

# Constrained Covering Arrays: Resolving invalid level combination constraints

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## Abstract

Covering arrays have been widely advocated as a mechanism to derive suites of test cases that ensure a pre-determined level of coverage at minimal cost [1,5]. Although these constructs have been extensively studied for over twenty years, relatively little of this effort has addressed the issue of resolving input parameter constraints [2,3]. This paper addresses that issue by extending the *replace* strategy presented in [3].

**Keywords:** Covering Array, Constraint Handling, Software Testing.

## 1. Introduction

**Definition 1:** A covering array  $CA(N; t, (v_1 \cdot v_2 \cdot \dots \cdot v_k))$  is an  $N \times k$  array such that the  $i$ -th column contains  $v_i$  distinct symbols. If a  $CA(N; t, (v_1 \cdot v_2 \cdot \dots \cdot v_k))$  has the property that for any  $t$  coordinate projection, all  $\prod_{i=1, \dots, t} v_i$  combinations of symbols exist, then it is a  $t$ -covering array and is optimal if  $N$  is minimal for fixed  $v_1 \cdot v_2 \cdot \dots \cdot v_k$  and  $t$ .

When used for testing, columns of such arrays correspond to input parameters and the symbols in each column are levels of the parameters. Each row of the array then represents a testing scenario. Because of the covering property of these constructs, the  $N$  scenarios ensure that every  $t$ -way interaction is exercised and so software testing practitioners find these constructs useful. However, a particular symbol from some column may preclude specific symbols from other columns. Such situations are sometimes referred to as *invalid level combination* constraints. In [2] the authors propose a construct which extends the definition given above to account for such constraints. We present a variant of that definition here.

**Definition 2:** A constrained covering array  $CCA(N; t, (v_1 \cdot v_2 \cdot \dots \cdot v_k), C)$  is an  $N \times k$  array, with symbol set  $v_1, v_2, \dots, v_k$  and constraints  $C$ , such that the tuples of any  $t$  coordinate projection are consistent with  $C$ .

## 2. Resolving invalid level combination constraints

In [5] the authors describe a network interface testing problem with invalid level combination constraints. The input parameters and corresponding levels are:

1. Switch Market: CANADA, USA, UK, MEXICO, FRANCE.
2. Originating Phone Type: RES, BUS, COIN, ISDN.
3. Receiving Phone Type: RES, BUS, COIN, ISDN.
4. Originating Interface: A, B.
5. Receiving Interface: A, B.

The constraints are:

1. ISDN lines can only use Interface B.
2. RES and BUS lines can only use Interface A on the near end but can use A or B on the far end.

The approach we propose to resolve these constraints is based on a technique proposed in [3]. The technique assumes that an *unconstrained* covering array is first created and then the following steps applied.

1. **Replace step:** Identify all rows that involve invalid level combinations. For each such row, generate clones so that coverage is preserved but invalid level combinations are removed.
2. **Resolve/repair step:** For each cloned row, express constrained parameters as a  $k$ -partite graph (see definition below) then search the graph for maximal cliques to resolve the constraints.

To illustrate, let us say that the unconstrained array contains the following row:

Switch Mkt	Orig Phone	Recv Phone	Orig Intfc	Recv Intfc
USA	ISDN	ISDN	A	A

Since this row violates our constraints, it is replaced by the following four clones which must then be repaired:

Switch Mkt	Orig Phone	Recv Phone	Orig Intfc	Recv Intfc
USA	ISDN	ISDN	missing	missing
USA	ISDN	missing	missing	A
USA	missing	ISDN	A	missing
USA	missing	missing	A	A

In [4] the authors describe a tool that implements a branch-and-bound algorithm that exploits the connection between covering arrays and  $k$ -partite graphs to find suitable symbols for these *missing* entries.

**Definition 3:** Let  $G = (V, E)$  denote an undirected graph where  $V$  denotes the set of vertices and  $E$  the set of edges.  $G$  is a  $k$ -partite graph if  $V$  can be partitioned into  $k$  disjoint sets  $V_1, V_2, \dots, V_k$  so that no two vertices within the same set are adjacent. That is, if  $v \in V_i$  and  $w \in V_j$  then  $(v, w) \in E \Rightarrow i \neq j$ . Furthermore,  $G$  is said to be a complete  $k$ -partite graph if all possible vertex pairs from  $V_1, V_2, \dots, V_k$  are adjacent.

Now, consider a  $\mathbf{CA}(N; t, (v_1 \cdot v_2 \cdot \dots \cdot v_k))$  and let  $G = (V, E)$  be a complete  $k$ -partite graph. The  $k$  columns of the array correspond to the  $k$  disjoint sets  $V_1, V_2, \dots, V_k$  of  $V$  so that each element from  $v_i$  corresponds to a vertex in  $V_i$ . In addition, each row of the array represents a subset of  $V$  so that the subgraph of  $G$  induced by these vertices is a maximal clique.

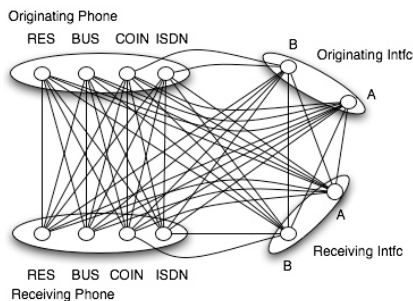
**Definition 4:** Given the undirected graph  $G$  above, for some subset  $Q \subseteq V$  of vertices, the induced subgraph  $G(Q)$  is said to be a clique if  $\forall v, w \in Q, v \neq w, (v, w) \in E$ . If  $G(Q)$  is of maximum size, we refer to it as a maximal clique.

So, by expressing the  $\mathbf{CA}(N; t, (v_1 \cdot v_2 \cdot \dots \cdot v_k))$  as a  $k$ -partite graph  $G$ , we may think of invalid level combination constraints as inducing a subgraph  $G' = (V', E')$  on  $G$  where,  $V_i \in V'$  iff column  $i$  participates in the constraints and  $E' = E \setminus \{(v, w) : v \text{ and } w \text{ correspond to invalid level combinations}\}$ .

*Note:* Any maximal clique of  $G'$  is a subgraph of a maximal clique of  $G$ .

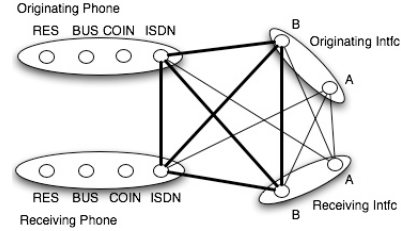
Hence, resolving constraints involves finding a maximal clique in  $G'$  to replace the subgraph of the maximal clique in  $G$  that contains invalid combinations.

The induced subgraph  $G'$  for the network interface testing problem is shown below.



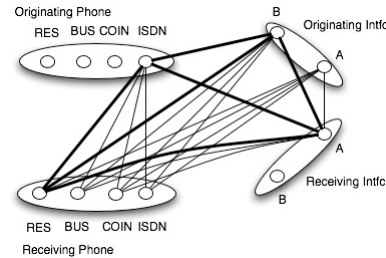
**Figure 1. Constrained subgraph**

Considering the first two clones, notice that for clone #1, Originating and Receiving Phone must be ISDN. Given this requirement, the search space may be pruned before attempting to find the maximal clique. So,  $G'$  may be reduced to the following graph:



**Figure 2. Constrained subgraph - clone #1**

In this case, the only maximal clique is identified by the thick lines and so clone #1 may be repaired by setting Originating and Receiving Interface to B.



**Figure 3. Constrained subgraph - clone #2**

For clone #2 the reduced  $G'$  is shown above. Notice that in this case there are several possible maximal cliques. The algorithm simply selects the first one found and so clone #2 may be repaired by setting Receiving Phone Type to RES and Originating Interface to B.

## References

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