0.5000	8.3178
5	7	4	9	0	9	0.5000	6.2383
6	14	1	31	0	31	0.5000	21.4876
7	14	1.7	43	0	43	0.5000	29.8053
8	14	2.2	31	2	33	0.5000	22.8739
9	14	2.8	31	0	31	0.5000	21.4876
10	14	4	19	0	19	0.5000	13.1698
11	27	1	55	1	56	0.5000	38.8162
12	27	1.7	40	4	44	0.5000	30.4985
13	27	2.2	21	0	21	0.5000	14.5561
14	27	2.8	21	1	22	0.5000	15.2492
15	27	4	15	1	16	0.5000	11.0904

Before fitting a logistic regression model, use the following steps to stack the Nready and Nnotready columns into a single column:

1. Select **Help > Sample Data Folder** and open Ingots2.jmp.
2. Select **Tables > Stack**.
3. Select Nready and NNotReady from the Select Columns list and click **Stack Columns**.

4. Click **OK**.

   This creates the new table in Figure 12.13. Label is the response (*Y*) column and Data is the frequency column.

This stacked data table is equivalent to the Ingots.jmp sample data table used in "Example of Nominal Logistic Regression".

Figure 12.13 Stacked Data Table

	Heat	Soak	Ntotal	P	Loss	Label	Data
1	7	1	10	0.5000	6.9315	Nready	10
2	7	1	10	0.5000	6.9315	Nnotready	0
3	7	1.7	17	0.5000	11.7835	Nready	17
4	7	1.7	17	0.5000	11.7835	Nnotready	0
5	7	2.2	7	0.5000	4.8520	Nready	7
6	7	2.2	7	0.5000	4.8520	Nnotready	0
7	7	2.8	12	0.5000	8.3178	Nready	12
8	7	2.8	12	0.5000	8.3178	Nnotready	0
9	7	4	9	0.5000	6.2383	Nready	9
10	7	4	9	0.5000	6.2383	Nnotready	0
11	14	1	31	0.5000	21.4876	Nready	31
12	14	1	31	0.5000	21.4876	Nnotready	0
13	14	1.7	43	0.5000	29.8053	Nready	43
14	14	1.7	43	0.5000	29.8053	Nnotready	0
15	14	2.2	33	0.5000	22.8739	Nready	31
16	14	2.2	33	0.5000	22.8739	Nnotready	2
17	14	2.8	31	0.5000	21.4876	Nready	31
18	14	2.8	31	0.5000	21.4876	Nnotready	0

Statistical Details for the Nominal and Ordinal Logistic Personalities

This section contains statistical details for logistic regression in the Fit Model platform.

- "Statistical Details for the Logistic Regression Model"
- "Statistical Details for Odds Ratios"
- "Statistical Details for Logistic Regression Statistical Tests"

Statistical Details for the Logistic Regression Model

This section contains details for the logistic regression models fit in the Fit Model platform.

Logistic regression fits nominal Y responses to a linear model of X terms. To be more precise, it fits probabilities for the response levels using a logistic function. For two response levels, the function is:

$$P(Y = r_1) = (1 + e^{-Xb})^{-1} \text{ where } r_1 \text{ is the first response level}$$

or equivalently:

$$\log\left(\frac{P(Y = r_1)}{P(Y = r_2)}\right) = Xb \text{ where } r_1 \text{ and } r_2 \text{ are the two responses levels, respectively}$$

Note: When Y is binary and has a nominal modeling type, you can set the Target Level in the Fit Model window to specify the level whose probability you want to model. In this section, the target level is designated as r_1.

For r nominal response levels, where $r > 2$, the model is defined by $r - 1$ linear model parameters of the following form:

$$\log\left(\frac{P(Y = j)}{P(Y = r)}\right) = Xb_j$$

The fitting principal of maximum likelihood means that the βs are chosen to maximize the joint probability attributed by the model to the responses that did occur. This fitting principal is equivalent to minimizing the negative log-likelihood (–LogLikelihood):

$$\text{Loss} = -\text{logLikelihood} = \sum_{i=1}^{n} -\log(\text{Prob}(i\text{th row has the } y_j\text{th response}))$$

Statistical Details for Odds Ratios

This section contains details for the odds ratios computed in the Fit Model platform.

For two response levels, the logistic regression model is specified as follows:

$$\log\left(\frac{\text{Prob}(Y = r_1)}{\text{Prob}(Y = r_2)}\right) = Xb \text{ where } r_1 \text{ and } r_1 \text{ are the two response levels}$$

Therefore, the *odds* are defined as follows:

$$\frac{\text{Prob}(Y = r_1)}{\text{Prob}(Y = r_2)} = \exp(X\beta) = \exp(\beta_0) \cdot \exp(\beta_1 X_1) \cdots \exp(\beta_i X_i)$$

Note that $\exp(\beta_i(X_i + 1)) = \exp(\beta_i X_i) \exp(\beta_i)$. This shows that if X_i changes by a unit amount, the odds is multiplied by $\exp(\beta_i)$, which we label the *unit odds ratio*. As X_i changes over its whole range, the odds are multiplied by $\exp((X_{\text{high}} - X_{\text{low}})\beta_i)$, which we label the *range odds ratio*. For binary responses, the log odds ratio for flipped response levels involves only changing the sign of the parameter. Therefore, you might want the reciprocal of the reported value to focus on the last response level instead of the first.

Two-level nominal effects are coded 1 and -1 for the first and second levels, so range odds ratios or their reciprocals would be of interest.

If no confidence intervals for the parameter estimates have been calculated when you select the Odds Ratio option, selecting the Odds Ratio option produces Wald-based confidence intervals for the odds ratio.

Selecting the Odds Ratio option produces profile likelihood-based confidence intervals for the odds ratios when all of the following conditions are met:

- the Likelihood Ratio Tests option has been selected
- the Confidence Intervals option has been selected
- the number of parameters for each response is less than 8
- the number of rows is less than 1000.

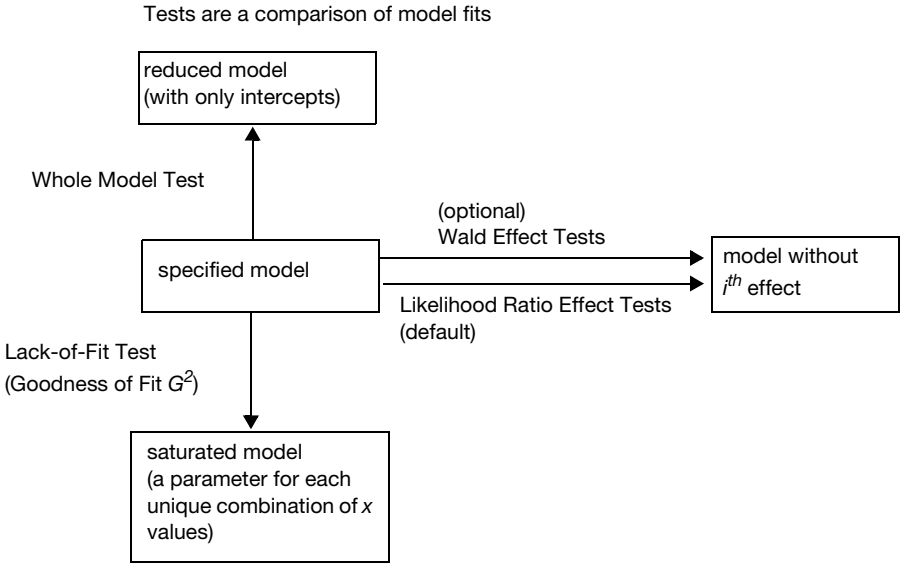
The method used for computing confidence intervals for the odds ratios is noted at the bottom of the Odds Ratios report.

Statistical Details for Logistic Regression Statistical Tests

In the Fit Model platform, all of the statistical tests in the Logistic Regression reports compare the fit of the specified model with subset or superset models, as illustrated in Figure 12.14. If a test shows significance, then the higher order model is justified.

- Whole model tests: if the specified model is significantly better than a reduced model without any effects except the intercepts.
- Lack of Fit tests: if a saturated model is significantly better than the specified model.
- Effect tests: if the specified model is significantly better than a model without a given effect.

Figure 12.14 Relationship of Statistical Tests



Chapter 13

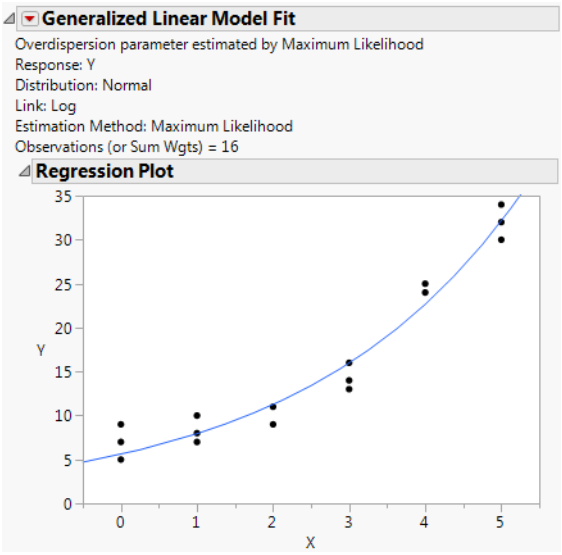
Generalized Linear Models

Fit Models for Nonnormal Response Distributions

The Generalized Linear Model personality of the Fit Model platform enables you to fit generalized linear models for responses with binomial, normal, Poisson, or exponential distributions. The platform provides reports similar to those that are provided for traditional linear models. The platform also accommodates separation in logistic regression models using the Firth correction.

Generalized linear models provide a unified way to fit responses that do not fit the usual requirements of traditional linear models. For example, frequency counts are often characterized as having a Poisson distribution and fit using a generalized linear model.

Figure 13.1 Example of a Generalized Linear Model Fit



Contents

Overview of the Generalized Linear Model Personality 539
Example of a Generalized Linear Model 540
Launch the Generalized Linear Model Personality 543
Generalized Linear Model Fit Report 545
Whole Model Test 546
Generalized Linear Model Fit Report Options 547
Additional Examples of the Generalized Linear Models Personality 550
Example of Using Contrasts in a Generalized Linear Model 550
Example of Poisson Regression with an Offset 551
Example of Normal Regression with a Log Link 552
Statistical Details for the Generalized Linear Model Personality 554
Statistical Details for Generalized Linear Model Construction 555
Statistical Details for Model Selection and Deviance 557

Overview of the Generalized Linear Model Personality

Traditional linear models are used extensively in statistical data analysis. However, there are situations that violate the assumptions of traditional linear models. In these situations, traditional linear models are not appropriate. Traditional linear models assume that the responses are continuous and normally distributed with constant variance across all observations. These assumptions might not be reasonable. For example, these assumptions are not reasonable if you want to model counts, or if the variance of the observed responses increases as the response increases. Another example of violating the assumptions of traditional linear models is when the mean of the response is restricted to a specific range of values, such as proportions that fall between 0 and 1.

For situations such as these that fall into a wider range of data analysis problems, generalized linear models can be applied. Generalized linear models are an extension of traditional linear models. A generalized linear model consists of a linear component, a link function, and a variance function. The link function, $g(\mu_i) = x'_i\beta$, is a monotonic and differentiable function that describes how the expected value of Y_i is related to the linear predictors. An example of generalized linear regression is Poisson regression, where $\log(\mu_i)$ is the link function. For a complete list of the generalized linear regression models available using the Generalized Linear Models personality of the Fit Model platform, see "Statistical Details for the Generalized Linear Model Personality".

Fitted generalized linear models can be summarized and evaluated using the same statistics as traditional linear models. The Fit Model platform provides parameter estimates, standard errors, goodness-of-fit statistics, confidence intervals, and hypothesis tests for generalized linear models. It should be noted that exact distribution theory is not always available or practical for generalized linear models. Therefore, some inference procedures are based on asymptotic results.

An important aspect of fitting generalized linear models is the selection of explanatory variables in the model. Changes in goodness-of-fit statistics are often used to evaluate the contribution of subsets of explanatory variables to a particular model. The *deviance* is defined as twice the difference between the maximum attainable value of the log-likelihood function and the value of the log-likelihood function at the maximum likelihood estimates of the regression parameters. The deviance is often used as a measure of goodness of fit. The maximum attainable log-likelihood is achieved with a model that has a parameter for every observation.

JMP PRO For variable selection and penalized methods in generalized linear modeling, you can use the Generalized Regression personality of the Fit Model platform in JMP Pro. See "Generalized Regression Models".

Example of a Generalized Linear Model

This example uses Poisson regression to model count data from a study of nesting horseshoe crabs. Each female crab had a male crab resident in her nest. The study investigated whether there were other males, called satellites, residing nearby. The data table contains a response variable listing the number of male satellites, as well as variables that describe the color, spine condition, weight, and carapace width of the female crab. You are interested in the relationship between the number of satellites and the variables that describe the female crabs.

1. Select **Help > Sample Data Folder** and open CrabSatellites.jmp.
2. Select **Analyze > Fit Model**.
3. Select satell and click **Y**.
4. Select color, spine, width, and weight and click **Add**.
5. From the Personality list, select **Generalized Linear Model**.
6. From the Distribution list, select **Poisson**.

   In the Link Function list, **Log** should be selected for you automatically.
7. Click **Run**.

Figure 13.2 Crab Satellite Results

Generalized Linear Model Fit

Response: satell
Distribution: Poisson
Link: Log
Estimation Method: Maximum Likelihood
Observations (or Sum Wgts) = 173

Whole Model Test

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	41.6030357	83.2061	7	<.0001*
Full	452.44163			
Reduced	494.044666			

Goodness Of Fit Statistic	ChiSquare	DF	Prob>ChiSq
Pearson	533.8165	165	<.0001*
Deviance	549.5856	165	<.0001*

AICc
921.7613

Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
color	3	9.2405455	0.0263*
spine	2	1.794722	0.4076
width	1	0.1168819	0.7324
weight	1	9.0438942	0.0026*

Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq	Lower CL	Upper CL
Intercept	-0.710184	0.9339406	0.5761557	0.4478	-2.532002	1.1287428
color[Light Med]	0.3273541	0.132167	5.7773064	0.0162*	0.0619978	0.5810457
color[Medium]	0.0625029	0.0737267	0.7238945	0.3949	-0.080871	0.2083795
color[Dark Med]	-0.186351	0.0963747	3.8096887	0.0510	-0.377505	0.0007691
spine[Both Good]	0.0210298	0.0947416	0.049346	0.8242	-0.163737	0.2084429
spine[One Worn/Broken]	-0.129342	0.1313469	1.0163566	0.3134	-0.399791	0.1168764
width	0.0167487	0.0489197	0.1168819	0.7324	-0.07991	0.1118218
weight	0.0004965	0.0001663	9.0438942	0.0026*	0.000172	0.0008234

The Whole Model Test shows that the difference between log-likelihoods for the full and reduced models is 41.6. The Prob>ChiSq value is equivalent to the *p*-value for a whole model *F* test. A *p*-value less than 0.0001 indicates that the model as a whole is significant. The report also contains the corrected Akaike Information Criterion (AICc), 921.7613. This value can be compared with other models to determine the best-fitting model for the data. Smaller AICc values indicate better fitting models.

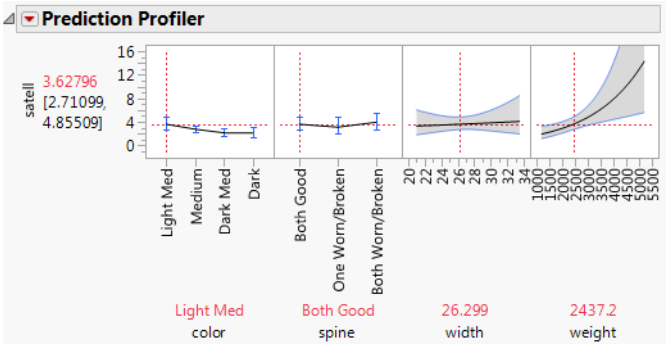
The Effects Tests report shows that weight is a significant factor in the model. Note that the *p*-value for weight, 0.0026, is the same in the Parameter Estimates table as well, since weight is a continuous variable.

The Effect Tests report also shows that the categorical variable color is significant. Color has four levels, Light Med, Medium, Dark Med, and Dark. The parameter estimates for the first three levels are shown in the Parameter Estimates table. The parameter estimate for Dark is the negative of the sum of the parameter estimates for the first three levels.

You can use the Prediction Profiler to further explore the model.

8. Click the red triangle next to Generalized Linear Model Fit and select **Profilers > Profiler**.

Figure 13.3 Prediction Profiler for Satell

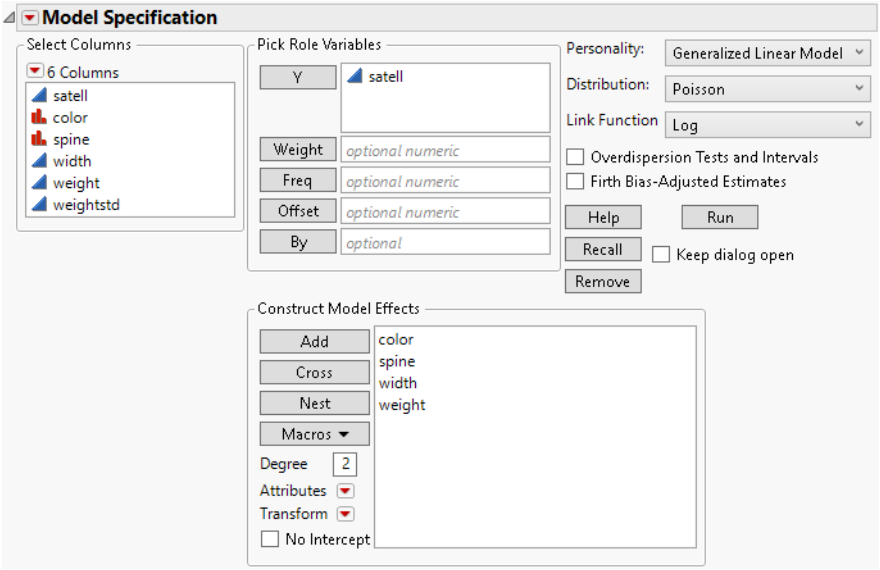


The confidence band on weight indicates that there is less variability in the model for smaller weight values than there is for larger weight values. The profiler enables you to easily explore the levels of the categorical variables. From the profiler, you can see that the predicted number of satellite crabs decreases as the color of the crab changes from light to dark.

Launch the Generalized Linear Model Personality

Launch the Generalized Linear Model personality by selecting **Analyze > Fit Model**, entering one or more columns for **Y**, and selecting **Generalized Linear Model** from the **Personality** menu.

Figure 13.4 Fit Model Launch Window with Generalized Linear Model Selected



For more information about aspects of the Fit Model window that are common to all personalities, see "Model Specification". For more information about the options in the Select Columns red triangle menu, see *Using JMP*. Information specific to the Generalized Linear Model personality is presented here.

If your model effects have missing values, you can treat these missing values as informative categories. Select the Informative Missing option from the Model Specification red triangle menu.

Tip: The No Intercept option is not available in the Generalized Linear Model personality of the Fit Model platform.

When you select Generalized Linear Model for Personality, the Fit Model launch window changes to include additional options. The following additional options are available in the Generalized Linear Model personality:

Distribution Specifies a probability distribution for the response variable.

Link Function Specifies a link function that relates the linear model to the response variable.

Overdispersion Tests and Intervals Specifies that an overdispersion parameter should be included in the model. Overdispersion occurs when the variance of the response is greater than would be expected by the theoretical variance of the response distribution. This can arise in Poisson and binomial response models. McCullagh and Nelder (1989) state that overdispersion is not uncommon in practice.

Note: This option adds a column for Overdispersion to the Goodness-of-Fit Statistics table in the Whole Model Test report.

Firth Bias-Adjusted Estimates Specifies that the Firth bias-adjusted method is used to fit the model. This maximum likelihood-based method has been shown to produce better estimates and tests than maximum likelihood-based models that do not use bias correction. In addition, bias-corrected MLEs ameliorate separation problems that tend to occur in logistic-type models. For more information about the separation problem in logistic regression, see Firth (1993) and Heinze and Schemper (2002).

Offset (Appears as a Pick Role Variables button.) Specifies a variable for the offset. An offset variable is one that is treated like a regression covariate whose parameter is fixed to be 1.0. Offset variables are most often used to scale the modeling of the mean in Poisson regression situations with a log link.

Response Specification for the Binomial Response Distribution

When you select Binomial as the Distribution, the response variable must be specified in one of the following ways:

- If your data are not summarized as frequencies of events, specify a single binary column as the response. The response column must be nominal.
- If your data are summarized as frequencies of events, specify a single binary column as the response and a frequency variable in the Freq role. The response column must be nominal, and the frequency variable contains the count of each response level.
- If your data are summarized as frequencies of events and number of trials, specify two continuous columns in this order: a count of the number of successes, and a count of the number of trials. Alternatively, you can specify the number of failures instead of the number of successes.

Generalized Linear Model Fit Report

By default, the Generalized Linear Model Fit report contains details about the model specification as well as the following reports:

Singularity Details (Appears only when there are linear dependencies among the model terms.) Shows a report that contains the linear functions that the model terms satisfy.

Regression Plot (Appears only when there is one continuous predictor and no more than one categorical predictor.) Shows a plot of the response on the vertical axis and the continuous predictor on the horizontal axis. A regression line is shown over the points. If there is a categorical predictor in the model, each level of the categorical predictor has a separate regression line and a legend appears next to the plot.

Whole Model Test Shows tests that compare the whole-model fit to the model that omits all the effects except the intercept parameters. This report also contains goodness-of-fit statistics and the corrected Akaike's Information Criterion (AICc) value. See "Whole Model Test".

Effect Summary An interactive report that enables you to add or remove effects from the model. See "Effect Summary Report".

Effect Tests The Effect Tests are joint tests that all the parameters for an individual effect are zero. If an effect has only one parameter, as with continuous effects, then the tests are the same as the tests in the Parameter Estimates table.

Note: Even if the Firth adjustment is used, the Effect Tests are based on the non-penalized likelihood function.

Parameter Estimates Shows the parameter estimates, standard errors, and associated hypothesis tests and confidence limits. Simple continuous effects have only one parameter. Models with complex classification effects have a parameter for each anticipated degree of freedom.

Note: If there are more than 1,000 observations, Wald-based confidence intervals are shown. Otherwise, profile-likelihood confidence intervals are shown.

Studentized Deviance Residual by Predicted Shows a plot of studentized deviance residuals on the vertical axis and the predicted response values on the horizontal axis.

Whole Model Test

In the Generalized Linear Model Fit report, the Whole Model Test table shows tests that compare the whole-model fit to the model that omits all the regression parameters except the intercept parameter. It also contains two goodness-of-fit statistics and the AICc value to assess model adequacy.

The Whole Model Test table shows these quantities:

Model Lists the model labels.

Difference The difference between the Full model and the Reduced model. This model is used to measure the significance of the regressors as a whole to the fit.

Full The complete model that includes the intercepts and all effects.

Reduced The model that includes only the intercept parameters.

–LogLikelihood The negative log-likelihood for the respective models. See "Likelihood, AICc, and BIC".

Note: When the Overdispersion Tests and Intervals option is selected in the launch window, the -LogLikelihood value is calculated using the quasi-likelihood approach.

L-R ChiSquare The likelihood ratio chi-square test statistic for the hypothesis that all regression parameters are zero. The test statistic is computed by taking twice the difference in negative log-likelihoods between the fitted model and the reduced model that has only an intercept.

DF The degrees of freedom (DF) for the Difference between the Full and Reduced model.

Prob>ChiSq The probability of obtaining a greater chi-square value if the specified model fits no better than the model that includes only an intercept.

Goodness of Fit Statistic Lists the two goodness-of-fit statistics: Pearson and Deviance.

ChiSquare The chi-square test statistic for the respective goodness-of-fit statistics.

DF The degrees of freedom for the respective goodness-of-fit statistics.

Prob>ChiSq The *p*-value for the respective goodness-of-fit statistics.

Overdispersion (Appears only when the Overdispersion Tests and Intervals option is selected in the launch window.) An estimate of the overdispersion parameter. See "Statistical Details for the Generalized Linear Model Personality".

AICc The corrected Akaike Information Criterion. See "Likelihood, AICc, and BIC".

Note: When the Overdispersion Tests and Intervals option is selected in the launch window, the AICc calculation does not include the overdispersion parameter.

Generalized Linear Model Fit Report Options

The Generalized Linear Model Fit red triangle menu contains the following options:

Custom Test Enables you to test a custom hypothesis. For more information about custom tests, see "Custom Test".

Contrast Enables you to test for differences in levels within a variable. If a contrast involves a covariate, you can specify the value of the covariate at which to test the contrast. For an example of the Contrast option, see "Example of Using Contrasts in a Generalized Linear Model".

Inverse Prediction (Available only for continuous *X* variables.) Enables you to predict an *X* value, given specific values for *Y* and the other *X* variables. For more information about the Inverse Prediction option, see "Inverse Prediction".

Covariance of Estimates Shows or hides a covariance matrix for all the effects in a model. The estimated covariance matrix of the parameter estimator is defined as follows:

$$\Sigma = -\mathbf{H}^{-1}$$

where **H** is the Hessian (or second derivative) matrix evaluated using the parameter estimates on the last iteration. Note that the dispersion parameter, whether estimated or specified, is incorporated into **H**. Rows and columns corresponding to aliased parameters are not included in Σ.

Correlation of Estimates Shows or hides a correlation matrix for all the effects in a model. The correlation matrix is the normalized covariance matrix. For each σ_{ij} element of Σ, the corresponding element of the correlation matrix is $\sigma_{ij}/\sigma_i\sigma_j$, where $\sigma_i = \sqrt{\sigma_{ii}}$.

Profilers Shows a submenu of the following profilers:

Profiler Shows or hides a prediction profiler for examining prediction traces for each X variable. For more information about the prediction profiler, see "Profiler".

Contour Profiler Shows or hides an interactive contour profiler. For more information about the contour profiler, see *Profilers*.

Surface Profiler Shows or hides a 3-D surface profiler. For more information about the surface profiler, see *Profilers*.

Diagnostic Plots Shows a submenu of plots of residuals, predicted values, and actual values. These plots enable you to search for outliers and determine the adequacy of your model. For more information about deviance, see "Statistical Details for Model Selection and Deviance". The following plots are available:

Studentized Deviance Residuals by Predicted Shows or hides a plot of studentized deviance residuals on the vertical axis and the predicted response values on the horizontal axis.

Studentized Pearson Residuals by Predicted Shows or hides a plot of the studentized Pearson residuals on the vertical axis and the predicted response values on the horizontal axis.

Deviance Residuals by Predicted Shows or hides a plot of the deviance residuals on the vertical axis and the predicted response values on the horizontal axis.

Pearson Residuals by Predicted Shows or hides a plot of the Pearson residuals on the vertical axis and the predicted response values on the horizontal axis.

Regression Plot (Available only when there is one continuous predictor and no more than one categorical predictor.) Shows or hides a plot of the response on the vertical axis and the continuous predictor on the horizontal axis. A regression line is shown over the points. If there is a categorical predictor in the model, each level of the categorical predictor has a separate regression line and a legend appears next to the plot.

Linear Predictor Plot (Available only when there is one continuous predictor and no more than one categorical predictor.) Shows or hides a plot of responses transformed by the inverse link function on the vertical axis and the continuous predictor on the horizontal axis. A transformed regression line is shown over the points. If there is a categorical predictor in the model, each level of the categorical predictor has a separate transformed regression line and a legend appears next to the plot.

Save Columns Shows a submenu of options that enable you to save certain quantities as new columns in the current data table. For more information about the residual formulas, see "Residual Formulas".

Prediction Formula Creates a formula column in the current data table that predicts the model.

Predicted Values Creates a column in the current data table that contains the values predicted by the model.

Mean Confidence Interval Creates columns in the current data table that contain the 95% confidence limits for the prediction equation for the model. These confidence limits reflect the variation in the parameter estimates.

Note: You can change the α level for the confidence limits in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Save Indiv Confid Limits Creates columns in the current data table that contain the 95% confidence limits for a given individual value for the model. These confidence limits reflect variation in the error and variation in the parameter estimates.

Note: You can change the α level for the confidence limits in the Fit Model window by selecting Set Alpha Level from the Model Specification red triangle menu.

Deviance Residuals Creates a column in the current data table that contains the deviance residuals.

Pearson Residuals Creates a column in the current data table that contains the Pearson residuals.

Studentized Deviance Residuals Creates a column in the current data table that contains the studentized deviance residuals.

Studentized Pearson Residuals Creates a column in the current data table that contains the studentized Pearson residuals.

Model Dialog Shows the completed launch window for the current analysis.

Effect Summary Shows or hides the Effect Summary report, which enables you to interactively update the effects in the model. See "Effect Summary Report".

See *Using JMP* for more information about the following options:

Local Data Filter Shows or hides the local data filter that enables you to filter the data used in a specific report.

Redo Contains options that enable you to repeat or relaunch the analysis. In platforms that support the feature, the Automatic Recalc option immediately reflects the changes that you make to the data table in the corresponding report window.

Platform Preferences Contains options that enable you to view the current platform preferences or update the platform preferences to match the settings in the current JMP report.

Save Script Contains options that enable you to save a script that reproduces the report to several destinations.

Save By-Group Script Contains options that enable you to save a script that reproduces the platform report for all levels of a By variable to several destinations. Available only when a By variable is specified in the launch window.

Additional Examples of the Generalized Linear Models Personality

This section contains examples using the Generalized Linear Models personality of the Fit Model platform.

- "Example of Using Contrasts in a Generalized Linear Model"
- "Example of Poisson Regression with an Offset"
- "Example of Normal Regression with a Log Link"

Example of Using Contrasts in a Generalized Linear Model

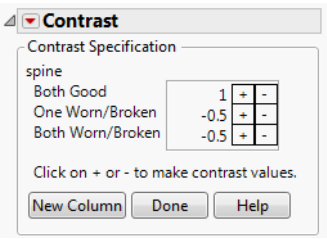
You can use contrasts in the Generalized Linear Model personality to compare differences in the levels of a variable. Suppose that you want to test whether female crabs with good spines attracted a different number of male crabs (satellites) than female crabs with worn or broken spines.

Note: This example continues the crab satellite example in "Example of a Generalized Linear Model".

1. Complete step 1 through step 7 of "Example of a Generalized Linear Model".
2. Click the red triangle next to Generalized Linear Model Fit and select **Contrast**. The Choose effects for contrast window appears.
3. Select spine, the variable of interest, and click **OK**.
4. To compare the crabs with good spines to crabs with worn or broken spines, click the **+** button beside Both Good and the **-** button beside both One Worn/Broken and Both Worn/Broken.

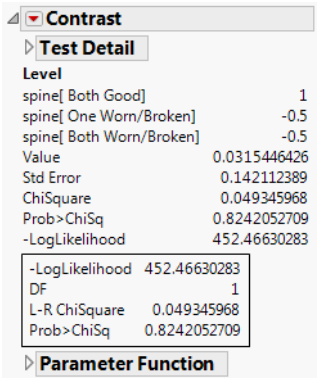
   This creates a contrast specification that compares the female crabs with good spines against the female crabs with worn or broken spines.

Figure 13.5 Contrast Specification Window



5. Click **Done**.

Figure 13.6 Contrast Report



The Prob>Chisq value, 0.8242, is much greater than 0.05, so we cannot conclude that there is a difference in satellite attraction based on spine condition.

Example of Poisson Regression with an Offset

In the Generalized Linear Model personality, you can specify an offset variable to scale the modeling of the mean in a Poisson regression model with a log link function. Offset variables are most often used to scale the modeling of the mean in Poisson regression situations with a log link.

The data table used in this example contains information about a certain type of damage caused by waves to the forward section of the hull. Hull construction engineers are interested in the risk of damage associated with three variables: ship type, the year in which the ship was constructed, and the block of years the ship was in service.

You use log(months of service) as the offset variable since you expect that the number of repairs will be proportional to the number of months in service.

To see how an offset variable is used, assume the linear component of the GLM is called η. Then, with a log link function, the model for the mean with the offset included is specified as follows:

$$\exp[\text{Log}(\text{months of service}) + \eta] = [(\text{months of service}) * \exp(\eta)].$$

To run this example, follow these steps:

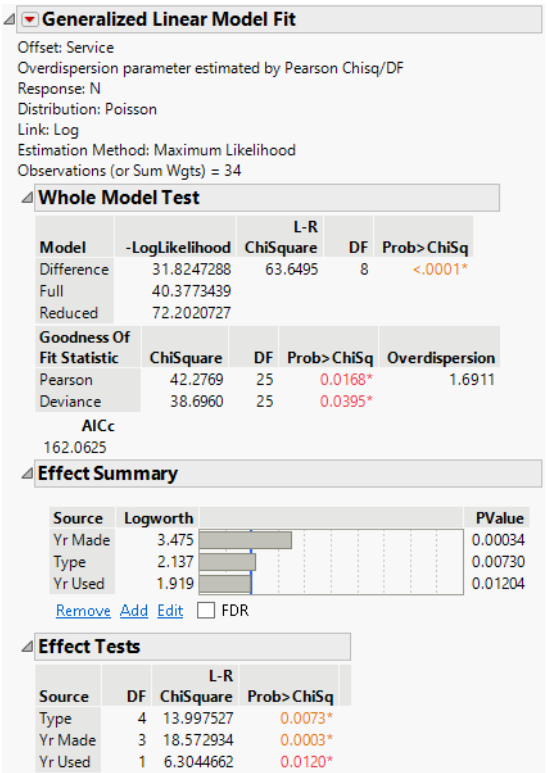
1. Select **Help > Sample Data Folder** and open Ship Damage.jmp.
2. Select **Analyze > Fit Model**.
3. From the Personality list, select **Generalized Linear Model**.
4. From the Distribution list, select **Poisson**.

In the Link Function list, **Log** should be selected for you automatically.

5. Select N and click **Y**.
6. Select Service and click **Offset**.
7. Select Type, Yr Made, Yr Used and click **Add**.
8. Click the check mark box for **Overdispersion Tests and Intervals**.
9. Click **Run**.

From the report, notice that all three effects (Type, Yr Made, Yr Used) are significant.

Figure 13.7 Partial Report for a Poisson with Offset Model



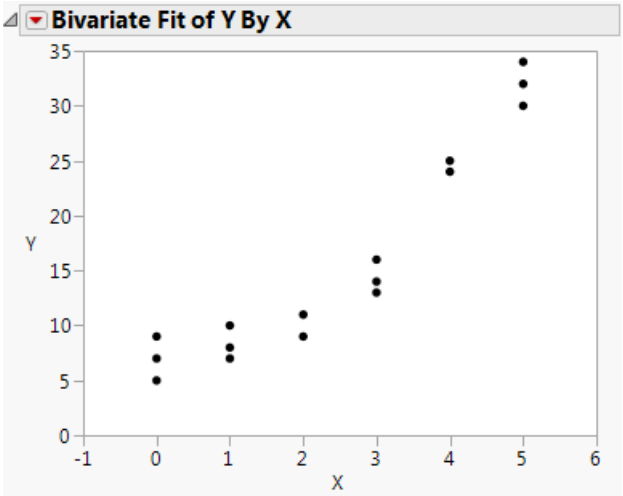
Example of Normal Regression with a Log Link

In this example, you are interested in fitting a generalized linear regression model with a normal distribution and a log link. You first explore the relationship between the explanatory and response variables to determine the appropriate link function to use in the Generalized Linear Model personality of the Fit Model platform.

1. Select **Help > Sample Data Folder** and open Nor.jmp.

2. Select **Analyze > Fit Y By X**.
3. Select Y and click **Y, Response**.
4. Select X, click **X, Factor**, and then click **OK**.

Figure 13.8 Y by X Results



You can see that Y varies nonlinearly with X and that the variance is approximately constant. Therefore, a normal distribution with a log link function is appropriate to model these data; that is, $\log(\mu_i) = \mathbf{x}_i'\beta$ so that $\mu_i = \exp(\mathbf{x}_i'\beta)$.

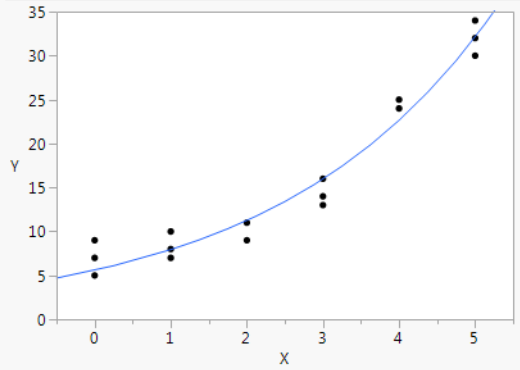
5. Select **Analyze > Fit Model**.
6. In the Personality list, select the **Generalized Linear Model**.
7. In the Distribution list, select **Normal**.
8. In the Link Function list, select **Log**.
9. Select Y and click **Y**.
10. Select X and click **Add**.
11. Click **Run**.

Figure 13.9 Model Results

Generalized Linear Model Fit

Overdispersion parameter estimated by Maximum Likelihood
Response: Y
Distribution: Normal
Link: Log
Estimation Method: Maximum Likelihood
Observations (or Sum Wgts) = 16

Regression Plot



Whole Model Test

Model	-LogLikelihood	L-R ChiSquare	DF	Prob>ChiSq
Difference	26.5218281	53.0437	1	<.0001*
Full	32.1782768			
Reduced	58.7001049			

Goodness Of Fit Statistic	ChiSquare	DF	Prob>ChiSq	Overdispersion
Pearson	52.3000	14	<.0001*	3.2687
Deviance	52.3000	14	<.0001*	

AICc
72.3566

Effect Tests

Source	DF	L-R ChiSquare	Prob>ChiSq
X	1	53.043656	<.0001*

Parameter Estimates

Term	Estimate	Std Error	L-R ChiSquare	Prob>ChiSq	Lower CL	Upper CL
Intercept	1.7213577	0.0893972	37.973777	<.0001*	1.5239809	1.8978651
X	0.3496015	0.0206493	53.043656	<.0001*	0.3081539	0.3944836

Statistical Details for the Generalized Linear Model Personality

This section contains statistical details for the Generalized Linear Model personality of the Fit Model platform.

- “Statistical Details for Generalized Linear Model Construction”
- “Statistical Details for Model Selection and Deviance”

Statistical Details for Generalized Linear Model Construction

To construct a generalized linear model, you must select response and explanatory variables for your data. You then must choose an appropriate link function and probability distribution for your response. Explanatory variables can be any combination of continuous variables, classification variables, and interactions. Some common examples of generalized linear models are listed in Table 13.1.

Table 13.1 Examples of Generalized Linear Models

Model	Response Variable	Distribution	Default Link Function
Traditional Linear Model	continuous	Normal	identity, $g(\mu) = \mu$
Logistic Regression	a count or a binary random variable	Binomial	logit, $g(\mu) = \log\left(\frac{\mu}{1-\mu}\right)$
Poisson Regression in Log Linear Model	a count	Poisson	log, $g(\mu) = \log(\mu)$
Exponential Regression	positive continuous	Exponential	$\frac{1}{\mu}$

The platform fits a generalized linear model to the data by maximum likelihood estimation of the parameter vector. In general, there is no closed-form solution for the maximum likelihood estimates of the parameters. Therefore, the platform estimates the parameters of the model numerically through an iterative fitting process using a technique pioneered by Nelder and Wedderburn (1972). The overdispersion parameter ϕ is estimated by dividing the Pearson goodness-of-fit statistic by its degrees of freedom. Covariances, standard errors, and confidence limits are computed for the estimated parameters based on the asymptotic normality of maximum likelihood estimators.

A number of link functions and probability distributions are available in the Generalized Linear Model personality of the Fit Model platform. Table 13.2 lists the built-in link functions.

Table 13.2 Built-in Link Functions

Link Function Name	Link Function Formula
Identity	$g(\mu) = \mu$
Logit	$g(\mu) = \log\left(\frac{\mu}{1-\mu}\right)$
Probit	$g(\mu) = \Phi^{-1}(\mu)$, where Φ is the standard normal cumulative distribution function
Log	$g(\mu) = \log(\mu)$
Reciprocal	$g(\mu) = \frac{1}{\mu}$
Power	$g(\mu) = \begin{cases} \mu^{\lambda} & \text{if } (\lambda \neq 0) \\ \log(\mu) & \text{if } \lambda = 0 \end{cases}$
Comp LogLog	$g(m) = \log(-\log(1-\mu))$

When you select the Power link function, a number box appears that enables you to enter the desired power.

Table 13.3 lists the variance functions associated with the available distributions for the response variable.

Table 13.3 Variance Functions for Response Distributions

Distribution	Variance Function
Normal	$V(\mu) = 1$
Binomial	$V(\mu) = \mu(1-\mu)$
Poisson	$V(\mu) = \mu$
Exponential	$V(\mu) = \mu^2$

Statistical Details for Model Selection and Deviance

An important aspect of generalized linear modeling is the selection of explanatory variables in the model. Changes in goodness-of-fit statistics are often used to evaluate the contribution of subsets of explanatory variables to a particular model. The *deviance* is defined to be twice the difference between the maximum attainable log-likelihood and the log-likelihood at the maximum likelihood estimates of the regression parameters. The deviance is often used as a measure of goodness of fit. The maximum attainable log-likelihood is achieved with a model that has a parameter for every observation. Table 13.4 lists the deviance formula for each of the available distributions for the response variable.

Table 13.4 Deviance Formulas for Response Distributions

Distribution	Deviance Formula
Normal	$\sum_i w_i(y_i - \mu_i)^2$
Binomial	$2\sum_i w_i m_i \left[y_i \log\left(\frac{y_i}{\mu_i}\right) + (1-y_i)\log\left(\frac{1-y_i}{1-\mu_i}\right)\right]$
Poisson	$2\sum_i w_i \left[y_i \log\left(\frac{y_i}{\mu_i}\right) - (y_i - \mu_i)\right]$
Exponential	$2\sum_i w_i \left[-\log\left(\frac{y_i}{\mu_i}\right) + \left(\frac{y_i - \mu_i}{\mu_i}\right)\right]$

The Pearson chi-square statistic is defined as follows:

$$X^2 = \sum_i \frac{w_i(y_i - \mu_i)^2}{V(\mu_i)}$$

where

y_i is the i^{th} response

μ_i is the corresponding predicted mean

$V(\mu_i)$ is the variance function

w_i is a known weight for the i^{th} observation

Note: If no weight is specified, $w_i = 1$ for all observations.

One strategy for variable selection is to fit a sequence of models. You start with a simple model that contains only an intercept term, and then include one additional explanatory variable in each successive model. You can measure the importance of the additional explanatory variable by the difference in deviance or fitted log-likelihood values between successive models. Asymptotic tests enable you to assess the statistical significance of the additional term.

When the distribution is non-normal, a normal critical value is used instead of a *t*-distribution critical value in inverse prediction.

Residual Formulas

Deviance

$$r_{Di} = \sqrt{d_i}(sign(y_i - \mu_i))$$

Studentized Deviance

$$r_{Di} = \frac{sign(y_i - \mu_i)\sqrt{d_i}}{\sqrt{\phi(1 - h_i)}}$$

Pearson

$$r_{Pi} = \frac{(y_i - \mu_i)}{\sqrt{V(\mu_i)}}$$

Studentized Pearson

$$r_{Pi} = \frac{y_i - \mu_i}{\sqrt{V(\mu_i)(1 - h_i)}}$$

where

$(y_i - \mu_i)$ is the raw residual

$\text{sign}(y_i - \mu_i)$ is 1 if $(y_i - \mu_i)$ is positive and -1 if $(y_i - \mu_i)$ is negative

d_i is the contribution to the total deviance from observation i

ϕ is the dispersion parameter

$V(\mu_i)$ is the variance function

h_i is the i^{th} diagonal element of the matrix $W_e^{(1/2)}X(X'W_eX)^{-1}X'W_e^{(1/2)}$, where W_e is the weight matrix used in computing the expected information matrix.

For more information about residuals and generalized linear models, see the GENMOD Procedure chapter in SAS Institute Inc. (2022a).

Appendix A

Statistical Details

Fitting Linear Models

This appendix discusses the different types of response models, their factors, their design coding, and parameterization. It also includes many other details of methods described in the main text.

The JMP system fits linear models to three different types of response models that are labeled continuous, ordinal, and nominal. Many details about the factor side are the same for the different response models, but JMP supports graphics and marginal profiles only for continuous responses—not for ordinal and nominal.

Different computer programs use different design-matrix codings, and thus parameterizations, to fit effects and construct hypothesis tests. JMP uses a different coding than the GLM procedure in the SAS system, although in most cases JMP and SAS GLM procedure produce the same results. The following sections describe the details of JMP coding and highlight those cases when it differs from that of the SAS GLM procedure.

Contents

The Response Models . . . 563
Continuous Responses . . . 563
Nominal Responses . . . 564
Ordinal Responses . . . 565
The Factor Models . . . 567
Continuous Factors . . . 567
Nominal Factors . . . 567
Ordinal Factors . . . 579
Frequencies . . . 585
The Usual Assumptions . . . 585
Assumed Model . . . 585
Relative Significance . . . 585
Multiple Inferences . . . 586
Validity Assessment . . . 586
Alternative Methods . . . 587
Key Statistical Concepts . . . 587
Uncertainty, a Unifying Concept . . . 587
The Two Basic Fitting Machines . . . 588
Likelihood, AICc, and BIC . . . 592
Power Calculations . . . 593
Computations for the LSN . . . 593
Computations for the LSV . . . 594
Computations for the Power . . . 595
Computations for the Adjusted Power . . . 596
Inverse Prediction with Confidence Limits . . . 597

The Response Models

The Fit Model platform fits linear models to three different types of responses: continuous, nominal, and ordinal. The models and methods available in the Fit Model platform are practical, are widely used, and suit the need for a general approach in a statistical software tool. As with all statistical software, you are responsible for learning the assumptions of the models that you choose to use, and the consequences if the assumptions are not met. See "The Usual Assumptions" in this chapter.

- "Continuous Responses"
- "Nominal Responses"
- "Ordinal Responses"

Continuous Responses

When the response column (the column assigned the Y role) is continuous, the Fit Model platform fits the value of the response directly. The basic model is that for each observation,

Y = (some function of the X's and parameters) + error

Statistical tests are based on the assumption that the error term in the model is normally distributed.

Fitting Principle for Continuous Response

The Fitting principle is called *least squares*. The least squares method estimates the parameters in the model to minimize the sum of squared errors. The errors in the fitted model, called *residuals*, are the difference between the actual value of each observation and the value predicted by the fitted model.

The least squares method is equivalent to the maximum likelihood method of estimation if the errors have a normal distribution. This means that the analysis estimates the model that gives the most likely residuals. The log-likelihood is a scale multiple of the sum of squared errors for the normal distribution.

Base Model for Continuous Responses

The simplest model for continuous measurement fits just one value to predict all the response values. This value is the estimate of the *mean*. The mean is just the arithmetic average of the response values. All other models are compared to this base model.

Nominal Responses

In the Fit Model platform, nominal responses are analyzed with a straightforward extension of the logit model. For a binary (two-level) response, a logit response model is specified as follows:

$$\log\left(\frac{P(y=1)}{P(y=2)}\right) = X\beta$$

The above can also be written as follows:

$$P(y=1) = F(X\beta)$$

where $F(x)$ is the cumulative distribution function of the standard logistic distribution:

$$F(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{1+e^x}$$

For r response levels, JMP fits the probabilities that the response is one of r different response levels given by the data values. The probability estimates must all be positive. For a given configuration of Xs, the probability estimates must sum to 1 over the response levels. The function that JMP uses to predict probabilities is a composition of a linear model and a multi-response logistic function. This is sometimes called a *log-linear* model because the logs of ratios of probabilities are linear models. JMP relates each response probability to the r^{th} probability and fit a separate set of design parameters to these r - 1 models.

$$\log\left(\frac{P(y=j)}{P(y=r)}\right) = X\beta_{(j)} \text{ for } j = 1, \ldots, r-1$$

Fitting Principle for Nominal Response

The fitting principle is called *maximum likelihood*. It estimates the parameters such that the joint probability for all the responses given by the data is the greatest obtainable by the model. Rather than reporting the joint probability (likelihood) directly, it is more manageable to report the total of the negative logs of the likelihood.

The uncertainty (negative log-likelihood) is the sum of the negative logs of the probabilities attributed by the model to the responses that actually occurred in the sample data. For a sample of size n, it is often denoted as H and written

$$H = \sum_{i=1}^{n} -\log(P(y=y_i))$$

If you attribute a probability of 1 to each event that did occur, then the sum of the negative logs is zero for a perfect fit.

The nominal model can take a lot of time and memory to fit, especially if there are many response levels. JMP tracks the progress of its calculations with an *iteration history*, which shows the negative log-likelihood values becoming smaller as they converge to the estimates.

Base Model for Nominal Responses

The simplest model for a nominal response is a set of constant response probabilities fitted as the occurrence rates for each response level across the whole data table. In other words, the probability that y is response level j is estimated by dividing the total sample count n into the total of each response level n_j. This probability is specified as follows:

$$p_j = \frac{n_j}{n}$$

All other models are compared to this base model. The base model serves the same role for a nominal response as the sample mean does for continuous models.

The R^2 statistic measures the portion of the uncertainty accounted for by the model, which is

$$1 - \frac{H(\text{full model})}{H(\text{base model})}$$

However, it is rare in practice to get an R^2 near 1 for categorical models.

Ordinal Responses

With an ordinal response (Y), as with nominal responses, the Fit Model platform fits probabilities that the response is one of r different response levels given by the data.

Ordinal data have an order like continuous data. The order is used in the analysis but the spacing or distance between the ordered levels is not used. If you have a numeric response but want your model to ignore the spacing of the values, you can assign the ordinal level to that response column. If you have a classification variable and the levels are in some natural order such as low, medium, and high, you can use the ordinal modeling type.

Ordinal responses are modeled by fitting a series of parallel logistic curves to the cumulative probabilities. Each curve has the same design parameters but a different intercept and is specified as follows:

$$\mathrm{P}(y \le j) = F(\alpha_j + \mathrm{X}\beta) \text{ for } j = 1, \ldots, r-1$$

where r response levels are present and $F(x)$ is the standard logistic cumulative distribution function

$$F(x) = \frac{1}{1+e^{-x}} = \frac{e^{x}}{1+e^{x}}$$

Another way to write this is in terms of an unobserved continuous variable, z, that causes the ordinal response to change as it crosses various thresholds

$$y = \begin{cases} r & \alpha_{r-1} \le z \\ j & \alpha_{j-1} \le z < \alpha_j \\ 1 & z \le \alpha_1 \end{cases}$$

where z is an unobservable function of the linear model and error

$$z = X\beta + \varepsilon$$

and ε has the logistic distribution.

These models are attractive because they recognize the ordinal character of the response, they need far fewer parameters than nominal models, and the computations are fast.

A different but mathematically equivalent way to envision an ordinal model is to think of a nominal model where, instead of modeling the odds, you model the cumulative probability. Instead of fitting functions for all but the last level, you fit only one function and slide it to fit each cumulative response probability.

Fitting Principle for Ordinal Response

The maximum likelihood fitting principle for an ordinal response model is the same as for a nominal response model. It estimates the parameters such that the joint probability for all the responses that occur is the greatest obtainable by the model. It uses an iterative method that is faster and uses less memory than nominal fitting.

Base Model

The simplest model for an ordinal response, like a nominal response, is a set of response probabilities fitted as the occurrence rates of the response in the whole data table.

The Factor Models

In the Fit Model platform, the way the x-variables (factors) are modeled to predict an expected value or probability is the subject of the factor side of the model.

The factors enter the prediction equation as a linear combination of x values and the parameters to be estimated. For a continuous response model, where i indexes the observations and j indexes the parameters, the assumed model for a typical observation, y_i, is written

$$y_i = \beta_0 + \beta_1 x_{1i} + \ldots + \beta_k x_{ki} + \varepsilon_i$$

where

y_i is the response

x_{ij} are functions of the data

ε_i is an unobservable realization of the random error

b_j are unknown parameters to be estimated.

The way the x's in the linear model are formed from the factor terms is different for each modeling type. The linear model x's can also be complex effects such as interactions or nested effects. Complex effects are discussed in detail later.

Continuous Factors

In the Fit Model platform, continuous factors are placed directly into the design matrix as regressors. If a column is a linear function of other columns, then the parameter for this column is marked *zeroed* or *nonestimable*. Continuous factors are centered by their mean when they are crossed with other factors (interactions and polynomial terms). Centering is suppressed if the factor has a Column Property of **Mixture** or **Coding**, or if the centered polynomials option is turned off when specifying the model. If there is a coding column property, the factor is coded before fitting.

Nominal Factors

In the Fit Model platform, nominal factors are transformed into indicator variables for the design matrix. SAS GLM constructs an indicator column for each nominal level. JMP constructs the same indicator columns for each nominal level except the last level. When the last nominal level occurs, a one is subtracted from all the other columns of the factor. For example, consider a nominal factor A with three levels coded for GLM and for JMP as shown below.

Table A.1 Nominal Factor A

	GLM			JMP	
A	**A1**	**A2**	**A3**	**A13**	**A23**
A1	1	0	0	1	0
A2	0	1	0	0	1
A3	0	0	1	–1	–1

In GLM, the linear model design matrix has linear dependencies among the columns, and the least squares solution uses a generalized inverse. The solution chosen happens to be such that the A3 parameter is set to zero.

In JMP, the linear model design matrix is coded so that it achieves full rank unless there are missing cells or other incidental collinearity. The parameter for the A effect for the last level is the negative sum of the other levels, which makes the parameters sum to zero over all the effect levels.

Interpretation of Parameters

Note: The parameter for a nominal level is interpreted as the differences in the predicted response for that level from the average predicted response over all levels.

The design column for a factor level is constructed as the zero-one indicator of that factor level minus the indicator of the last level. This is the coding that leads to the parameter interpretation above.

Table A.2 Interpreting Parameters

JMP Parameter Report	How to Interpret	Design Column Coding
Intercept	mean over all levels	$\mathbf{1}'$
A[1]	$\alpha_1 - 1/3(\alpha_1 + \alpha_2 + \alpha_3)$	(A==1) – (A==3)
A[2]	$\alpha_2 - 1/3(\alpha_1 + \alpha_2 + \alpha_3)$	(A==2) – (A==3)

Interactions and Crossed Effects

Interaction effects with both GLM and JMP are constructed by taking a direct product over the rows of the design columns of the factors being crossed. For example, the GLM code

```
PROC GLM;
  CLASS A B;
  MODEL A B A*B;
```

yields this design matrix:

Table A.3 Design Matrix

		A			B			AB								
A	**B**	1	2	3	1	2	3	11	12	13	21	22	23	31	32	33
A1	**B1**	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0
A1	**B2**	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0
A1	**B3**	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0
A2	**B1**	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0
A2	**B2**	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0
A2	**B3**	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0
A3	**B1**	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0
A3	**B2**	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0
A3	**B3**	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1

Using the JMP Fit Model command and requesting a factorial model for columns A and B produces the following design matrix. Note that A13 in this matrix is A1–A3 in the previous matrix. However, A13B13 is A13*B13 in the current matrix.

Table A.4 Current Matrix

		A		B					
A	**B**	**13**	**23**	**13**	**23**	**A13 B13**	**A13 B23**	**A23 B13**	**A23 B23**
A1	**B1**	1	0	1	0	1	0	0	0
A1	**B2**	1	0	0	1	0	1	0	0

Table A.4 Current Matrix *(Continued)*

A1	**B3**	1	0	–1	–1	–1	–1	0	0
A2	**B1**	0	1	1	0	0	0	1	0
A2	**B2**	0	1	0	1	0	0	0	1
A2	**B3**	0	1	–1	–1	0	0	–1	–1
A3	**B1**	–1	–1	1	0	–1	0	–1	0
A3	**B2**	–1	–1	0	1	0	–1	0	–1
A3	**B3**	–1	–1	–1	–1	1	1	1	1

The JMP coding saves memory and some computing time for problems with interactions of factors with few levels.

The expected values of the cells in terms of the parameters for a three-by-three crossed model are:

Table A.5 Three-by-Three Crossed Model

	B1	**B2**	**B3**
A1	$\mu + \alpha_1 + \beta_1 + \alpha\beta_{11}$	$\mu + \alpha_1 + \beta_2 + \alpha\beta_{12}$	$\mu + \alpha_1 - \beta_1 - \beta_2 - \alpha\beta_{11} - \alpha\beta_{12}$
A2	$\mu + \alpha_2 + \beta_1 + \alpha\beta_{21}$	$\mu + \alpha_2 + \beta_2 + \alpha\beta_{22}$	$\mu + \alpha_2 - \beta_1 - \beta_2 - \alpha\beta_{21} - \alpha\beta_{22}$
A3	$\mu - \alpha_1 - \alpha_2 + \beta_1 - \alpha\beta_{11} - \alpha\beta_{21}$	$\mu - \alpha_1 - \alpha_2 + \beta_2 - \alpha\beta_{12} - \alpha\beta_{22}$	$\mu - \alpha_1 - \alpha_2 - \beta_1 - \beta_2 + \alpha\beta_{11} + \alpha\beta_{12} + \alpha\beta_{21} + \alpha\beta_{22}$

Nested Effects

Nested effects in GLM are coded the same as interaction effects because GLM determines the right test by what is not in the model. Any effect not included in the model can be soaked up by a containing interaction (or, equivalently, nested) effect.

Nested effects in JMP are coded differently. JMP uses the terms inside the parentheses as grouping terms for each group. For each combination of levels of the nesting terms, JMP constructs the effect on the outside of the parentheses. The levels of the outside term do not need to line up across the levels of the nesting terms. Each level of nest is considered separately with regard to the construction of design columns and parameters.

Table A.6 Nested Effects

				B(A)					
				A1	A1	A2	A2	A3	A3
A	B	A13	A23	B13	B23	B13	B23	B13	B23
A1	B1	1	0	1	0	0	0	0	0
A1	B2	1	0	0	1	0	0	0	0
A1	B3	1	0	–1	–1	0	0	0	0
A2	B1	0	1	0	0	1	0	0	0
A2	B2	0	1	0	0	0	1	0	0
A2	B3	0	1	0	0	–1	–1	0	0
A3	B1	–1	–1	0	0	0	0	1	0
A3	B2	–1	–1	0	0	0	0	0	1
A3	B3	–1	–1	0	0	0	0	–1	–1

Least Squares Means across Nominal Factors

Least squares means are the predicted values corresponding to some combination of levels, after setting all the other factors to some neutral value. The neutral value for direct continuous regressors is defined as the sample mean. The neutral value for an effect with uninvolved nominal factors is defined as the average effect taken over the levels (which happens to result in all zeros in our coding). Ordinal factors use a different neutral value in "Ordinal Least Squares Means". The least squares means might not be estimable, and if not, they are marked nonestimable. The least squares means in JMP agree with those in SAS PROC GLM (Goodnight and Harvey 1978) in all cases except when a weight is used. When a weight variable is used, JMP uses a weighted mean and SAS PROC GLM uses an unweighted mean for its neutral values.

Effective Hypothesis Tests

Generally, the hypothesis tests produced by JMP agree with the hypothesis tests of most other trusted programs, such as SAS PROC GLM (Hypothesis types III and IV). The following two sections describe where there are differences.

In SAS PROC GLM, the hypothesis tests for Types III and IV are constructed using the general form of estimable functions and finding functions that involve only the effects of interest and effects contained by the effects of interest (Goodnight 1978).

The same tests are constructed in JMP. However, because there is a different parameterization, an effect can be tested (assuming full rank for now) by doing a joint test on all the parameters for that effect. The tests do not involve containing interaction parameters because the coding has made them uninvolved with the tests on their contained effects.

If there are missing cells or other singularities, the JMP tests are different from GLM tests. There are several ways to describe them:

- JMP tests are equivalent to testing that the least squares means are different, at least for main effects. If the least squares means are nonestimable, then the test cannot include some comparisons and therefore loses degrees of freedom. For interactions, JMP is testing that the least squares means differ by more than just the marginal pattern described by the containing effects in the model.
- JMP tests an effect by comparing the SSE for the model with that effect to the SSE for the model without that effect. JMP parameterizes the model so that this method makes sense.
- JMP implements the *effective hypothesis tests* described by Hocking (1985, pp. 80–89, 163–166), although JMP uses structural rather than cell-means parameterization. Effective hypothesis tests start with the hypothesis desired for the effect and include "as much as possible" of that test. Of course, if there are containing effects with missing cells, then this test has to drop part of the hypothesis because the complete hypothesis would not be estimable. The effective hypothesis drops as little of the complete hypothesis as possible.
- The differences among hypothesis tests in JMP and GLM (and other programs) that relate to the presence of missing cells are not considered interesting tests anyway. If an interaction is significant, the test for the contained main effects is not interesting. If the interaction is not significant, then it can always be dropped from the model. Some tests are not even unique. If you relabel the levels in a missing cell design, then the GLM Type IV tests can change.

The following section continues this topic in finer detail.

Singularities and Missing Cells in Nominal Effects

Consider the case of linear dependencies among the design columns. With JMP coding, this does not occur unless there is insufficient data to fill out the combinations that need estimating, or unless there is some type of confounding or collinearity of the effects.

With linear dependencies, a least squares solution for the parameters might not be unique and some tests of hypotheses cannot be tested. The strategy chosen for JMP is to set parameter estimates to zero in sequence as their design columns are found to be linearly dependent on previous effects in the model. A special column in the report shows what parameter estimates are zeroed and which parameter estimates are estimable. A separate *singularities* report shows what the linear dependencies are.

In cases of singularities the hypotheses tested by JMP can differ from those selected by GLM. Generally, JMP finds fewer degrees of freedom to test than GLM because it holds its tests to a higher standard of marginality. In other words, JMP tests always correspond to tests across least squares means for that effect, but GLM tests do not always have this property.

For example, consider a two-way model with interaction and one missing cell where A has three levels, B has two levels, and the A3B2 cell is missing.

Table A.7 Two-Way Model with Interaction

A B	A1	A2	B1	A1B1	A2B1	
A1 B1	1	0	1	1	0	
A2 B1	0	1	1	0	1	
A3 B1	–1	–1	1	–1	–1	
A1 B2	1	0	–1	–1	0	
A2 B2	0	1	–1	0	–1	
A3 B2	–1	–1	–1	1	1	Suppose this interaction is missing.

The expected values for each cell are:

Table A.8 Expected Values

	B1	B2
A1	$\mu + \alpha_1 + \beta_1 + \alpha\beta_{11}$	$\mu + \alpha_1 - \beta_1 - \alpha\beta_{11}$
A2	$\mu + \alpha_2 + \beta_1 + \alpha\beta_{21}$	$\mu + \alpha_2 - \beta_1 - \alpha\beta_{21}$
A3	$\mu - \alpha_1 - \alpha_2 + \beta_1 - \alpha\beta_{11} - \alpha\beta_{21}$	$\mu - \alpha_1 - \alpha_2 - \beta_1 + \alpha\beta_{11} + \alpha\beta_{21}$

Obviously, any cell with data has an expectation that is estimable. The cell that is missing has an expectation that is nonestimable. In fact, its expectation is precisely that linear combination of the design columns that is in the singularity report

$$\mu - \alpha_1 - \alpha_2 - \beta_1 + \alpha\beta_{11} + \alpha\beta_{21}$$

Suppose that you want to construct a test that compares the least squares means of B1 and B2. In this example, the average of the rows in the above table give these least squares means.

$$\begin{aligned} \text{LSM(B1)} = & (1/3)(\mu + \alpha_1 + \beta_1 + \alpha\beta_{11} + \\ & \mu + \alpha_2 + \beta_1 + \alpha\beta_{21} + \\ & \mu - \alpha_1 - \alpha_2 + \beta_1 - \alpha\beta_{11} - \alpha\beta_{21}) \\ = & \ \mu + \beta_1 \end{aligned}$$

$$\begin{aligned} \text{LSM(B2)} = & (1/3)(\mu + \alpha_1 + -\beta_1 - \alpha\beta_{11} + \\ & \mu + \alpha_2 + -\beta_1 - \alpha\beta_{21} + \\ & \mu - \alpha_1 - \alpha_2 - \beta_1 + \alpha\beta_{11} + \alpha\beta_{21}) \\ = & \ \mu - \beta_1 \end{aligned}$$

$$\text{LSM(B1)} - \text{LSM(B2)} = 2\beta_1$$

Note that this shows that a test on the β_1 parameter is equivalent to testing that the least squares means are the same. But because β_1 is not estimable, the test is not testable, meaning there are no degrees of freedom for it.

Now, construct the test for the least squares means across the A levels.

$$\begin{aligned} \text{LSM(A1)} = & (1/2)(\mu + \alpha_1 + \beta_1 + \alpha\beta_{11} + \mu + \alpha_1 - \beta_1 - \alpha\beta_{11}) \\ = & \ \mu + \alpha_1 \end{aligned}$$

$$\begin{aligned} \text{LSM(A2)} = & (1/2)(\mu + \alpha_2 + \beta_1 + \alpha\beta_{21} + \mu + \alpha_2 - \beta_1 - \alpha\beta_{21}) \\ = & \ \mu + \alpha_2 \end{aligned}$$

$$\begin{aligned} \text{LSM(A3)} = & (1/2)(\mu - \alpha_1 - \alpha_2 + \beta_1 - \alpha\beta_{11} - \alpha\beta_{21} + \\ & \mu - \alpha_1 - \alpha_2 - \beta_1 + \alpha\beta_{11} + \alpha\beta_{21}) \\ = & \ \mu - \alpha_1 - \alpha_2 \end{aligned}$$

$$\text{LSM(A1)} - \text{LSM(A3)} = 2\alpha_1 + \alpha_2$$

$$\text{LSM(A2)} - \text{LSM(A3)} = 2\alpha_2 + \alpha_1$$

Neither of these turn out to be estimable, but there is another comparison that is estimable; namely comparing the two A columns that have no missing cells.

$$\text{LSM(A1)} - \text{LSM(A2)} = \alpha_1 - \alpha_2$$

This combination is indeed tested by JMP using a test with 1 degree of freedom, although there are two parameters in the effect.

The estimability can be verified by taking its inner product with the singularity combination, and checking that it is zero:

Table A.9 Verification

	singularity	**combination**
parameters	**combination**	**to be tested**
m	1	0
$\mathbf{a_1}$	–1	1
$\mathbf{a_2}$	–1	–1
$\mathbf{b_1}$	–1	0
$\mathbf{ab_{11}}$	1	0
$\mathbf{ab_{21}}$	1	0

It turns out that the design columns for missing cells for any interaction always knocks out degrees of freedom for the main effect (for nominal factors). Thus, there is a direct relation between the non-estimability of least squares means and the loss of degrees of freedom for testing the effect corresponding to these least squares means.

How does this compare with what GLM does? GLM and JMP do the same test when there are no missing cells. That is, they effectively test that the least squares means are equal. But when GLM encounters singularities, it focuses out these cells in different ways, depending on whether they are Type III or Type IV. For Type IV, it looks for estimable combinations that it can find. These might not be unique, and if you reorder the levels, you might get a different result. For Type III, it does some orthogonalization of the estimable functions to obtain a unique test. But the test might not be very interpretable in terms of the cell means.

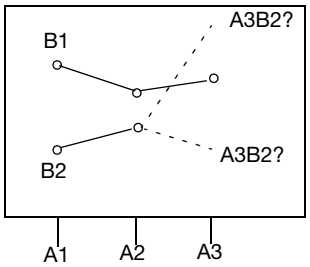
The JMP approach has several points in its favor, although at first it might seem distressing that you might lose more degrees of freedom than with GLM:

1. The tests are philosophically linked to LSMs.

2. The tests are easy computationally, using reduction sum of squares for reparameterized models.
3. The tests agree with Hocking's "Effective Hypothesis Tests".
4. The tests are *whole marginal tests*, meaning they always go completely across other effects in interactions.

The last point needs some elaboration: Consider a graph of the expected values of the cell means in the previous example with a missing cell for A3B2.

Figure A.1 Expected Values of the Cell Means



The graph shows expected cell means with a missing cell. The means of the A1 and A2 cells are profiled across the B levels. The JMP approach says you cannot test the B main effect with a missing A3B2 cell, because the mean of the missing cell could be anything, as allowed by the interaction term. If the mean of the missing cell was the higher value shown, the B effect would likely test significant. If it were the lower, it would likely test nonsignificant. The point is that you do not know. That is what the least squares means are saying when they are declared nonestimable. That is what the hypotheses for the effects should be saying too—that you do not know.

If you want to test hypotheses involving margins for subsets of cells, then that is what GLM Type IV does. In JMP, you would have to construct these tests yourself by partitioning the effects with a lot of calculations or by using contrasts.

JMP and GLM Hypotheses

GLM works differently than JMP and produces different hypothesis tests in situations where there are missing cells. In particular, GLM does not recognize any difference between a nesting and a crossing in an effect, but JMP does. Suppose that you have a three-layer nesting of A, B(A), and C(A B) with different numbers of levels as you go down the nested design.

Figure A.10 shows the test of the main effect A in terms of the GLM parameters. The first set of columns is the test done by JMP. The second set of columns is the test done by GLM Type IV. The third set of columns is the test equivalent to that by JMP; it is the first two columns that have been multiplied by a matrix:

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

to be comparable to the GLM test. The last set of columns is the GLM Type III test. The difference is in how the test distributes across the containing effects. In JMP, it seems more top-down hierarchical. In GLM Type IV, the test seems more bottom-up. In practice, the test statistics are often similar.

Table A.10 Comparison of GLM and JMP Hypotheses

Parameter	JMP Test for A		GLM-IV Test for A		JMP Rotated Test		GLM-III Test for A	
u	0	0	0	0	0	0	0	0
a1	0.6667	-0.3333	1	0	1	0	1	0
a2	–0.3333	0.6667	0	1	0	1	0	1
a3	–0.3333	-0.3333	-1	-1	-1	-1	-1	-1
a1b1	0.1667	-0.0833	0.2222	0	0.25	0	0.2424	0
a1b2	0.1667	-0.0833	0.3333	0	0.25	0	0.2727	0
a1b3	0.1667	-0.0833	0.2222	0	0.25	0	0.2424	0
a1b4	0.1667	-0.0833	0.2222	0	0.25	0	0.2424	0
a2b1	-0.1667	0.3333	0	0.5	0	0.5	0	.5
a2b2	-0.1667	0.3333	0	0.5	0	0.5	0	.5
a3b1	-0.1111	-0.1111	-0.3333	-0.3333	-0.3333	-0.3333	-0.3333	-0.3333
a3b2	-0.1111	-0.1111	-0.3333	-0.3333	-0.3333	-0.3333	-0.3333	-0.3333
a3b3	-0.1111	-0.1111	-0.3333	-0.3333	-0.3333	-0.3333	-0.3333	-0.3333

Table A.10 Comparison of GLM and JMP Hypotheses *(Continued)*

a1b1c1	0.0833	-0.0417	0.1111	0	0.125	0	0.1212	0
a1b1c2	0.0833	-0.0417	0.1111	0	0.125	0	0.1212	0
a1b2c1	0.0556	-0.0278	0.1111	0	0.0833	0	0.0909	0
a1b2c2	0.0556	-0.0278	0.1111	0	0.0833	0	0.0909	0
a1b2c3	0.0556	-0.0278	0.1111	0	0.0833	0	0.0909	0
a1b3c1	0.0833	-0.0417	0.1111	0	0.125	0	0.1212	0
a1b3c2	0.0833	-0.0417	0.1111	0	0.125	0	0.1212	0
a1b4c1	0.0833	-0.0417	0.1111	0	0.125	0	0.1212	0
a1b4c2	0.0833	-0.0417	0.1111	0	0.125	0	0.1212	0
a2b1c1	-0.0833	0.1667	0	0.25	0	0.25	0	0.25
a2b1c2	-0.0833	0.1667	0	0.25	0	0.25	0	0.25
a2b2c1	-0.0833	0.1667	0	0.25	0	0.25	0	0.25
a2b2c2	-0.0833	0.1667	0	0.25	0	0.25	0	0.25
a3b1c1	-0.0556	-0.0556	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667
a3b1c2	-0.0556	-0.0556	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667
a3b2c1	-0.0556	-0.0556	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667
a3b2c2	-0.0556	-0.0556	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667
a3b3c1	-0.0556	-0.0556	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667
a3b3c2	-0.0556	-0.0556	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667	-0.1667

From the perspective of the JMP parameterization, the tests for A are:

Table A.11 Tests for A

parameter	**GLM–IV test**		**JMP test**	
m	0	0	0	0

Table A.11 Tests for A *(Continued)*

a13	2	1	1	0
a23	1	2	0	1
a1:b14	0	0	0	0
a1:b24	0.11111	0	0	0
a1:b34	0	0	0	0
a2:b12	0	0	0	0
a3:b13	0	0	0	0
a3:b23	0	0	0	0
a1b1:c12	0	0	0	0
a1b2:c13	0	0	0	0
a1b2:c23	0	0	0	0
a1b3:c12	0	0	0	0
a1b4:c12	0	0	0	0
a2b1:c13	0	0	0	0
a2b2:c12	0	0	0	0
a3b1:c12	0	0	0	0
a3b2:c12	0	0	0	0
a3b3:c12	0	0	0	0

So from the JMP perspective, the GLM test looks a little strange, putting a coefficient on the a1b24 parameter.

Ordinal Factors

In the Fit Model platform, factors marked with the ordinal modeling type are coded differently than nominal factors. The parameter estimates are interpreted differently, the tests are different, and the least squares means are different.

For ordinal factors, the first level of the factor is a control or baseline level, and the parameters measure the effect on the response as the ordinal factor is set to each succeeding level. The ordinal factor coding is appropriate for factors that contain levels that represent various doses, where the first dose is zero. The following table shows an example of a three-level ordinal factor:

Table A.12 Ordinal Factors

Term	Coded Column		
A	**a2**	**a3**	
A1	0	0	control level, zero dose
A2	1	0	low dose
A3	1	1	higher dose

The pattern for the design is such that the lower triangle is ones with zeros elsewhere.

For a simple main-effects model, this can be specified as follows:

$$y = \mu + \alpha_2 X_{(a \le 2)} + \alpha_3 X_{(a \le 3)} + \varepsilon$$

noting that μ is the expected response at $A = 1$, $\mu + \alpha_2$ is the expected response at $A = 2$, and $\mu + \alpha_2 + \alpha_3$ is the expected response at $A = 3$. Thus, α_2 estimates the effect moving from $A = 1$ to $A = 2$ and α_3 estimates the effect moving from $A = 2$ to $A = 3$.

If all the parameters for an ordinal main effect have the same sign, then the response effect is monotonic across the ordinal levels.

Ordinal Interactions

The ordinal interactions, as with nominal effects, are produced with a horizontal direct product of the columns of the factors. Consider an example with two ordinal factors A and B, where each factor has three levels. The ordinal coding in JMP produces the design matrix shown next. The pattern for the interaction is a block lower-triangular matrix of lower-triangular matrices of ones.

Table A.13 Ordinal Interactions

						A*B			
						A2		A3	
A	**B**	A2	A3	B2	B3	B2	B3	B2	B3
A1	**B1**	0	0	0	0	0	0	0	0
A1	**B2**	0	0	1	0	0	0	0	0
A1	**B3**	0	0	1	1	0	0	0	0
A2	**B1**	1	0	0	0	0	0	0	0
A2	**B2**	1	0	1	0	1	0	0	0
A2	**B3**	1	0	1	1	1	1	0	0
A3	**B1**	1	1	0	0	0	0	0	0
A3	**B2**	1	1	1	0	1	0	1	0
A3	**B3**	1	1	1	1	1	1	1	1

Note: When you test to see whether there is no effect, there is not much difference between nominal and ordinal factors for simple models. However, there are major differences when interactions are specified. We recommend that you use nominal rather than ordinal factors for most models.

Hypothesis Tests for Ordinal Crossed Models

To see what the parameters mean, examine this table of the expected cell means in terms of the parameters, where μ is the intercept, α_2 is the parameter for level A2, and so on.

Table A.14 Expected Cell Means

	B1	**B2**	**B3**
A1	μ	$\mu + \beta_2$	$\mu + \beta_2 + \beta_3$
A2	$\mu + \alpha_2$	$\mu + \alpha_2 + \beta_2 + \alpha\beta_{22}$	$\mu + \alpha_2 + \beta_2 + \beta_3 + \alpha\beta_{22} + \alpha\beta_{23}$

Table A.14 Expected Cell Means *(Continued)*

A3	$\mu + \alpha_2 + \alpha_3$	$\mu + \alpha_2 + \alpha_3 + \beta_2 + \alpha\beta_{22} + \alpha\beta_{32}$	$\mu + \alpha_2 + \alpha_3 + \beta_2 + \beta_3 + \alpha\beta_{22} + \alpha\beta_{23} + \alpha\beta_{32} + \alpha\beta_{33}$

Note that the main effect test for A is really testing the A levels holding B at the first level. Similarly, the main effect test for B is testing across the top row for the various levels of B holding A at the first level. This is the appropriate test for an experiment where the two factors are both doses of different treatments. The main question is the efficacy of each treatment by itself, and fewer points are devoted to looking for *drug interactions* when doses of both drugs are applied. In some cases, it might even be dangerous to apply large doses of each drug.

Note that each cell's expectation can be obtained by adding all the parameters associated with each cell that is to the left and above it, inclusive of the current row and column. The expected value for the last cell is the sum of all the parameters.

Though the hypothesis tests for effects contained by other effects differs with ordinal and nominal codings, the test of effects not contained by other effects is the same. In the crossed design above, the test for the interaction would be the same no matter whether A and B were fit nominally or ordinally.

Ordinal Least Squares Means

As stated previously, least squares means are the predicted values corresponding to some combination of levels, after setting all the other factors to some neutral value. JMP defines the neutral value for an effect with uninvolved ordinal factors as the effect at the first level, meaning the control, or *baseline* level.

This definition of least squares means for ordinal factors maintains the idea that the hypothesis tests for contained effects are equivalent to tests that the least squares means are equal.

Singularities and Missing Cells in Ordinal Effects

With the ordinal coding, you are saying that the first level of the ordinal effect is the baseline. It is thus possible to get good tests on the main effects even when there are missing cells in the interactions—even if you have no data for the interaction.

Example with Missing Cell

The example is the same as above, with two observations per cell except that the A3B2 cell has no data. You can now compare the results when the factors are coded nominally with results when they are coded ordinally. The model fit is the same, as seen in Figure A.2.

Table A.15 Observations

Y	A	B
12	1	1
14	1	1
15	1	2
16	1	2
17	2	1
17	2	1
18	2	2
19	2	2
20	3	1
24	3	1

Figure A.2 Summary Information for Nominal Factors (Left) and Ordinal Factors (Right)

Summary of Fit

RSquare	0.891732
RSquare Adj	0.805118
Root Mean Square Error	1.48324
Mean of Response	17.2
Observations (or Sum Wgts)	10

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	90.60000	22.6500	10.2955
Error	5	11.00000	2.2000	Prob > F
C. Total	9	101.60000		0.0125*

Summary of Fit

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Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	90.60000	22.6500	10.2955
Error	5	11.00000	2.2000	Prob > F
C. Total	9	101.60000		0.0125*

The parameter estimates are very different because of the different coding. Note that the missing cell affects estimability for the nominal parameters but not for the ordinal parameters.

Figure A.3 Parameter Estimates for Nominal Factors (Left) and Ordinal Factors (Right)

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob>|t|
Intercept	Biased	18.083333	0.74162	24.38	<.0001*
A[1]	Biased	-3.833333	0.856349	-4.48	0.0065*
A[2]	Biased	-0.333333	0.856349	-0.39	0.7131
B[1]	Biased	-0.75	0.74162	-1.01	0.3583
A[1]*B[1]	Biased	-0.5	1.048809	-0.48	0.6537
A[2]*B[1]	Zeroed	0	0	.	.

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob>|t|
Intercept		13	1.048809	12.40	<.0001*
A[2-1]		4	1.48324	2.70	0.0429*
A[3-2]		5	1.48324	3.37	0.0199*
B[2-1]		2.5	1.48324	1.69	0.1527
A[2-1]*B[2-1]		-1	2.097618	-0.48	0.6537
A[3-2]*B[2-1]	Zeroed	0	0	.	.

The singularity details show the linear dependencies (and also identify the missing cell by examining the values).

Figure A.4 Singularity Details for Nominal Factors (Left) and Ordinal Factors (Right)

Singularity Details

Term	Details
Intercept	=A[1] + A[2] + B[1] - A[1]*B[1] - A[2]*B[1]

Singularity Details

Term	Details
A[3-2]*B[2-1]	= 0

The effect tests lose degrees of freedom for nominal. In the case of B, there is no test. For ordinal, there is no loss because there is no missing cell for the *base* first level.

Figure A.5 Effects Tests for Nominal Factors (Left) and Ordinal Factors (Right)

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F	
A	2	1	24.500000	11.1364	0.0206*	LostDFs
B	1	0	0.000000	.	.	LostDFs
A*B	2	1	0.500000	0.2273	0.6537	LostDFs

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F	
A	2	2	81.333333	18.4848	0.0049*	
B	1	1	6.250000	2.8409	0.1527	
A*B	2	1	0.500000	0.2273	0.6537	LostDFs

The least squares means are also different. The nominal LSMs are not all estimable, but the ordinal LSMs are. You can verify the values by looking at the cell means. Note that the A*B LSMs are the same for the two. Figure A.6 shows least squares means for nominal and ordinal factors.

Figure A.6 Least Squares Means for Nominal Factors (Left) and Ordinal Factors (Right)

Effect Details

A

Least Squares Means Table

Level	Least Sq Mean		Std Error	Mean
1	14.250000		0.74161985	14.2500
2	17.750000		0.74161985	17.7500
3	0.000000	NonEstimable	.	22.0000

B

Least Squares Means Table

Level	Least Sq Mean		Std Error	Mean
1	17.333333		0.60553007	17.3333
2	0.000000	NonEstimable	.	17.0000

A*B

Least Squares Means Table

Level	Least Sq Mean		Std Error
1,1	13.000000		1.0488088
1,2	15.500000		1.0488088
2,1	17.000000		1.0488088
2,2	18.500000		1.0488088
3,1	22.000000		1.0488088
3,2	0.000000	NonEstimable	.

Effect Details

A

Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
1	13.000000	1.0488088	14.2500
2	17.000000	1.0488088	17.7500
3	22.000000	1.0488088	22.0000

B

Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
1	13.000000	1.0488088	17.3333
2	15.500000	1.0488088	17.0000

A*B

Least Squares Means Table

Level	Least Sq Mean		Std Error
1,1	13.000000		1.0488088
1,2	15.500000		1.0488088
2,1	17.000000		1.0488088
2,2	18.500000		1.0488088
3,1	22.000000		1.0488088
3,2	0.000000	NonEstimable	.

Frequencies

The impact of frequencies, including those with non-integer values, on an analysis is explained by their effect on the loss function. Suppose that you want to estimate the parameter θ using response values y_i and predictors $x_{i1}, x_{i2},..., x_{in}$. Suppose that the loss function, assuming no frequency variable, is given by the following:

$$L(\theta|\underset{\sim}{y}) = \sum_{i=1}^{n} L(\theta|y_i, x_{i1}, x_{i2},\ldots,x_{in})$$

If frequencies f_i are defined, then the loss function is:

$$L(\theta|\underset{\sim}{y}, \underset{\sim}{f}) = \sum_{i=1}^{n} f_i L(\theta|y_i, x_{i1}, x_{i2},\ldots,x_{in})$$

Calculations for all inference-base quantities, such as parameter estimates, standard errors, hypothesis tests, and confidence intervals, are based on this form of the loss function.

The Usual Assumptions

Before you put your faith in statistics, reassure yourself that you know both the value and the limitations of the techniques that you use. Statistical methods are just tools—they cannot guard you from incorrect science (invalid statistical assumptions) or bad data.

Assumed Model

Most statistics are based on the assumption that the model is correct. To the extent that your model might not be correct, you must attenuate your credibility in the statistical reports that result from the model.

Relative Significance

Many statistical tests do not evaluate the model in an absolute sense. Significant test statistics might be saying only that the model fits better than some reduced model, such as the mean. The model can appear to fit the data but might not describe the underlying physical model well at all.

Multiple Inferences

Often the value of the statistical results is not that you believe in them directly, but rather that they provide a key to some discovery. To confirm the discovery, you might need to conduct further studies. Otherwise, you might just be sifting through the data.

For example, if you conduct enough analyses, you can find 5% significant effects in 5% of your studies, even if the factors have no predictive value. Similarly, to the extent that you use your data to shape your model (instead of testing the correct model for the data), you are corrupting the significance levels in your report. The random error then influences your model selection and leads you to believe that your model is better than it really is.

Validity Assessment

There are a variety of techniques and patterns to assess the validity of the model:

- Model validity can be checked against a saturated version of the factors with Lack of Fit tests. The Fit Model platform presents these tests automatically if your contain replicated x values in a non-saturated model.
- You can check the distribution assumptions for a continuous response by looking at plots of residuals and studentized residuals from the Fit Model platform. Or, use the **Save** commands in the platform pop-up menu to save the residuals in data table columns. Then use the **Analyze > Distribution** on these columns to look at a histogram with its normal curve and the normal quantile plot. The residuals are not quite independent, but you can informally identify severely non-normal distributions.
- The best all-around diagnostic tool for continuous responses is the leverage plot because it shows the influence of each point on each hypothesis test. If you suspect that there is a mistaken value in your data, this plot helps determine whether a statistical test is heavily influenced by a single point.
- It is a good idea to scan your data for outlying values and examine them to see whether they are valid observations. You can spot univariate outliers in the Distribution platform reports and plots. Bivariate outliers appear in Fit Y by X scatterplots and in the Multivariate scatterplot matrix. You can see trivariate outliers in a three-dimensional plot produced by the **Graph > Scatterplot 3D**. Higher dimensional outliers can be found with Principal Components or Scatterplot 3D, and with Mahalanobis and jack-knifed distances computed and plotted in the Multivariate platform.

Alternative Methods

The statistical literature describes special nonparametric and robust methods, but JMP implements only a few of them at this time. These methods require fewer distributional assumptions (nonparametric), and then are more resistant to contamination (robust). However, they are less conducive to a general methodological approach, and the small sample probabilities on the test statistics can be time consuming to compute.

If you are interested in linear rank tests and need only normal large sample significance approximations, you can analyze the ranks of your data to perform the equivalent of a Wilcoxon rank-sum or Kruskal-Wallis one-way test.

If you are uncertain that a continuous response adequately meets normal assumptions, you can change the modeling type from continuous to ordinal and then analyze safely, even though this approach sacrifices some richness in the presentations and some statistical power as well.

Key Statistical Concepts

There are two key concepts that unify classical statistics and encapsulate statistical properties and fitting principles into forms that you can visualize:

- a unifying concept of uncertainty
- two basic fitting machines

These two ideas help unlock the understanding of statistics with intuitive concepts that are based on the foundation laid by mathematical statistics.

Statistics is to science what accounting is to business. It is the craft of weighing and balancing observational evidence. Statistical tests are like credibility audits. But statistical tools can do more than that. They are instruments of discovery that can show unexpected things about data and lead to interesting new ideas. Before using these powerful tools, you need to understand a bit about how they work.

Uncertainty, a Unifying Concept

When you do accounting, you total money amounts to get summaries. When you look at scientific observations in the presence of uncertainty or noise, you need some statistical measurement to summarize the data. Just as money is additive, uncertainty is additive if you choose the right measure for it.

The best measure is not the direct probability because to get a joint probability, you have to assume that the observations are independent and then multiply probabilities rather than add them. It is easier to take the log of each probability because then you can sum them and the total is the log of the joint probability.

However, the log of a probability is negative because it is the log of a number between 0 and 1. In order to keep the numbers positive, JMP uses the negative log of the probability. As the probability becomes smaller, its negative log becomes larger. This measure is called uncertainty, and it is measured in reverse fashion from probability.

In business, you want to maximize revenues and minimize costs. In science, you want to minimize uncertainty. Uncertainty in science plays the same role as cost plays in business. All statistical methods fit models such that uncertainty is minimized.

It is not difficult to visualize uncertainty. Just think of flipping a series of coins where each toss is independent. The probability of tossing a head is 0.5, and -log(0.5) is 1 for base 2 logarithms. The probability of tossing h heads in a row is defined as follows:

$$p = \left(\frac{1}{2}\right)^h$$

Solving for h produces the following:

$$h = -\log_2 p$$

You can think of the uncertainty of some event as the number of consecutive “head” tosses you have to flip to get an equally rare event.

Almost everything we do statistically has uncertainty, -log$p$, at the core. Statistical literature refers to uncertainty as *negative log-likelihood*.

The Two Basic Fitting Machines

An amazing fact about statistical fitting is that most of the classical methods reduce to using two simple machines, the spring and the pressure cylinder.

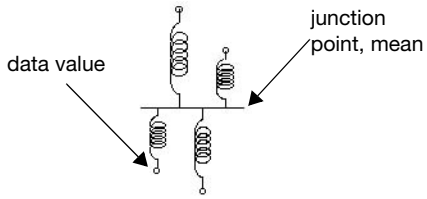
Springs

First, springs are the machine of fit for a continuous response model (Farebrother 1987). Suppose that you have n points and that you want to know the expected value (mean) of the points. Envision what happens when you lay the points out on a scale and connect them to a common junction with springs (Figure A.7). When you let go, the springs wiggle the junction point up and down and then bring it to rest at the mean. This is what must happen according to physics.

If the data are normally distributed with a mean at the junction point where springs are attached, then the physical energy in each point's spring is proportional to the uncertainty of the data point. All you have to do to calculate the energy in the springs (the uncertainty) is to compute the sum of squared distances of each point to the mean.

To choose an estimate that attributes the least uncertainty to the observed data, the spring settling point is chosen as the estimate of the mean. That is the point that requires the least energy to stretch the springs and is equivalent to the least squares fit.

Figure A.7 Connect Springs to Data Points

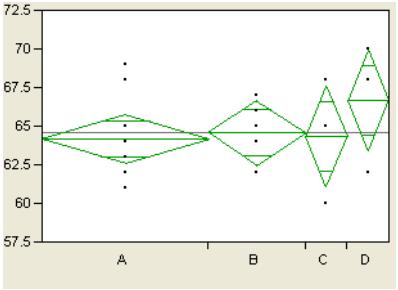


That is how you fit one mean or fit several means. That is how you fit a line, or a plane, or a hyperplane. That is how you fit almost any model to continuous data. You measure the energy or uncertainty by the sum of squares of the distances that you must stretch the springs.

Statisticians put faith in the normal distribution because it is the one that requires the least faith. It is, in a sense, the most random. It has the most non-informative shape for a distribution. It is the one distribution that has the most expected uncertainty for a given variance. It is the distribution whose uncertainty is measured in squared distance. In many cases it is the limiting distribution when you have a mixture of distributions or a sum of independent quantities. It is the distribution that leads to test statistics that can be measured fairly easily.

When the fit is constrained by hypotheses, you test the hypotheses by measuring this same spring energy. Suppose you have responses from four different treatments in an experiment, and you want to test if the means are significantly different. First, envision your data plotted in groups as shown in Figure A.8, but with springs connected to a separate mean for each treatment. Then exert pressure against the spring force to move the individual means to the common mean. Presto! The amount of energy that constrains the means to be the same is the test statistic that you need. That energy is the main ingredient in the *F* test for the hypothesis that tests whether the means are the same.

Figure A.8 A Oneway Plot for a Continuous Response Variable



Pressure Cylinders

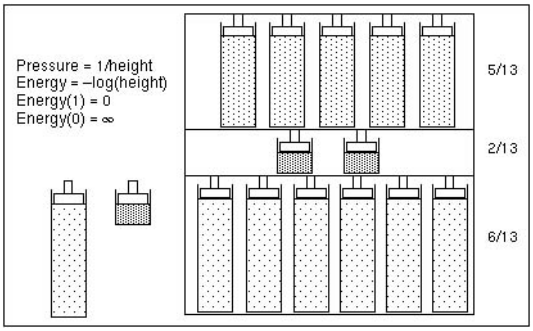
What if your response is categorical instead of continuous? For example, suppose that the response is the country of origin for a sample of cars. For your sample, there are probabilities for the three response levels, American, European, and Japanese. You can set these probabilities for country of origin to some estimate and then evaluate the uncertainty in your data. This uncertainty is found by summing the negative logs of the probabilities of the responses given by the data. It is defined as follows:

$$H = \sum h_{y(i)} = -\sum \log p_{y(i)}$$

The idea of springs illustrates how a mean is fit to continuous data. When the response is categorical, statistical methods estimate the response probabilities directly and choose the estimates that minimize the total uncertainty of the data. The probability estimates must be nonnegative and sum to 1. You can picture the response probabilities as the composition along a scale whose total length is 1. For each response observation, load into its response area a gas pressure cylinder, such as a tire pump. Let the partitions between the response levels vary until an equilibrium of lowest potential energy is reached. The sizes of the partitions that result then estimate the response probabilities.

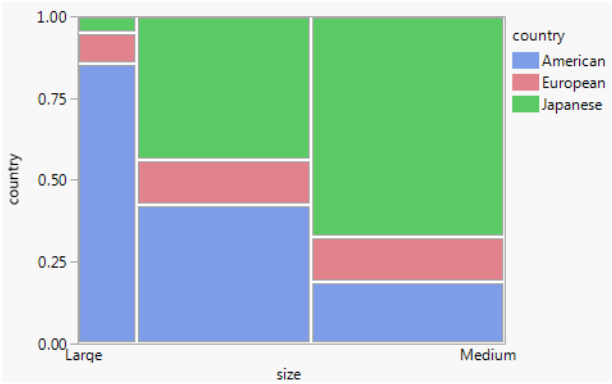
Figure A.9 shows what the situation looks like for a single category such as the medium size cars (see the mosaic column from Carpoll.jmp labeled medium in Figure A.10). Suppose there are thirteen responses (cars). The first level (American) has six responses, the next has two, and the last has five responses. The response probabilities become 6/13, 2/13, and 5/13, respectively, as the pressure against the response partitions balances out to minimize the total energy.

Figure A.9 Effect of Pressure Cylinders in Partitions



As with springs for continuous data, you can divide your sample by some factor and fit separate sets of partitions. Then test that the response rates are the same across the groups by measuring how much additional energy you need to push the partitions to be equal. Imagine the pressure cylinders for car origin probabilities grouped by the size of the car. The energy required to force the partitions in each group to align horizontally tests whether the variables have the same probabilities. Figure A.10 shows these partitions.

Figure A.10 A Mosaic Plot for Categorical Data



Likelihood, AICc, and BIC

Many statistical models in JMP are fit using a technique called *maximum likelihood*. This technique seeks to estimate the parameters of a model, which we denote generically by β, by maximizing the likelihood function. The likelihood function, denoted L(β), is the product of the probability density functions (or probability mass functions for discrete distributions) evaluated at the observed data values. Given the observed data, maximum likelihood estimation seeks to find values for the parameters, β, that maximize L(β).

Rather than maximize the likelihood function L(β), it is more convenient to work with the negative of the natural logarithm of the likelihood function, *-Log* L(β). The problem of maximizing L(β) is reformulated as a minimization problem where you seek to minimize the negative log-likelihood (*-LogLikelihood* = *-Log* L(β)). Therefore, smaller values of the negative log-likelihood or twice the negative log-likelihood (*-2LogLikelihood*) indicate better model fits.

You can use the value of negative log-likelihood to choose between models and to conduct custom hypothesis tests that compare models fit using different platforms in JMP. This is done through the use of likelihood ratio tests. One reason that *-2LogLikelihood* is reported in many JMP platforms is that the distribution of the difference between the full and reduced model *-2LogLikelihood* values is asymptotically Chi-square. The degrees of freedom associated with this likelihood ratio test are equal to the difference between the numbers of parameters in the two models (Wilks 1938).

The corrected Akaike's Information Criterion (AICc) and the Bayesian Information Criterion (BIC) are information-based criteria that assess model fit. Both are based on *-2LogLikelihood*.

AICc is defined as follows:

$$\text{AICc} = -2LogLikelihood + 2k + 2k(k+1)/(n-k-1)$$

where k is the number of estimated parameters in the model and n is the number of observations used in the model. This value can be used to compare various models for the same data set to determine the best-fitting model. The model having the smallest value, as discussed in Akaike (1974), is usually the preferred model.

BIC is defined as follows:

$$\text{BIC} = -2LogLikelihood + k\ln(n)$$

where k is the number of estimated parameters in the model and n is the number of observations used in the model. When comparing the BIC values for two models, the model with the smaller BIC value is considered better.

In general, BIC penalizes models with more parameters more than AICc does. For this reason, it leads to choosing more parsimonious models, that is, models with fewer parameters, than does AICc. For a detailed comparison of AICc and BIC, see Burnham and Anderson (2004).

Simplified Formulas for AICc and BIC in Least Squares Regression

In the case of least squares regression, the AICc and BIC can also be calculated based on the sum of squared errors (SSE). In terms of SSE, AICc and BIC are defined as follows:

$$\text{AICc} = n\ln\left(\frac{SSE}{n}\right) + 2k + 2k(k+1)/(n-k-1) + n\ln(2\pi) + n$$

$$\text{BIC} = n\ln\left(\frac{SSE}{n}\right) + k\ln(n) + n\ln(2\pi) + n$$

where k is the number of estimated parameters in the model, n is the number of observations used in the model, and SSE is the error sum of squares in the model.

Power Calculations

The next sections give formulas for computing the least significant number (LSN), least significant value (LSV), power, and adjusted power in the Fit Model platform. With the exception of LSV, these computations are provided for each effect, and for a collection of user-specified contrasts (under Custom Test and LS Means Contrast). LSV is computed only for a single linear contrast. In the details below, the *hypothesis* refers to the collection of contrasts of interest.

- "Computations for the LSN"
- "Computations for the LSV"
- "Computations for the Power"
- "Computations for the Adjusted Power"

Computations for the LSN

The least significant number (LSN) solves for N in the equation:

$$\alpha = 1 - FDist\left[\frac{N\delta^2}{df_{Hyp}\sigma^2}, df_{Hyp}, N - df_{Hyp} - 1\right]$$

where

FDist is the cumulative distribution function of the central *F* distribution

df_{Hyp} represents the degrees of freedom for the hypothesis

σ^2 is the error variance

δ^2 is the squared effect size

For retrospective analyses, δ^2 is estimated by the sum of squares for the hypothesis divided by *n*, the size of the current sample. If the test is for an effect, then $\delta^2$is estimated by the sum of squares for that effect divided by the number of observations in the current study. For retrospective studies, the error variance σ^2 is estimated by the mean square error. These estimates, along with an α value of 0.05, are entered into the Power Details window as default values.

When you are conducting a prospective analysis to plan a future study, consider determining the sample size that will achieve a specified power (see "Computations for the Power").

Computations for the LSV

The least significant value (LSV) is computed only for a single linear contrast.

Test of a Single Linear Contrast

Consider the one-degree-freedom test $L\beta = 0$, where *L* is a row vector of constants. The test statistic for a *t* test for this hypothesis is:

$$\frac{Lb}{s\sqrt{L(X'X)^{-1}L'}}$$

where *s* is the root mean square error. We reject the hypothesis at significance level α if the absolute value of the test statistic exceeds the 1 - $\alpha/2$ quantile of the t distribution, $t_{1-\alpha/2}$, with degrees of freedom equal to those for error.

To find the least significant value, denoted $(Lb)^{LSV}$, we solve for *Lb*:

$$(Lb)^{LSV} = t_{1-\alpha/2}s\sqrt{L(X'X)^{-1}L'}$$

Test of a Single Parameter

In the special case where the linear contrast tests a hypothesis setting a single β_i equal to 0, this reduces to the following:

$$b_i^{LSV} = t_{1-\alpha/2} s \sqrt{(X'X)_{ii}^{-1}} = t_{1-\alpha/2} StdError(b_i)$$

Test of a Difference in Means

In a situation where the test of interest is a comparison of two group means, the literature talks about the *least significant difference (LSD)*. In the special case where the model contains only one nominal variable, the formula for testing a single linear contrast reduces to the formula for the LSD:

$$\text{LSD} = t_{1-\alpha/2} s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

However, in JMP, the parameter associated with a level for a nominal effect measures the difference between the mean of that level and the mean for all levels. So, the LSV for such a comparison is half the LSD for the differences of the means.

Note: If you are testing a contrast across the levels of a nominal effect, keep in mind how JMP codes nominal effects. Namely, the parameter associated with a given level measures the difference to the average for all levels.

Computations for the Power

Suppose that you are interested in computing the power of a test of a linear hypothesis, based on significance level α and a sample size of N. You want to detect an effect of size δ.

To calculate the power, begin by finding the critical value for an α-level F test of the linear hypothesis. This is given by solving for F_C in the equation

$$\alpha = 1\text{–}FDist[F_C, df_{Hyp}, N - df_{Model} - 1]$$

Here, df_{Hyp} represents the degrees of freedom for the hypothesis, df_{Model} represents the degrees of freedom for the model, and N is the proposed (or actual) sample size.

Then calculate the noncentrality parameter associated with the desired effect size. The noncentrality parameter is defined as follows:

$$\lambda = (N\delta^2)/\sigma^2$$

where σ^2 is a proposed (or estimated) value of the error variance.

Given an effect of size δ, the test statistic has a non-central F distribution, where the distribution function is denoted *FDist* below, with noncentrality parameter λ. To obtain the power of your test, calculate the probability that the test statistic exceeds the critical value:

$$\text{Power} = 1 - FDist\left[F_C, df_{Hyp}, N - df_{Model} - 1, \frac{N\delta^2}{\sigma^2}\right]$$

In obtaining retrospective power for a study with n observations, JMP estimates the noncentrality parameter $\lambda = (n\delta^2)/\sigma^2$ by $\hat{\lambda} = SS_{Hyp}/\hat{\sigma}^2$, where SS_{Hyp} represents the sum of squares due to the hypothesis.

Computations for the Adjusted Power

The adjusted power calculation (Wright and O'Brien 1988) is relevant only for retrospective power analysis. Adjusted power calculates power using a noncentrality parameter estimate that has been adjusted to remove the positive bias that occurs when parameters are simply replaced by their sample estimates.

The estimate of the noncentrality parameter, λ, obtained by estimating δ and σ by their sample estimates, is calculated as follows:

$$\hat{\lambda} = SS_{Hyp}/MSE$$

Wright and O'Brien (1988) explain that an unbiased estimate of the noncentrality parameter is given by the following:

$$[\hat{\lambda}(df_{Error} - 2)/df_{Error}] - df_{Hyp} = \frac{\hat{\lambda}(N - df_{Model} - 1 - 2)}{N - df_{Model} - 1} - df_{Hyp}$$

The expression on the right illustrates the calculation of the unbiased noncentrality parameter when a sample size N, different from the study size n, is proposed for a retrospective power analysis. Here, df_{Hyp} represents the degrees of freedom for the hypothesis and df_{Model} represents the degrees of freedom for the whole model.

Unfortunately, this adjustment to the noncentrality estimate can lead to negative values. Negative values are set to zero, reintroducing some slight bias. The adjusted noncentrality estimate is

$$\hat{\lambda}_{\text{adj}} = \text{Max}\left[0, \frac{\hat{\lambda}(N - df_{Model} - 1 - 2)}{N - df_{Model} - 1} - df_{Hyp}\right]$$

The adjusted power is

$$\text{Power}_{\text{adj}} = 1 - FDist[\text{F}_{\text{C}}, df_{Hyp}, N - df_{Model} - 1, \hat{\lambda}_{\text{adj}}]$$

Confidence limits for the noncentrality parameter are constructed as described in Dwass (1955):

$$\text{Lower CL for } \lambda = \text{Max}\left[0, \left[\left(\sqrt{SS_{\text{Hyp}}/MSE}\right) - \sqrt{df_{Hyp}\text{F}_{\text{C}}}\right]^2\right]$$

$$\text{Upper CL for } \lambda = \left[\left(\sqrt{SS_{\text{Hyp}}/MSE}\right) - \sqrt{df_{Hyp}\text{F}_{\text{C}}}\right]^2$$

Confidence limits for the power are obtained by substituting these confidence limits for λ into the following equation:

$$\text{Power} = 1 - FDist[\text{F}_{\text{C}}, df_{Hyp}, N - df_{Model} - 1, \lambda]$$

Inverse Prediction with Confidence Limits

Inverse prediction estimates a value of an independent variable from a response value. In bioassay problems, inverse prediction with confidence limits is especially useful. In JMP, you can request inverse prediction estimates for continuous and binary response models. If the response is continuous, you can request confidence limits for an individual response or an expected response.

The confidence limits are computed using Fieller's theorem (1954), which is based on the following logic. The goal is predicting the value of a single regressor and its confidence limits given the values of the other regressors and the response.

- Let **b** estimate the parameters β so that we have **b** distributed as N(β,V).
- Let **x** be the regressor values of interest, with the i^{th} value to be estimated.
- Let **y** be the response value.

We desire a confidence region on the value of **x**[i] such that $\beta' x = y$ with all other values of **x** given.

The inverse prediction is

$$x[i] = \frac{y - \beta'_{(i)} x_{(i)}}{\beta[i]}$$

where the subscript (*i*) in parentheses indicates that the i^{th} component is omitted. A confidence interval can be formed from the relation

$$(y - b'x)^2 < t^2 x'Vx$$

where *t* is the *t* value for the specified confidence level.

The equation

$$(y - b'x)^2 - t^2 x'Vx = 0$$

can be written as a quadratic in terms of $z = x[i]$:

$$gz^2 + hz + f = 0$$

where

$$g = b[i]^2 - t^2 V[i, i]$$

$$h = -2yb[i] + 2b[i]b'_{(i)}x_{(i)} - 2t^2 V[i, (i)]'x_{(i)}$$

$$f = y^2 - 2yb'_{(i)}x_{(i)} + (b'_{(i)}x_{(i)})^2 - t^2 x_{(i)}'V_{(i)}x_{(i)}$$

Depending on the values of g, h, and f, the set of values satisfying the inequality, and hence the confidence interval for the inverse prediction, can have a number of forms:

- an interval of the form (ϕ_1, ϕ_2), where $\phi_1 < \phi_2$
- two disjoint intervals of the form $(-\infty, \phi_1) \cup (\phi_2, \infty)$, where $\phi_1 < \phi_2$
- the entire real line, $(-\infty, \infty)$
- only one of $-(\infty, \phi)$ or (ϕ, ∞)

In the case where the Fieller confidence interval is the entire real line, Wald intervals are presented.

Note: The Fit Y by X logistic platform and the Fit Model Nominal Logistic personalities use *t* values when computing confidence intervals for inverse prediction. The Fit Model Generalized Linear Model personality, as well as PROC PROBIT in SAS/STAT, use *z* values, which give different results.

Appendix B

References

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