Component Integrated Importance: Modeling Complex Aging Systems

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ABSTRACT

One objective of studying complex system reliabilities using block diagrams is to determine which individual components contribute most significantly to the reliability of the assembled system. A well-known method is to compute the Birnbaum component importance (BCI) of every component, then prioritize reliability improvement efforts on the components that have larger importance values. In the literature and practice, there are two versions of BCI, one is static and the other is dynamic. The static version computes BCI of individual components at a given time t, while the dynamic version computes BCI of individual components at many time points and forms curves for individual components. Both BCI versions (static and dynamic) assume the system is new. However, component importance can change dramatically as systems age. We will present the details of our study. We will discuss two alternatives that consider component importance measure dynamics that involve system aging. One published alternative is known as the Component Integrated Importance [2]. The other is conditional BCI.

1 INTRODUCTION

The study of component importance measurement arises in the context of resource allocation when engineers have limited resources to improve system reliabilities. For example, in Birnbaum [1], an application was to improve the system reliability while keeping the cost low. The solution was to find the component with the largest ratio of its reliability importance and the cost of increasing its reliability. The reliability importance is referred as the Birnbaum component importance (BCI).

Extensions to BCI have focused on different criteria, depending upon the objective or purpose of desired optimality. This paper will be no exception, as a vast amount of literature has been dedicated to the subject in the areas of methodology development and industrial applications. See example [4], [5], [6], and references therein. This paper will explore one particular direction, which considers the system aging, such that an importance measure is conditional on the system age. We start the discussion by revisiting BCI first.

The computation of BCI at any given time consists of the structure of a system and the reliability of individual components at the given time. The result is called static BCI: the computation represents a single snapshot of the component importance. Simple comparison among components is relatively trivial, and not necessarily informative, by lining up the importance, with or without other considerations, such as cost functions.

When BCI for a component is computed over a grid of time values, and the computed values are connected and drawn, the result is known as a dynamic BCI graph. That graph reveals how the importance of a component evolves over time. An overlay of multiple dynamic BCIs for different components can help to compare components. Considering the cost function should be straightforward as well.

The reliability of a component at any given time is the expectation that the component will still function at that time. The expectation is taken when the system is brand new and has not yet been put into use. From that perspective, despite the name of dynamic BCI, the view on component importance is still arguably static. As soon as the system starts to run, the computed importance measurements may or may not remain unchanged, after the structure of the system and conditional reliability function of individual components change. The conditional reliability function of a component at any given time is the reliability function given that component has been functioning through that time. The conditional reliability function is also known as the residual (remaining) life reliability function, which means the reliability function for the rest of the life.

In this paper, we will discuss in detail the value and impact of system aging upon component importance, how to evaluate the impact of aging on component importance, and how to visualize it. We will discuss two approaches. The first one simply replaces reliability function of individual components by their conditional reliability functions. We call it Remaining Life BCI, or conditional BCI, interchangeably. The computation needs to introduce a new quantity, the survival time t_0, which is the duration since the system started to function. Given t_0, one can compute the static version of BCI for the Remaining Life, or the dynamic version of BCI. The visual comparison of component importance for different system ages may require three dimensional graphics, which are time, survival time t_0, and importance measurement. The second approach is based upon the Component Integrated

Importance [2] (CII). The CII approach in this paper introduces the hazard functions of individual components into the computation. The definition of CII in the original paper is specialized for discrete states. We apply the definition to our context, and obtain an equivalent formula in our context. The visual comparison of this component importance only requires two dimensional graphics, which are time and importance measurement.

2 NOTATIONS

We use $R(t)$, or sometimes R, to denote the reliability function of time t. The Weibull reliability function uses the following parameterization: $R(t;\alpha,\beta) = \exp\left(-(t/\alpha)^{\beta}\right)$. We use numerical subscripts to denote components, such as α_1 denotes the scale parameter of component 1, and $R_1(t)$ denotes the reliability function of component 1. We use symbol S as a subscript to denote the system. For example, $R_S(t)$ denotes the system reliability function. Therefore, the dynamic BCI for component 1 can be denoted by $BCI_1(t) = \partial R_S(t)/\partial R_1(t)$. Or simply, by BCI_1. The Remaining Life reliability function at given time t_0, which is the conditional reliability function that the product is functioning through t_0, is defined as: $R(t|t > t_0;\alpha,\beta) = R(t;\alpha,\beta)/R(t_0;\alpha,\beta)$. Or simply $R(t|t_0)$. The dynamic version of Remaining Life BCI for component 1 is $BCI_1(t|t_0) = \partial R_S(t|t_0)/\partial R_1(t|t_0)$. The calculation of CII is simple in our context. For example, the CII for component 1 can be expressed as $CII_1(t) = f_1(t) \cdot BCI_1(t)$, in which $f_1(t)$is the failure density function of component 1. We will explain how we obtain the expression in a later section.

3 A TRIVIAL EXAMPLE

In this section, we illustrate the three definitions. Imagine a series system of two components. Both components have a Weibull reliability function, but different parameters. Parameters are $\alpha_1 = 200, \beta_1 = 0.2, \alpha_2 = 300, \beta_2 = 0.5$. Moreover, let us assume time unit is in hours to facilitate our discussion. Figure 1 is an overlay of the dynamic BCIs of the two components.

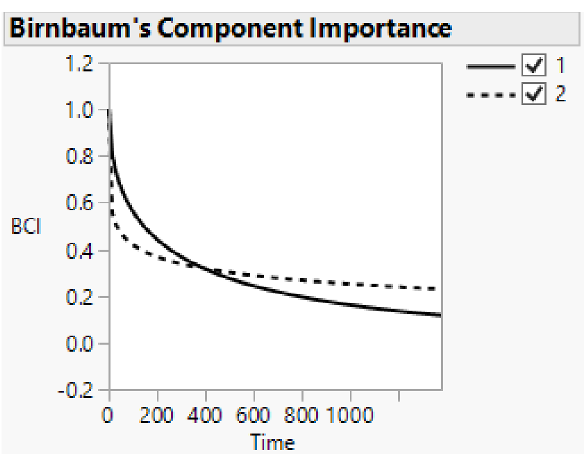

Figure 1: Dynamic BCI of a Series System

For the series system, both components decrease values in dynamic BCIs. Roughly before 400 hours, the first component has larger BCI values, and vice versa after that time point. The first result in the appendix shows how to calculate the intersection.

However, the view changes dramatically, if the system has run for 20 hours, and it is still running. In order to assess the component importance of a running system, we use the Remaining Life reliability functions of components to compute all relevant quantities, and Figure 2 is a graph of dynamic BCI using those quantities.

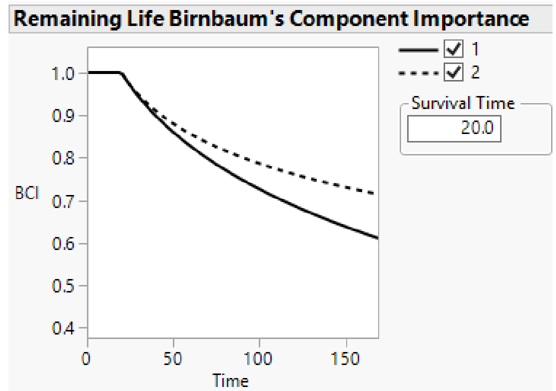

Figure 2: Remaining Life BCI of a Series System

In Figure 2, the flat line at the top that goes from time 0 to 20 hours represents the system has been functioning through 20 hours. At 20 hours, which is the present time, the remaining curves are forward assessment about component importance, using BCI. We can see both curves decrease, and the second component has higher importance values all the time after 20 hours.

That is a very different perspective from Figure 1. And that perspective remains unchanged as the survival time increase. In other words, the second component always has a larger forward looking BCI value after 20 hours. The second result in the appendix shows how to compute the survival time that suffices the condition that component 2 always has larger BCI values.

On the other hand, the CII result shows a similar phenomenon. Because the pattern is not clear in a linear-linear scale, we use a log-linear scale to illustrate, i.e. the vertical axis is in a log scale, and the horizontal axis is in a linear scale. We can observe the intersection of two CIIs is around 20 hours. Even in the original linear-linear scale plot, it is evident the second component has larger CII values after some earlier time point than 400 hours.

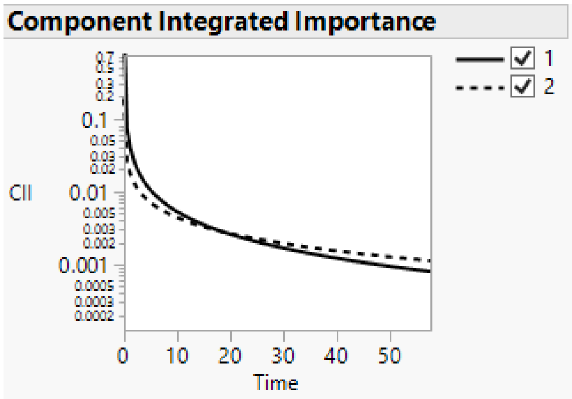

Figure 3: CII of a Series System

The third result in the appendix shows the time coordinate of the intersection is identical to the survival time that is discovered by previously using Remaining Life BCI graph. The value is around 20. It is interesting to observe that Remaining Life BCI has to constantly adjust the survival time to discover interesting component importance dynamics, while CII can use a single graph.

4 THE FORMULATION OF CII

The definition of BCI was clearly stated in [1], and restated many times thereafter in the literature. The Remaining Life BCI should be easy to understand as well as mentioned previously. We will now address the original CII and our adoption.

In the explanation of [2], a CII function is indexed by three things: the identification i of the component, the state m of the component, the state l that the component is transitioning to. The formulation of the function is a multiplication of three pieces: the probability of component i being in state m; the transition rate of component i from state m to state l; the impact on system performance as component i transiting from state m to state l. Or in symbolic expression, the formula is:

$$CII(i,m,l) = P(i,m) \cdot \lambda(i,m,l) \cdot \Delta Q(i,m,l).$$

In order to adopt the concept in our context, we need to realize that, here, there are only two possible states at any time, for any component, which are functioning or not functioning. The probability that a component is functioning at a given time equals the reliability value at that time. The transition rate of a component, at a given time, from functioning to not functioning, is known as the hazard rate of the component in our context. Hazard rate is also known as the instantaneous failure rate. The hazard rate function is usually denoted by $h(t)$. Its calculation is simply a conditional density: $h(t) = f(t)/R(t)$.

We identify the last piece in the multiplication as simply BCI in our context. Notice the description of the piece is the impact on system performance change which is calculated as the change of probability of the system in a certain state as the component changes. Let us revisit the BCI calculation and see why that description is equivalent to the BCI concept.

In order to calculate BCI, we first write the system reliability function in this form: $R_S = R_i \cdot R_{S|i} + (1 - R_i) \cdot R_{S|\bar{\imath}}$. Notice $R_{S|i}$ is the system reliability when the component i is functioning, and $R_{S|\bar{\imath}}$ is the system reliability when the component i is not functioning. We use the symbol i with or without the bar on its top to indicate the component is or is not functioning. The computation of BCI for component i is thereafter: $BCI_i = \partial R_S / \partial R_i = R_{S|i} - R_{S|\bar{\imath}}$. That is exactly what was described as system performance change, in terms of reliability, as component i changes from functioning to not functioning.

By putting the reliability function, hazard rate function, and BCI function of component i back into the CII formula, we obtain: $CII_i = R_i \cdot h_i \cdot BCI_i = f_i \cdot BCI_i$, which is what we have stated in section 2.

5 A MORE REALISTIC EXAMPLE

In this section, we give a more realistic example. The example is based upon a NASA aircraft design example [3]. The original example was more realistic and complex. We largely simplify it to serve the purpose here.

Figure 4 illustrates the reliability block diagram of the design. The diagram consists of a cockpit, a control system in the cockpit integrated with other dependent components , namely the four engines, and two wings. There are two engines on each wing. In order to guarantee that the plane can safely finish the planned flight, it is required to have at least one engine functioning on each wing, which is denoted by two “OR” gates in the diagram. All other components are required to function, which is required by the “AND” gate in the diagram.

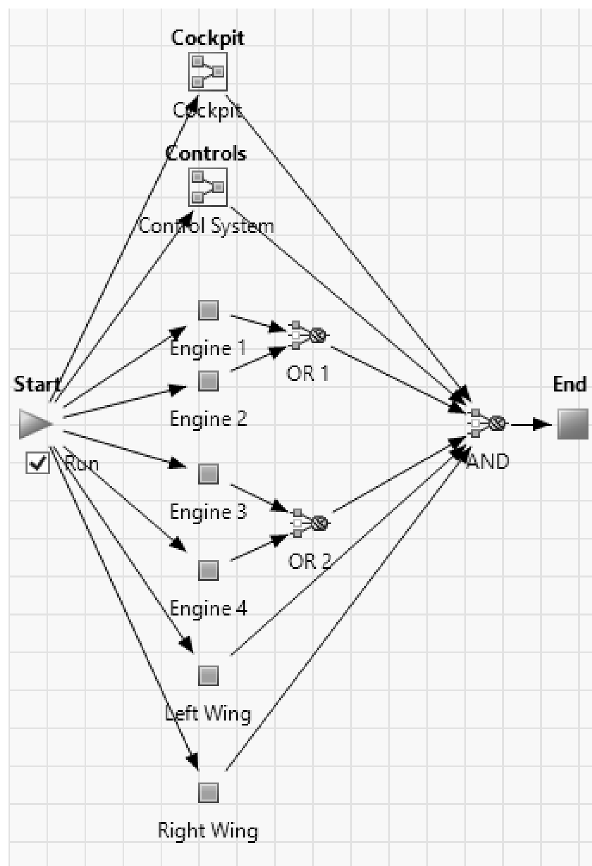

Figure 4: Reliability Block Diagram of an Aircraft

Figure 5 through 7 are graphs of BCIs, Remaining Life BCIs, and CIIs. Duplicated components, which are engines and wings, have identical function values at any given time. So we choose to only display one unique component of each type.

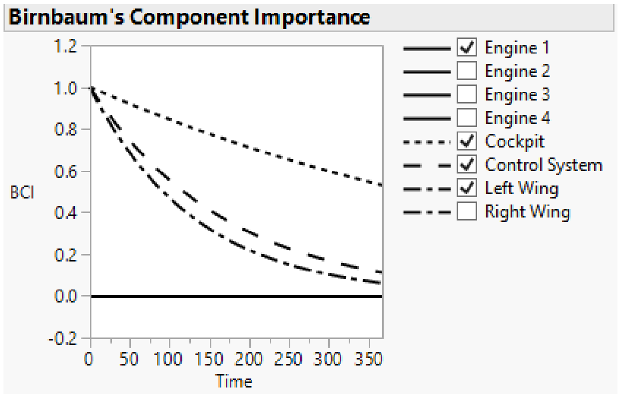

Figure 5: Aircraft BCI

According to BCIs in Figure 5, the cockpit has the highest BCI values, the control system the second, the wings the third, and the engines the last. Notice that the BCI curves of control system and wing are relatively close. Therefore, if the improvement cost are the same for the control system and wings, one may have difficulty prioritizing assets for improvement of the system reliability.

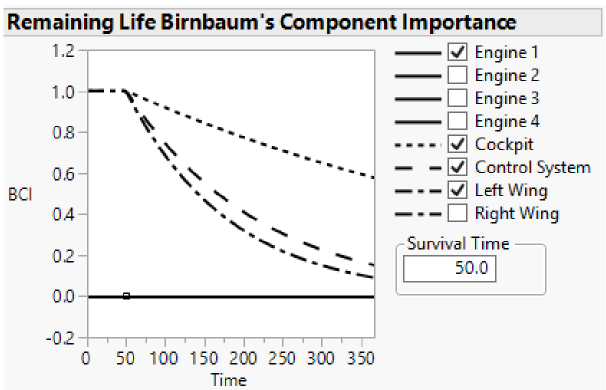

Figure 6: Aircraft Remaining Life BCI

We adjusted the survival time in Figure 6, and hoped to discover unexpected changes in relative positions among Remaining Life BCI curves. However, for this example, we have not discovered valuable findings as we have shown in section 3.

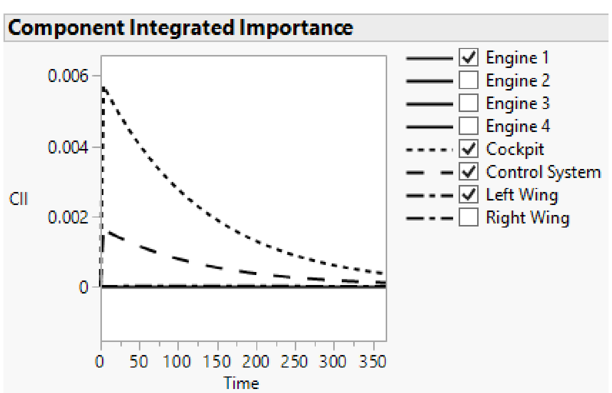

Figure 7: Aircraft CII

In Figure 7, the CII curve for the wings is reduced in importance and is now is similar to that of the engine. Now, if the improvement cost for the control system and wings are the same, the control system appears to be more important, according to the CII criterion. To convince us that CII provides a useful alternative perspective in comparing to BCI, we draw an overlay of failure density functions of the component. Recall, CII is a multiplication of density and BCI. Figure 8 is the overlay.

According to Figure 8, it is intriguing to see that the wings are even less likely to experience failure than the engines. If one has to improve the reliability of wing, it may not be even possible.

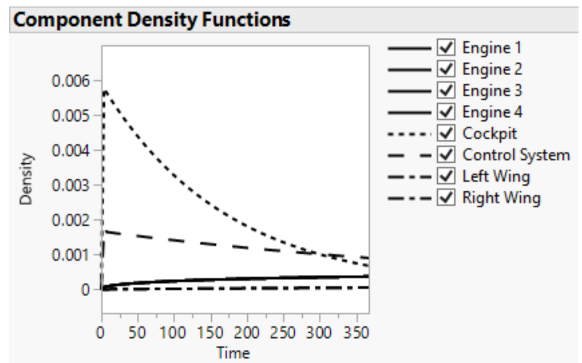

Figure 8: Failure Density Functions of Aircraft Components

6 OPTIMALITY IN CII

In terms of optimality, CII is equivalent to BCI with the reciprocal of component failure density functions which are built into the cost functions, λ_i's, in [1]. That is equivalent to thinking that the component that is more likely to failure is more likely to be improved. That is not necessarily always true. But when it is true, that is when CII becomes more valuable than BCI.

7 CONCLUSION

In this paper, we bring together the well-known work of BCI, our development of Remaining Life BCI, and our adoption and modification of CII that suits our application. By comparing them through two examples, we understand the limitations of BCI. BCI emphasizes the structural importance of individual components. BCI does not consider the aging of a system that can dramatically change the structure of the system, the reliabilities of individual components, and the projections of Remaining Life reliabilities of those components. In addition, Remaining Life BCI, helps to adapt BCI concepts into an aging system. However, to have a comprehensive view, the Remaining Life BCI requires one to manually scan over the system ages. On the contrary, CII provides a single, overall view of the impact of aging on a system. We propose CII as an alternative, rather than a replacement, to BCI.

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APPENDIX: RESULTS AND SKETCHES OF PROOFS

In this section, we list several results and sketches of proofs about the two-component series system in section 3. Although we give detailed parameter values for all four parameters, we only assume the shape parameters are different in this section, i.e. $\beta_1 \neq \beta_2$.

1. The BCI function of a component coins the reliability function of the other component.
   To prove, we first write down the system reliability function $R_S = R_1 \cdot R_2$. Then, $BCI_1 = \partial R_S / \partial R_1 = R_2$.
2. Two BCI functions have two intersections, one is at $t = 0$.
   To find the other solution, equate BCI_1 and BCI_2, which is equivalent to equating R_1 and R_2, according to the first result. With $\beta_1 \neq \beta_2$, solve the equation to get two solutions: $t_1 = 0$ and $t_2 = \left(\alpha_2^{\beta_2}/\alpha_1^{\beta_1}\right)^{1/(\beta_2-\beta_1)}$.
3. Two remaining life BCI functions of the system in section 3 can have one and only one intersection, which is the survival time t_0, such that the remaining life BCI of a component will always be above the remaining life BCI of the other component.
   To find the solution t_0, without loss of generality, assume $\beta_1 < \beta_2$, and solve the inequality $BCI_1(t|t_0) \leq BCI_2(t|t_0)$. That is equivalent to solve $R_2(t|t_0;\alpha_2,\beta_2) \leq R_1(t|t_0;\alpha_1,\beta_1)$, according to the first result. The equality holds at $t = t_0$ for certain. To prove that is the only solution, we need to show the inequality holds for all $t > t_0$. Write the expressions explicitly, and rearrange the terms, the inequality is equivalent to this expression: $\alpha_2^{\beta_2} t^{\beta_1} - \alpha_1^{\beta_1} t^{\beta_2} < \alpha_2^{\beta_2} t_0^{\beta_1} - \alpha_1^{\beta_1} t_0^{\beta_2}$, the right hand side of which is a constant. A sufficient condition that the inequality holds is that the left hand side (LHS) is monotone decreasing for $t > t_0$. That means the following two inequality must hold simultaneously. The first inequality is that first derivative of LHS is less than zero, i.e. $\alpha_2^{\beta_2}\beta_1 t^{\beta_1-1} - \alpha_1^{\beta_1}\beta_2 t^{\beta_2-1} < 0$. And the second inequality is $t > t_0$. The condition that makes both inequality hold is $t_0 = \left(\frac{\alpha_2^{\beta_2}\beta_1}{\alpha_1^{\beta_1}\beta_2}\right)^{\frac{1}{\beta_2-\beta_1}}$.
4. The two CII functions of system in section 3 has one and only one intersection. And the intersection is at $t = \left(\frac{\alpha_2^{\beta_2}\beta_1}{\alpha_1^{\beta_1}\beta_2}\right)^{\frac{1}{\beta_2-\beta_1}}$. The result coins t_0 in the third result.
   To find the intersection, we equate two CII functions as $f_1(t) \cdot R_2(t) = f_2(t) \cdot R_1(t)$, using the definition in section 2 and the first result in this appendix. Expand the equation by detailed expressions of density and reliability functions and solve for t, we can get the result easily.

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